Equatorial adjustment revisited:

understanding dynamics of tropical atmosphere with moist-convective rotating shallow water model

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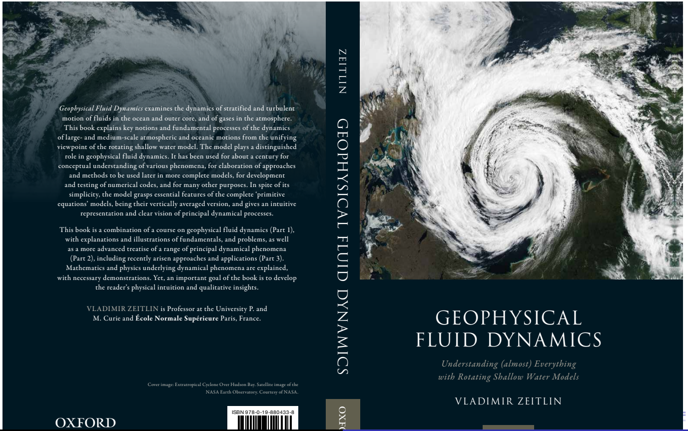
Physics at the Equator,
Lyon, France, October 2019

Rotating Shallow Water (RSW): allows to understand almost everything in GFD



Geophysical Fluid Dynamics examines the dynamics of stratified and turbulent motion of fluids in the ocean and outer core, and of gases in the atmosphere. This book explains key notions and fundamental processes of the dynamics of large- and medium-scale atmospheric and oceanic motions from the unifying viewpoint of the rotating shallow water model. The model plays a distinguished role in geophysical fluid dynamics. It has been used for about a century for conceptual understanding of various phenomena, for elaboration of approaches and methods to be used later in more complete models, for development and testing of numerical codes, and for many other purposes. In spite of its simplicity, the model grasps essential features of the complete 'primitive equations' models, being their vertically averaged version, and gives an intuitive representation and clear vision of principal dynamical processes.

This book is a combination of a course on geophysical fluid dynamics (Part 1), with explanations and illustrations of fundamentals, and problems, as well as a more advanced treatise of a range of principal dynamical phenomena (Part 2), including recently arisen approaches and applications (Part 3). Mathematics and physics underlying dynamical phenomena are explained, with necessary demonstrations. Yet, an important goal of the book is to develop the reader's physical intuition and qualitative insights.

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Cover image: Extratropical Cyclone Over Hudson Bay. Satellite image of the NASA Earth Observatory. Courtesy of NASA.

OXFORD

ISBN 978-0-19-880433-8

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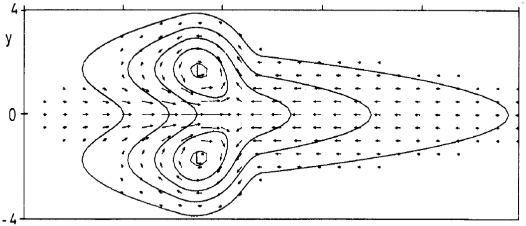
Understanding (almost) Everything
with Rotating Shallow Water Models

VLADIMIR ZEITLIN

Recalling classics

(Linearized) RSW model on the equatorial β- plane: classical tool for understanding large-scale circulation patterns in tropics. Heating due to moist convection: mass sink (Matsuno, 1966; Gill, 1980).

Gill's mechanism of large-scale response to localized heating - folklore in tropical meteorology and climatology:



RSW model

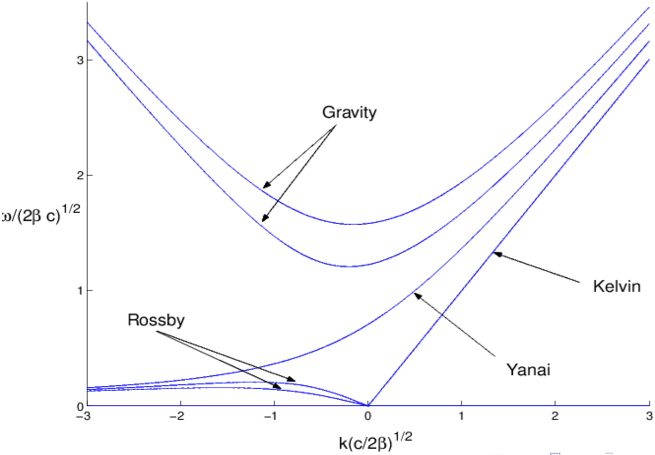
RSW equations on the tangent plane $(x,\ y)$ to a rotating planet (no topography, no dissipation):

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + f(y)\, \hat{\boldsymbol{z}} \times \boldsymbol{v} + g\nabla h = 0\,,$$

