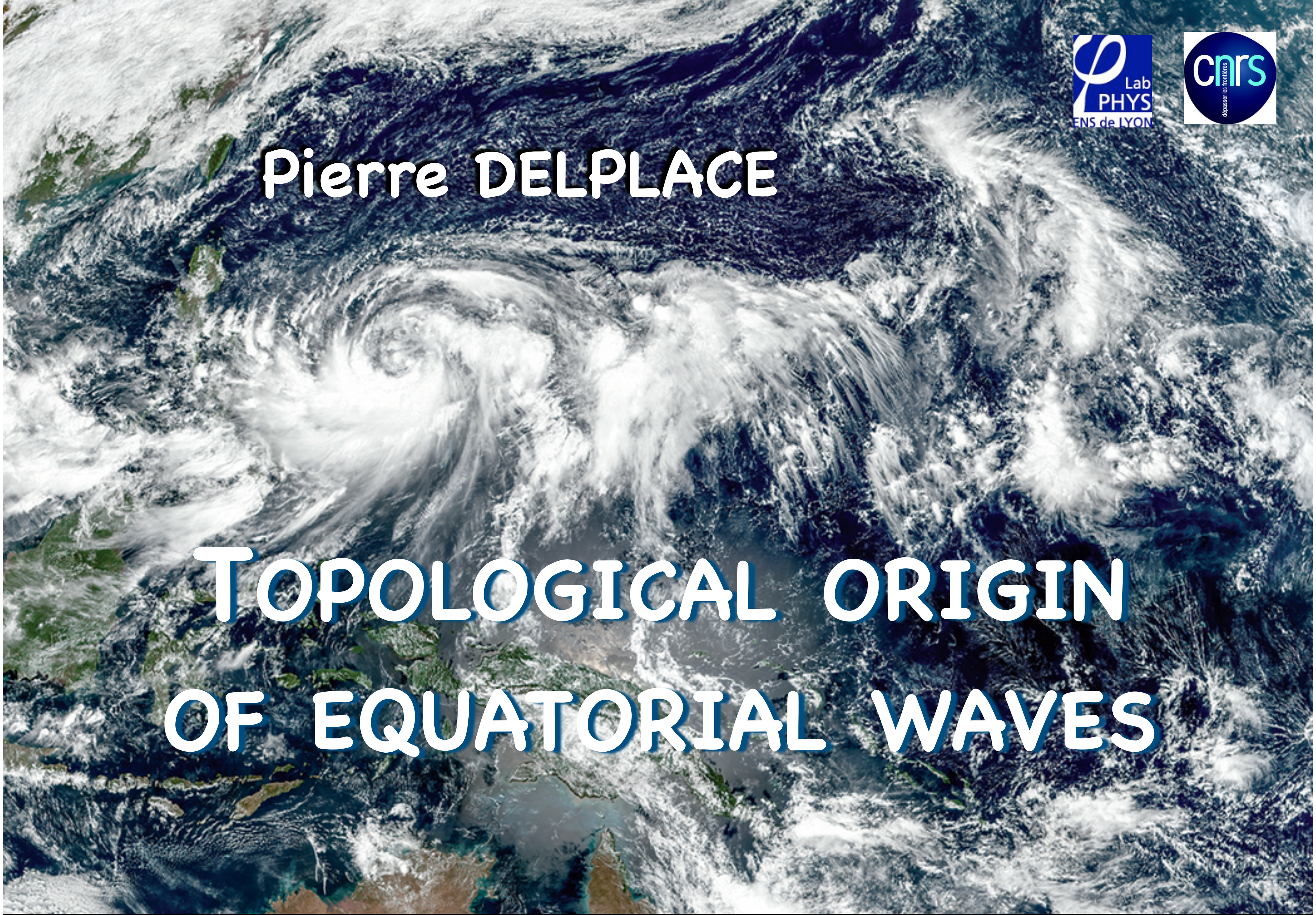