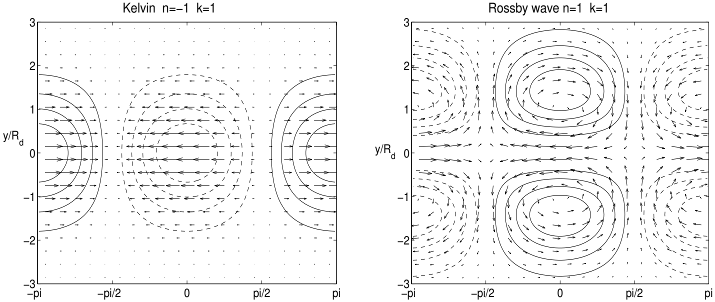
$$\partial_t h + \nabla \cdot (\boldsymbol{v} h) = 0\,.$$

$\boldsymbol{v} = (u,\ v)$ - horizontal velocity, h - geopotential height.
Mid-latidude tangent plane: $f(y) = f_{0+}\beta y$
Equatorial tangent plane: $f(y) = \beta y$.
Bottom topography: $h \to h - b(x, y)$ in the 2nd equation, easy.
Linearized equations → equatorial waves.

Dispersion diagram for equatorial waves in RSW



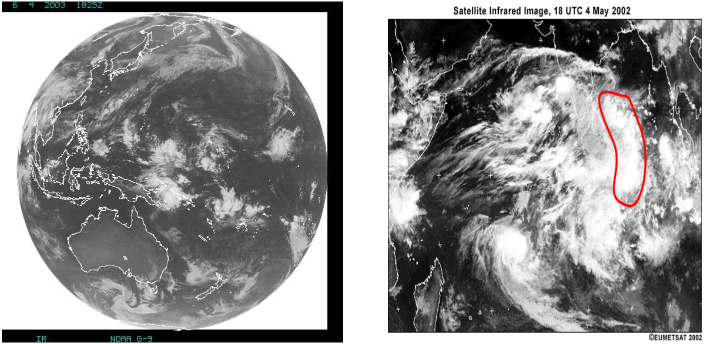
Long low-frequency equatorial waves



Unidirectional: Rossby wave (RW) → West, Kelvin wave (KW) → East.

Gill: RW signal to the West, KW signal to the East of source.

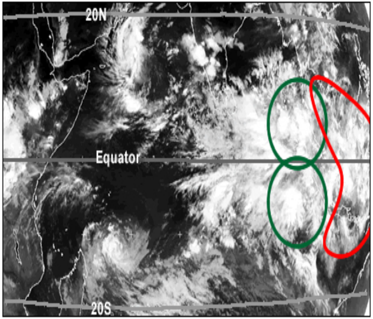
Looking from space



Left panel: Rossby wave. *Right panel:* Kelvin wave.

Simultaneous generation Rossby and Kelvin waves over the oceanic warmpool, à la Gill

Satellite Infrared Image, 18 UTC 7 May 2002



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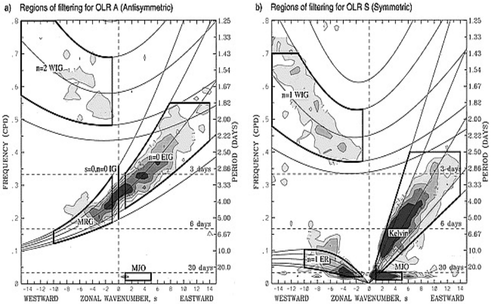
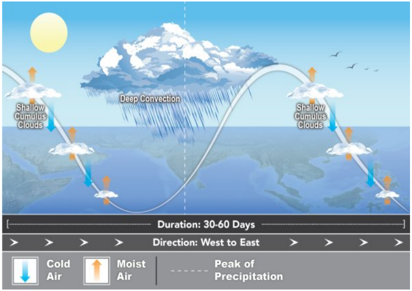
Equatorial wave spectrum vs OLR data (Wheeler & Kiladis, 1999).

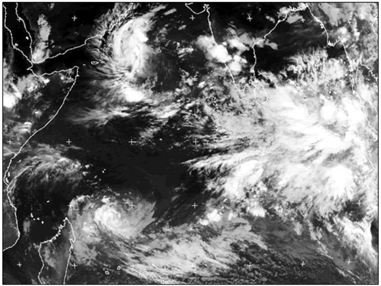
OLR data superimposed onto dispersion diagram of equatorial waves: good agreement, except the enigmatic slow eastward-propagating motions: MJO



Madden-Julian Oscillation (MJO)



Satellite Infrared Image, 18 UTC 7 May 2002



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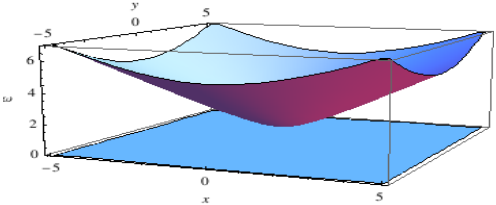
Periodic enhanced convection pattern moving slowly eastward over Indo-Pacific warm-pool, dying out in the Pacific.

Dispersion relation on the f-plane



Spectral gap between slow (vortex) and fast (inertia-gravity waves (IGW)) motions.

Geostrophic balance in mid-latitudes

- β neglected, balance between pressure and Coriolis forces

  $$f_0 u = -g\partial_y h, \quad f_0 v = g\partial_x h$$

  $\rightarrow h$ - stream-function, vorticity $\zeta = \nabla^2 h$,

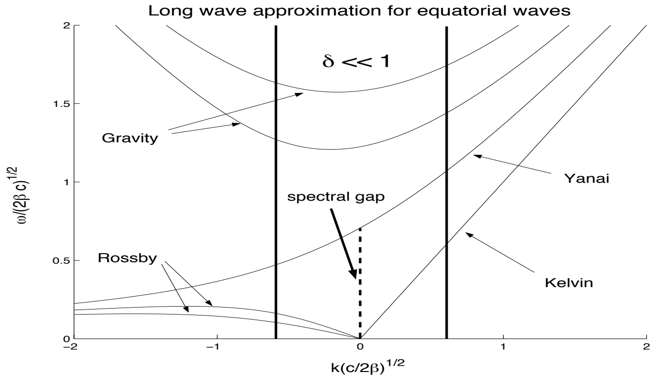
  $$\frac{d(\zeta - h)}{dt} = 0, \quad \frac{d}{dt} = \partial_t + u\partial_x + v\partial_y$$

  .
- β-correction $\rightarrow$ evolution of vorticity. QG equation:

  $$\frac{d(\zeta - h)}{dt} = -\beta\partial_x h$$

  Linearization - Rossby waves,
- Geostrophic adjustment: relaxation by emission of IGW of initial perturbation towards balanced state obeying QG .

Slow vs fast motions in tropics



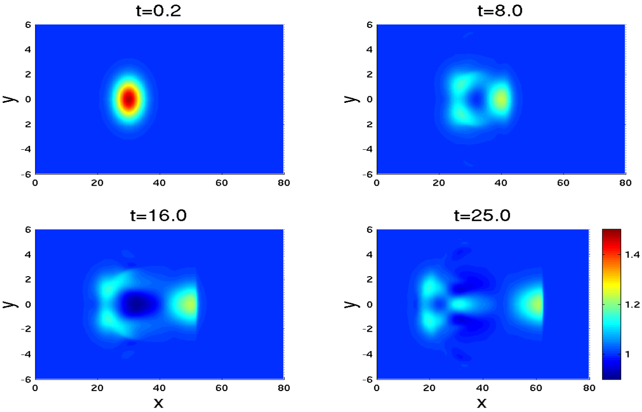
Geostrophic balance in tropics

Observations:

- Ill-defined for meridional velocity: $\beta y\, v = g \partial_x h$.
- Yet RW are in $\sim$ geostrophic equilibrium,
- Zonal velocity and h in KW are in geostrophic balance.
- Balance is leading-order approximation at large zonal to meridional aspect ratios (used by Gill, 1980)

Adjustment of localized pressure anomalies with large aspect ratios produces RW and KW (LeSommer, Reznik & Zeitlin, 2004) $\Leftrightarrow$ Gill's mechanism.

Adjustment of a large-scale pressure anomaly in tropics: (LeSommer, Reznik & Zeitlin, 2004)



Open questions/Answers in what follows

Questions:

1. How adjustment scenario depend on the aspect ratio?
2. How the effects of moisture modify the adjustment scenario?
3. Is Gill's mechanism universal?

Answers:

1. Substantially
2. Strongly
3. No

Getting RSW from Primitive Equations

Adiabatic, hydrostatic PE with pseudo-height vertical coordinate $\bar{z} = z_0(1 - \left(\frac{p}{p_s}\right)^{\frac{R}{c_p}})$ on the tangent plane:

$$\frac{\partial \boldsymbol{v}_h}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}_h + f(y)\hat{\boldsymbol{z}} \times \boldsymbol{v}_h = -\nabla_h \phi,$$

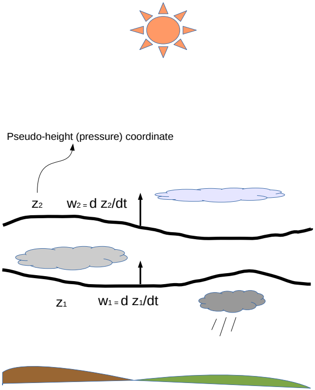
$$-g\frac{\theta}{\theta_0} + \frac{\partial \phi}{\partial \bar{z}} = 0,$$

$$\frac{d}{dt}\theta \equiv \frac{\partial \theta}{\partial t} + \boldsymbol{v} \cdot \nabla \theta = 0; \quad \nabla \cdot \boldsymbol{v} = 0.$$

θ - potential temperature, ϕ - geopotential, p - pressure, $\boldsymbol{v}_h$ - horizontal velocity, $\boldsymbol{v} = (\boldsymbol{v}_h, w)$.

Vertical averaging between a pair of material surfaces + columnar motion hypotheses → RSW.