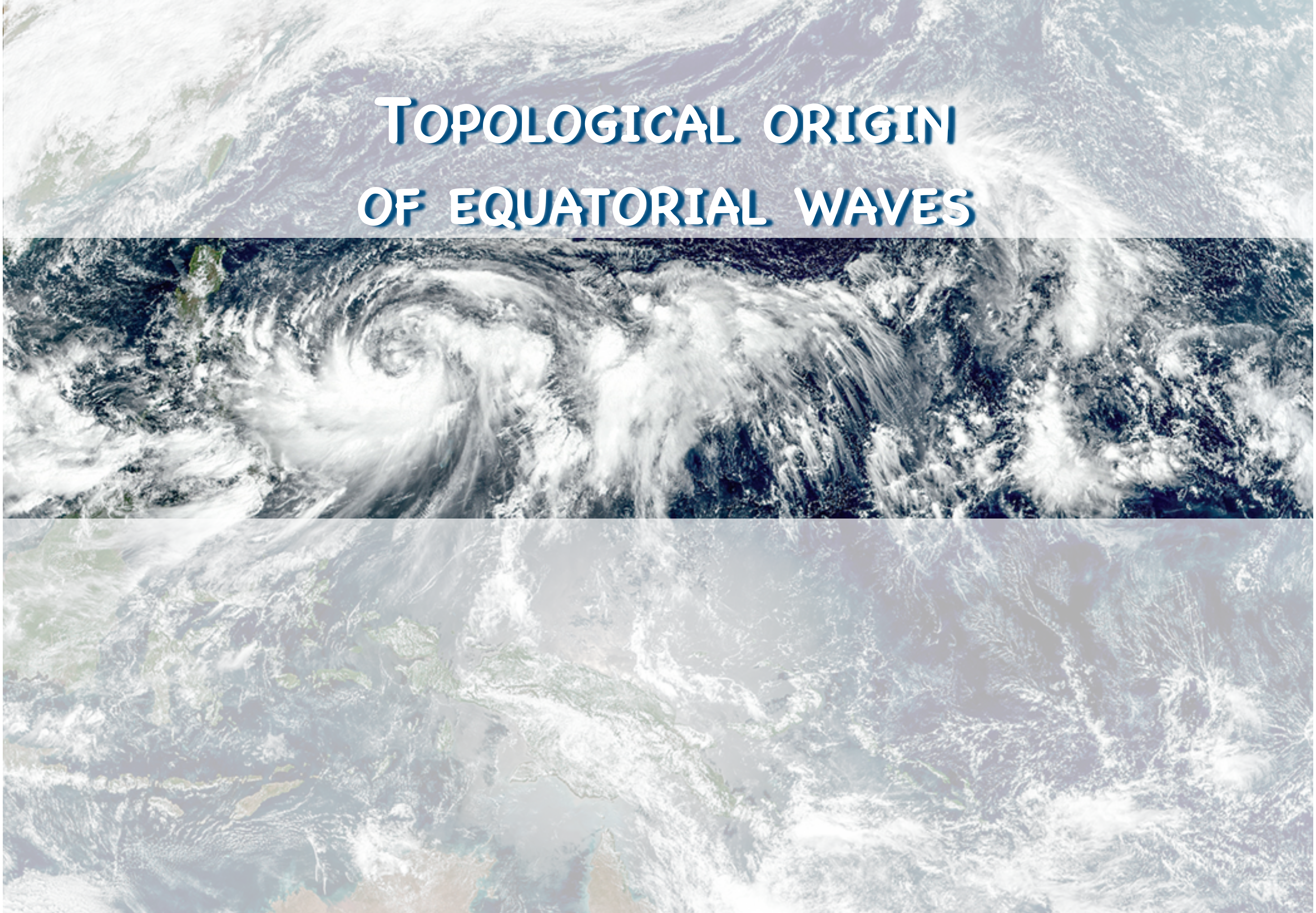