Averaging over layers between material surfaces



Incorporating moisture

Advection of moisture q in adiabatic primitive equations $\frac{d}{dt}q = 0$ $\Rightarrow$ conservation law for the bulk amount of q in the air column $Q_i = \int_{z_{i-1}}^{z_i} q\, dz$:

$$\partial_t Q_i + \boldsymbol{\nabla} \cdot (Q_i \boldsymbol{v}_i) = 0.$$

"Dry" system: pot. temperature θ and q are advected:

$$\frac{d}{dt}\theta = 0, \quad \frac{d}{dt}q = 0.$$

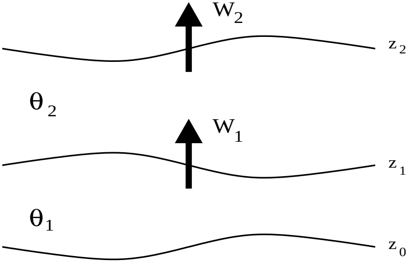
Moist-convective system: linearized equivalent potential temperature is $\approx$ advected (L - latent heat of condensation, c_p - specific heat of the air):

$$\frac{d}{dt}\left(\theta + \frac{L}{c_p}q\right) \approx 0,$$

Adding convective fluxes in RSW derivation

$w = \frac{dz}{dt} \to w = \frac{dz}{dt} + \mathcal{W}$, vertical velocity due to convective flux $\mathcal{W}$ to be defined and linked to other variables → convective shallow water (Matsuno-Gill philosophy pushed further)

$$w_0 = \frac{dz_0}{dt}, \quad w_1 = \frac{dz_1}{dt} + \mathcal{W}_1, \quad w_2 = \frac{dz_2}{dt} + \mathcal{W}_2.$$



Linking convection to condensation: moist-convective RSW

Condensation sink in bulk humidity:

$$\partial_t Q_i + \boldsymbol{\nabla} \cdot (Q_i \boldsymbol{v}_i) = -P_i,$$

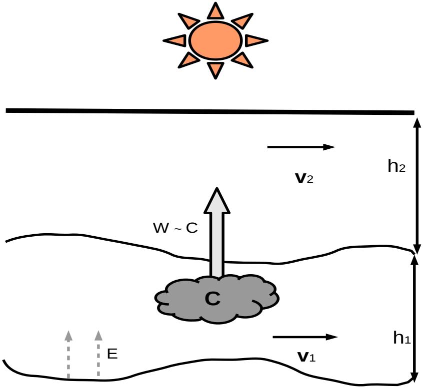
Conservation of equivalent potential temperature in pseudo-height coordinates →

$$\theta_{i+1} = \theta(z_i) + \frac{L}{c_p} q(z_i) \approx \theta_i + \frac{L}{c_p} q(z_i) > \theta_i,$$

Averaging over the layer →

$$\mathcal{W}_i = \beta_i P_i, \quad \beta_i = \frac{L}{c_p(\theta_{i+1} - \theta_i)} > 0.$$

Sketch of the moist-convective (mcRSW) model



2-layer mcRSW model with moist lower layer (Lambaerts, Lapeyre, Zeitlin & Bouchut, 2011)

$$\begin{cases} \partial_t \boldsymbol{v}_1 + (\boldsymbol{v}_1 \cdot \boldsymbol{\nabla})\boldsymbol{v}_1 + f(y)\hat{\boldsymbol{z}} \times \boldsymbol{v}_1 = -g\boldsymbol{\nabla}(h_1 + h_2), \\ \partial_t \boldsymbol{v}_2 + (\boldsymbol{v}_2 \cdot \boldsymbol{\nabla})\boldsymbol{v}_2 + f(y)\hat{\boldsymbol{z}} \times \boldsymbol{v}_2 = -g\boldsymbol{\nabla}(h_1 + \alpha h_2) + \frac{\boldsymbol{v}_1 - \boldsymbol{v}_2}{h_2}\beta P, \\ \partial_t h_1 + \boldsymbol{\nabla} \cdot (h_1 \boldsymbol{v}_1) = -\beta P, \\ \partial_t h_2 + \boldsymbol{\nabla} \cdot (h_2 \boldsymbol{v}_2) = +\beta P, \\ \partial_t Q + \boldsymbol{\nabla} \cdot (Q \boldsymbol{v}_1) = -P + E, \end{cases}$$

Condensation: relaxational parametrisation with relaxation time τ and saturation threshold Q^s ($\mathcal{H}$ - step function):

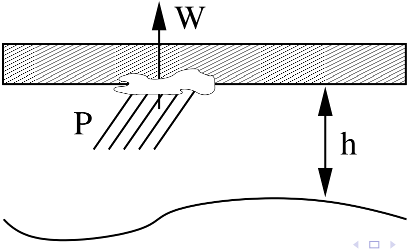
$$P = \frac{Q - Q^s}{\tau}\mathcal{H}(Q - Q^s)$$

Surface evaporation - standard: $E = \propto |\boldsymbol{v}_1|(Q^s - Q)\mathcal{H}(Q^s - Q)$.