Pierre DELPLACE

TOPOLOGICAL ORIGIN OF EQUATORIAL WAVES

Physics at the equator: from the lab to the stars *October 16-18 2019 | ENS de Lyon (France)*

Topological origin of equatorial waves



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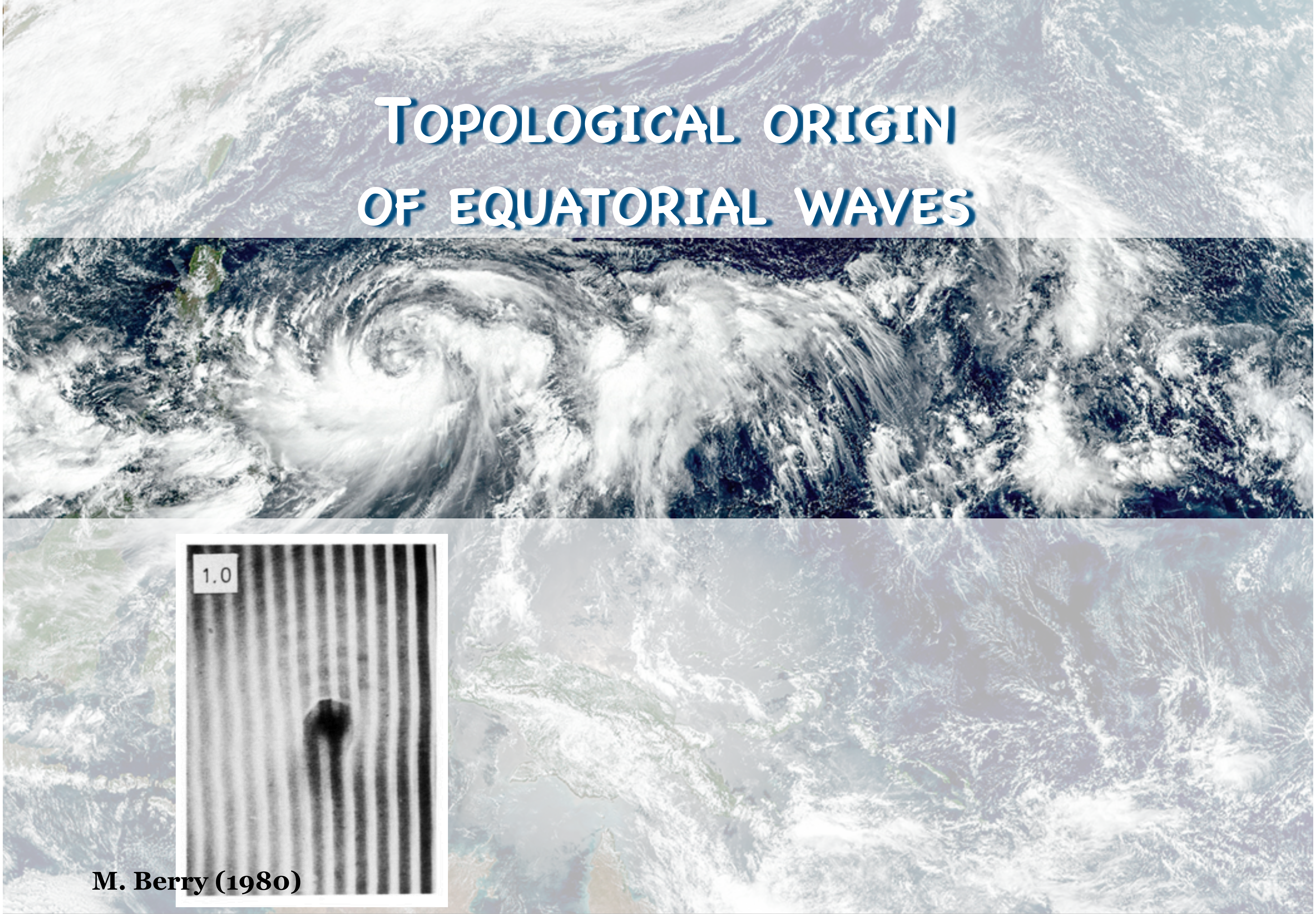
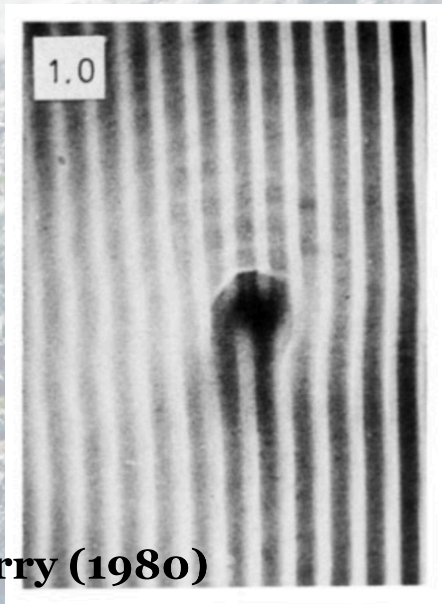