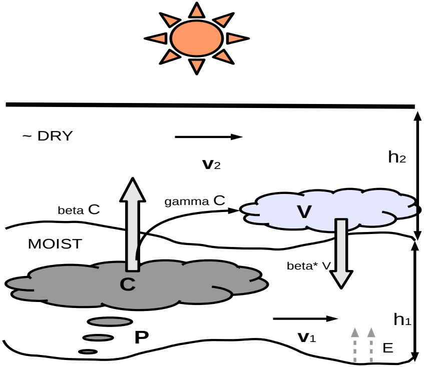
One-layer reduction (Bouchut, Lambaerts, Lapeyre & Zeitlin, 2009)

Infinitely deep upper layer → simplified one-layer model:

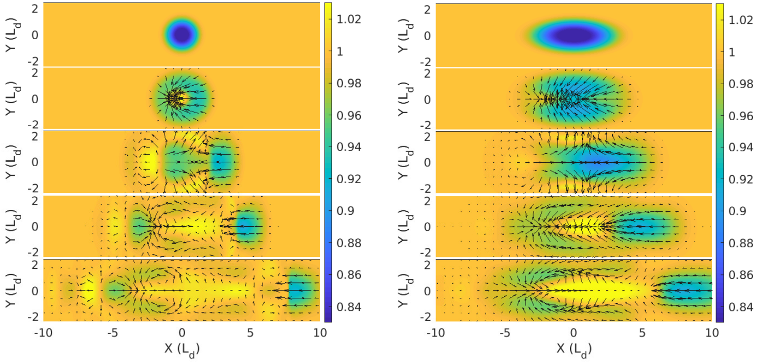
$$\begin{cases} \partial_t \boldsymbol{v}_1 + (\boldsymbol{v}_1 \cdot \nabla)\boldsymbol{v}_1 + f(y)\hat{\boldsymbol{z}} \times \boldsymbol{v}_1 = -g\nabla h_1, \\ \partial_t h_1 + \nabla \cdot (\boldsymbol{v}_1 h_1) = -\beta P, \\ \partial_t Q + \nabla \cdot (Q\boldsymbol{v}_1) = -P + E. \end{cases}$$



Improving mcRSW (Rostami & Zeitlin, 2018)

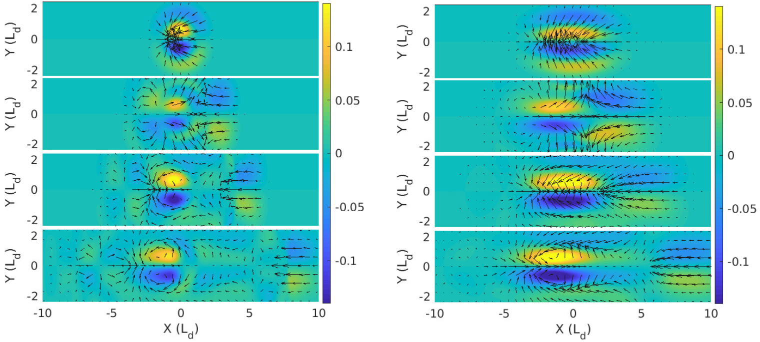


Adjustment of circular vs elongated anomaly. Pressure



Left: “Dry" adjustment of a circular pressure anomaly.
$t = 0, 1, 3, 5, 9$ $[1/\beta L_d]$. *Right:* Same for aspect ratio $a = 2.5$.

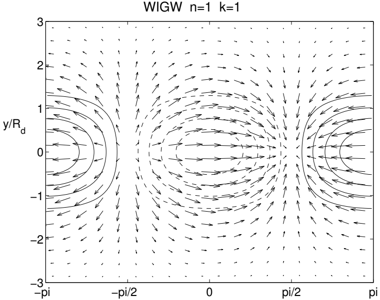
Adjustment of circular vs elongated anomaly. Vorticity



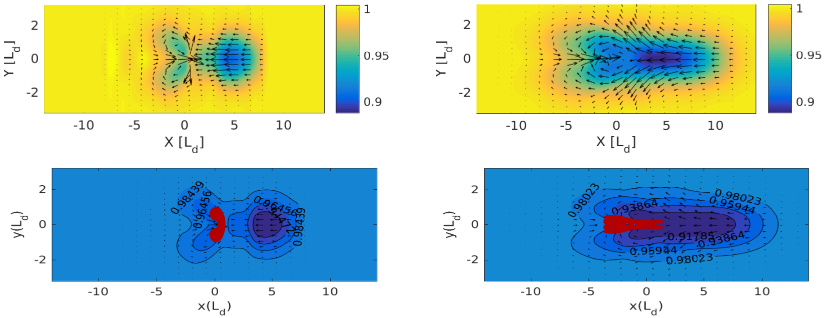
Left: “Dry" adjustment of a circular pressure anomaly. Relative vorticity (colors), and velocity (arrows): $t = 1, 3, 5, 9\ \ [1/\beta L_d]$. *Right:* Same for anomaly with aspect ratio $a = 2.5$.