M. Berry (1980)

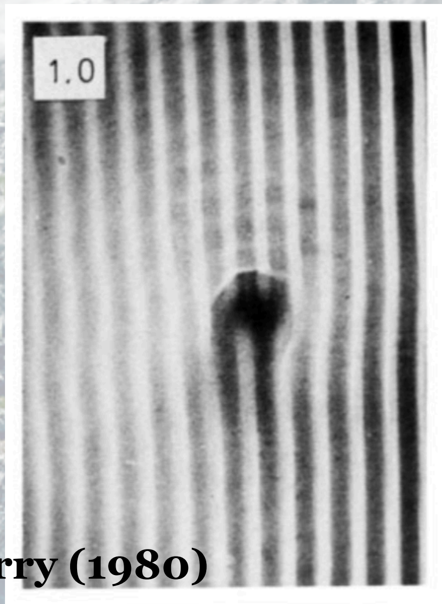
Topological origin of equatorial waves



M. Berry (1980)

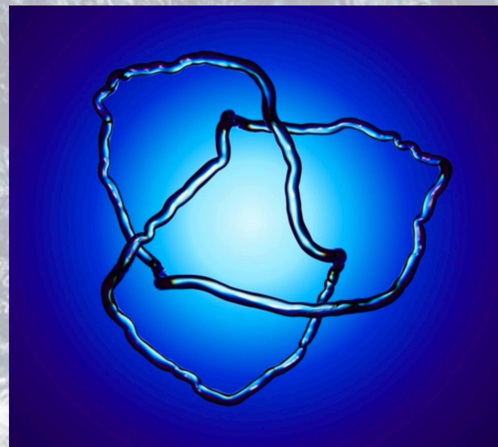
M. Berry (2000)

Topological origin of equatorial waves



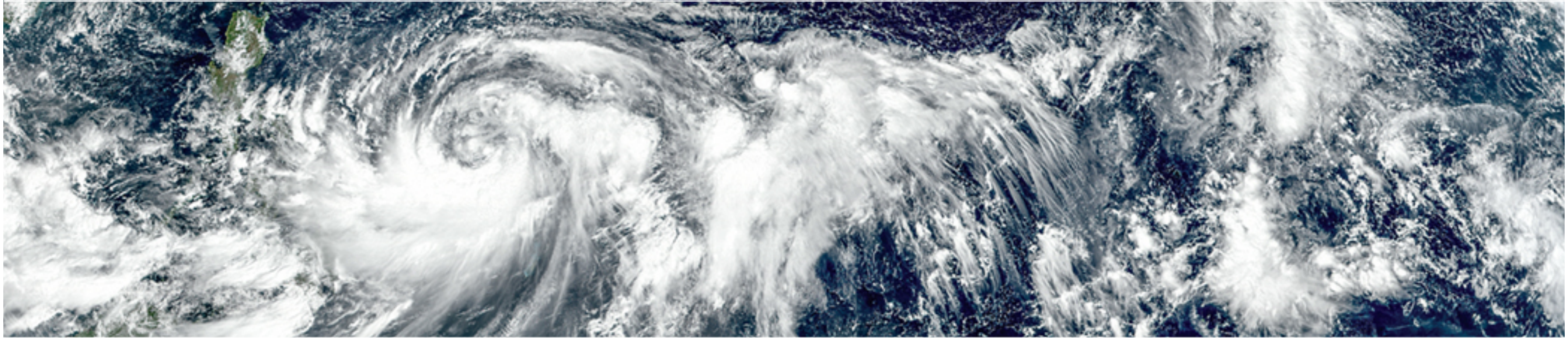
M. Berry (1980)