Equatorial westward-moving inertia-gravity wave

Adjustment of anomalies with small aspect ratio in the western sector is dominated by large-scale westward-moving inertia-gravity wave:

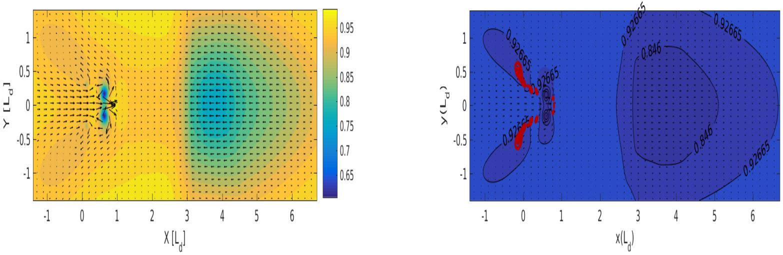


“Moist” circular vs elongated weak depressions



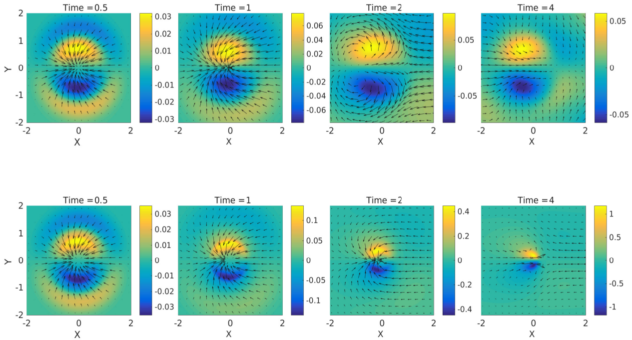
Pressure and velocity (top), and condensation/heating (red) at $t = 7\ [1/\beta L_d]$. $max\,|\Delta H/H| = 0.1$, $a = 1$ (*left*) and $a = 5$ (*right*).

"Moist" adjustment of stronger circular depressions, and emergence of intense dipoles



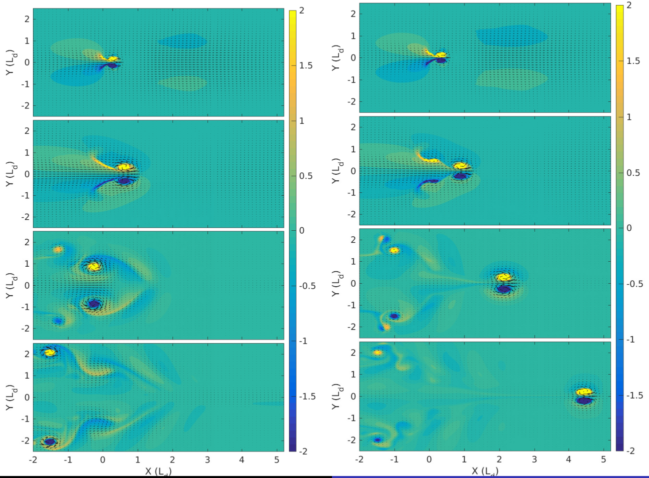
Pressure and velocity (*left*), pressure and condensation/heating (*right*): moist adjustment of a circular depression with $max\,|\Delta H/H| = 0.15$.

“Dry” vs “moist” adjustment of circular depressions



Vorticity and velocity at the initial stages of “dry” (*top*) and moist adjustments of a circular depression with $max\, |\Delta H/H| = 0.1$.

The fate of the dipole: $max\,|\Delta H/H| = 0.15$ vs 0.18



Dynamical explanation of a slowly eastward-moving dipole - a modon?

Coherent, essentially nonlinear, eastward-moving dipolar structures (nonlinear solitary Rossby waves), the modons are known in QG approximation in midlatitudes (Larichev & Reznik, 1976).
Were extrapolated to the full sphere (Tribbia, 1984; Verkeley, 1984).
Yet, full nonlinear QG not available on the equatorial beta plane.
Hint: atmospheric tropical motions at large scales have small signature in pressure (Charney, 1963). Confirmed by data analysis (Yano *et al*, 2009).

Charney balance for tropical motions in RSW

Pressure variations in tropics are weak ⇒ quasi-barotropic scaling: Pressure perturbation parameter λ: $h = H(1 + \lambda\eta)$
"Vortex" scaling (single velocity and single spatial scales):
$(x, y) \sim L, \ (u, v) \sim V, \ t \sim \frac{L}{V} \Rightarrow$

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \overline{\beta} y \hat{\mathbf{z}} \wedge \mathbf{v} + \frac{gH\lambda}{V^2} \nabla \eta = 0\,, \quad \overline{\beta} = \beta L^2 / V.$$

Assumption

$$\lambda \to 0, \text{ and } \frac{gH\lambda}{V^2} = \mathcal{O}(1) \Rightarrow V << \sqrt{gH}, \Rightarrow$$

typical velocity is much smaller than the phase velocity of the Kelvin waves $c_K = \sqrt{gH}$.