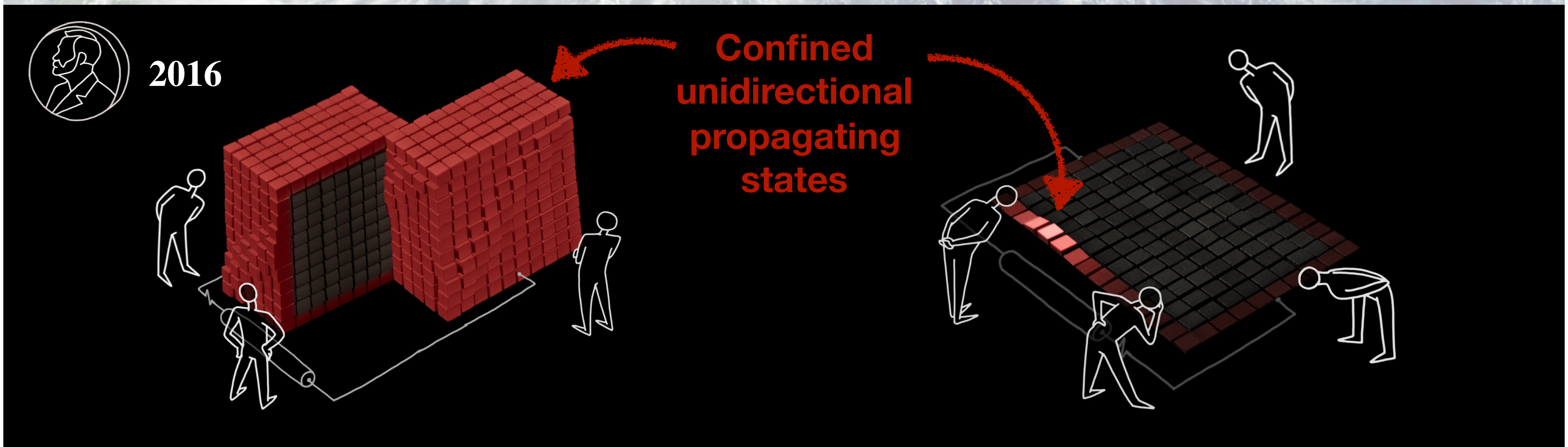
M. Berry (2000)