Vorticity equation

Rescaled equations

$$\begin{aligned} \partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} + \overline{\beta}\, y\, \hat{\mathbf{z}} \wedge \mathbf{v} + \nabla\eta &= 0\,, \\ \lambda(\partial_t\eta + \mathbf{v}\cdot\nabla\eta) + (1+\lambda\eta)\,\nabla\cdot\mathbf{v} &= 0. \end{aligned}$$

Equivalent to long-wave approximation in oceanography.
Leading order in λ: $\nabla\cdot\mathbf{v}_0 = 0 \Rightarrow$ streamfunction ψ for $(u_0,\, v_0)$.

Vorticity equation à la QG

Cross-differentiation of momentum equations:

$$\nabla^2\psi_t + \mathcal{J}(\psi, \nabla^2\psi) + \overline{\beta}\psi_x = 0.$$

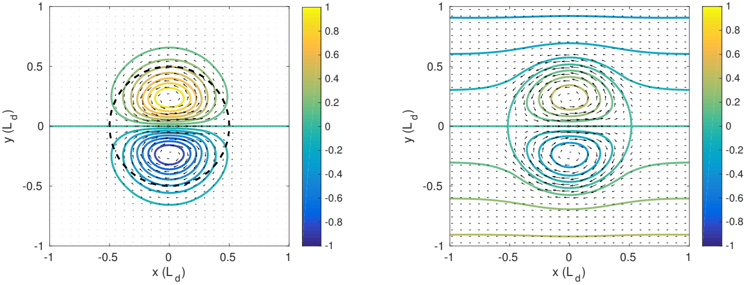
Steady-moving modon solutions of the asymptotic vorticity equation

Matched *external* and *internal* solutions with respect to a circle $r = a$ in polar coordinates (r, θ):

$$\begin{cases} \psi_{ext} = -\frac{Ua}{K_1(pa)} K_1(pr) \sin\theta, \quad r > a, \\ \psi_{int} = \left[\frac{Up^2}{k^2 J_1(ka)} J_1(kr) - \frac{r}{k^2}(1 + U + Uk^2) \right] \sin\theta, \quad r < a, \end{cases}$$

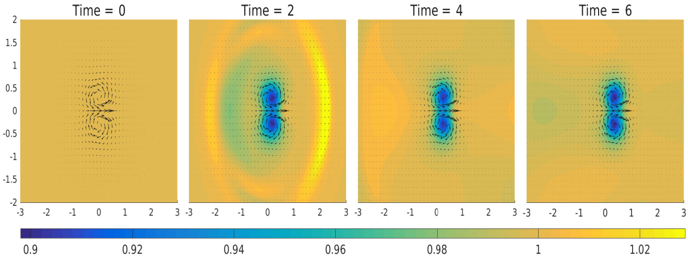
J_1 and K_1 - Bessel functions, p is real, and $p^2 = \overline{\beta}/U$, so $U > 0$, and the motion is eastward. Each pair $(a, p) \rightarrow$ series of eigenvalues k arising from matching conditions, the lowest corresponds to a dipole.

Phase portrait of an asymptotic equatorial modon



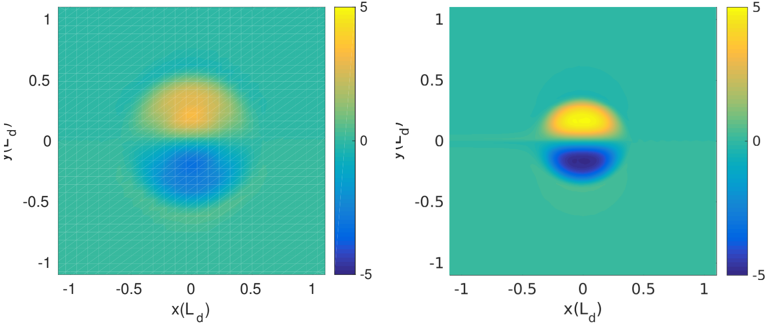
Streamlines and velocity field of an asymptotic modon in stationary (*left*) and co-moving (*right*) frames. Dashed/green circle: separatrix of radius a.

Adjustment of asymptotic modons, and emergence of "true" modons



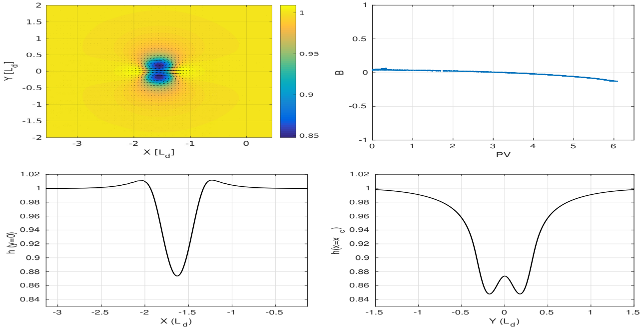
Evolution of pressure and velocity given by numerical simulation initialized with the velocity field of an asymptotic modon.

Comparison of asymptotic and "true" modons



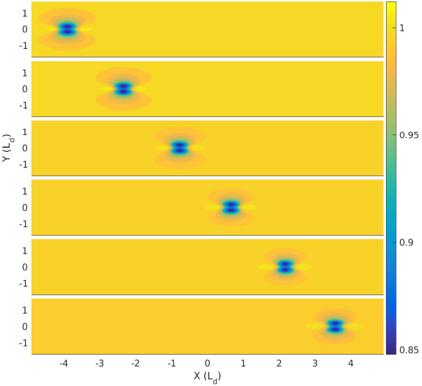
Vorticity of the asymptotic (left) and corresponding "true" (right) modons.

Coherence of the adjusted “true“ modon



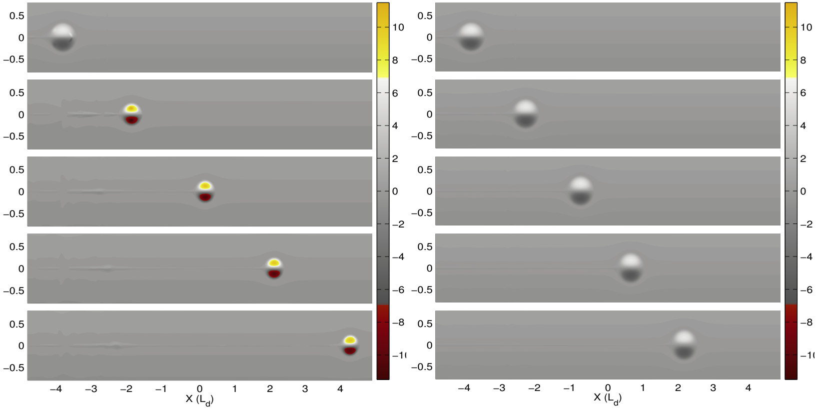
Thickness, Bernoulli function vs potential vorticity in the co-moving frame, zonal and meridional sections of the modon at $t = 15\,[1/\beta L_d]$.

Evolution of a "true" equatorial modon

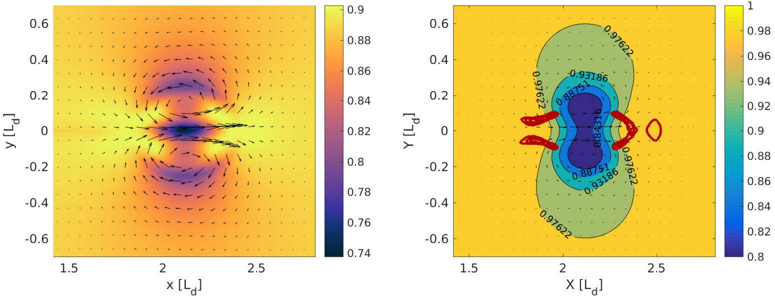


Snapshots of h at $t = 0, 10, 20, 30, 40, 50\ [1/(\beta L_d)]$

Moist-convective vs "dry" evolution of the same modon

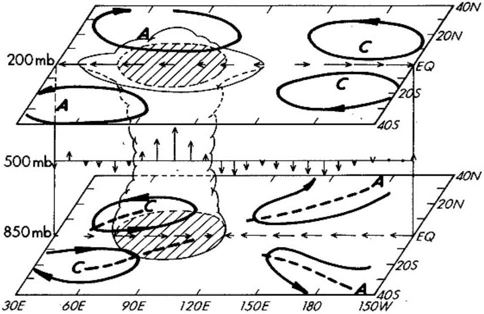


Moisture and condensation patterns of the modon



Velocity and water vapor (left), pressure and condensation (right) at $t = 30\ [1/(\beta L_d)]$. Remark: Charney regime + moist-convective heating ↔ “WTG” approximation (Sobel, Nilsson & Polvani, 2001).

Dynamical structure of MJO (Zhang, 2005)



Twin cyclones in the lower layer: Rossby wave? - but moving eastward → idea: MJO ↔ a modon
(first Yano & Tribbia, 2017, at planetary scale on the sphere).

Conclusions

- “Dry” djustment is strongly influenced by aspect ratio and scale of initial perturbation
- “’Moist” adjustment produces specific coherent structures
- Slowly eastward-moving equatorial modons arise in some parameter regimes

Perspective

What's next?

- Using two- (or three-) layer models, to include vertical structure
- Using improved mcRSW to describe liquid water and precipitation
- Using Thermal Shallow Water to including horizontal temperature gradients and atmosphere-ocean coupling through relaxation to sea-surface temperature.
- Including real-life topography and mean winds
- Trying data assimilation

References

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