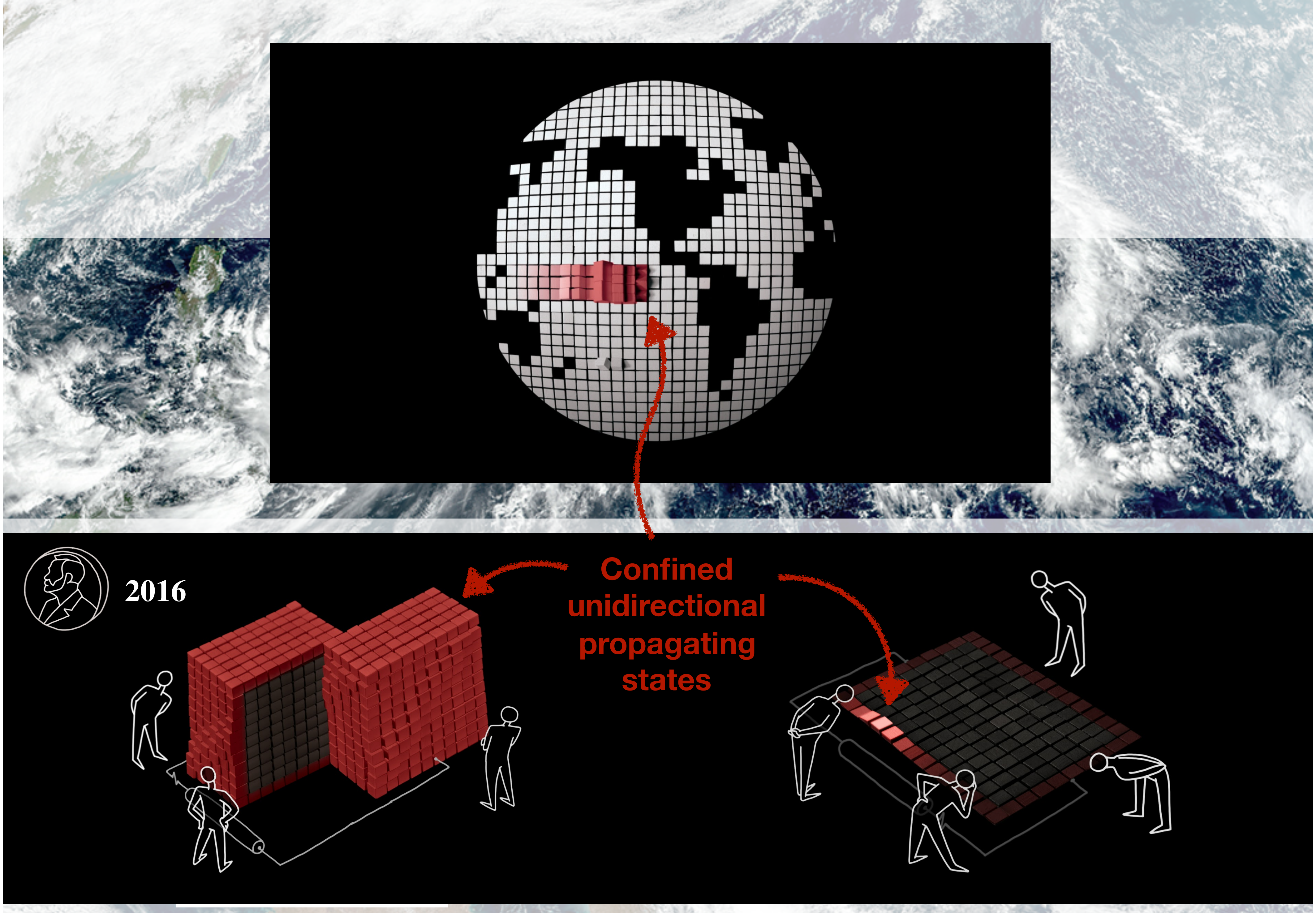
Irvine's Lab, Chicago

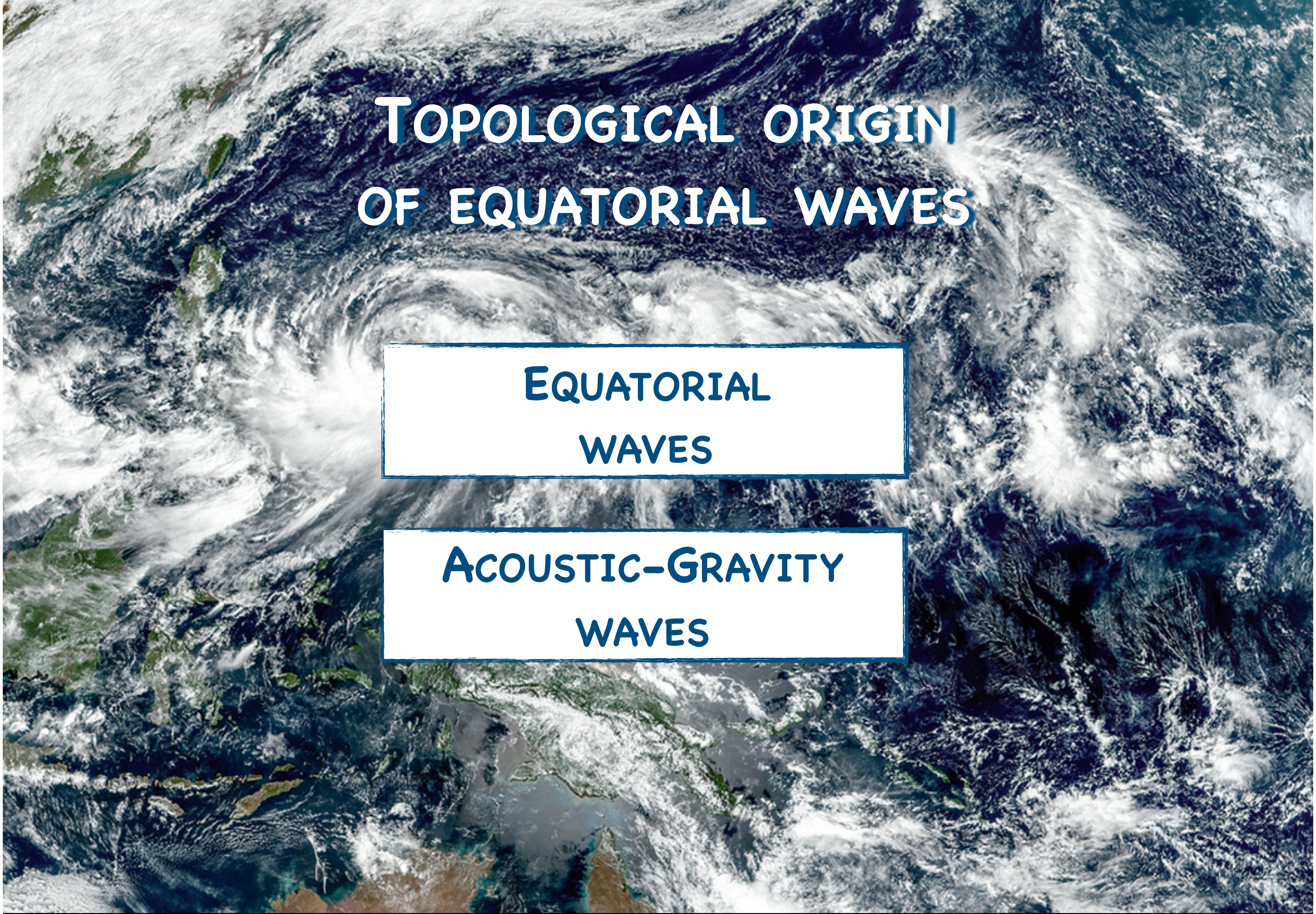
Topological origin of equatorial waves





Physics at the equator: from the lab to the stars *October 16-18 2019 | ENS de Lyon (France)*


2016
Confined
unidirectional
propagating
states

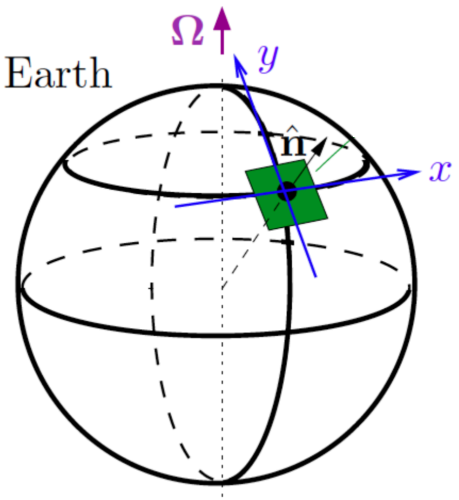

Topological origin
of equatorial waves
Equatorial
waves
Acoustic-Gravity
waves

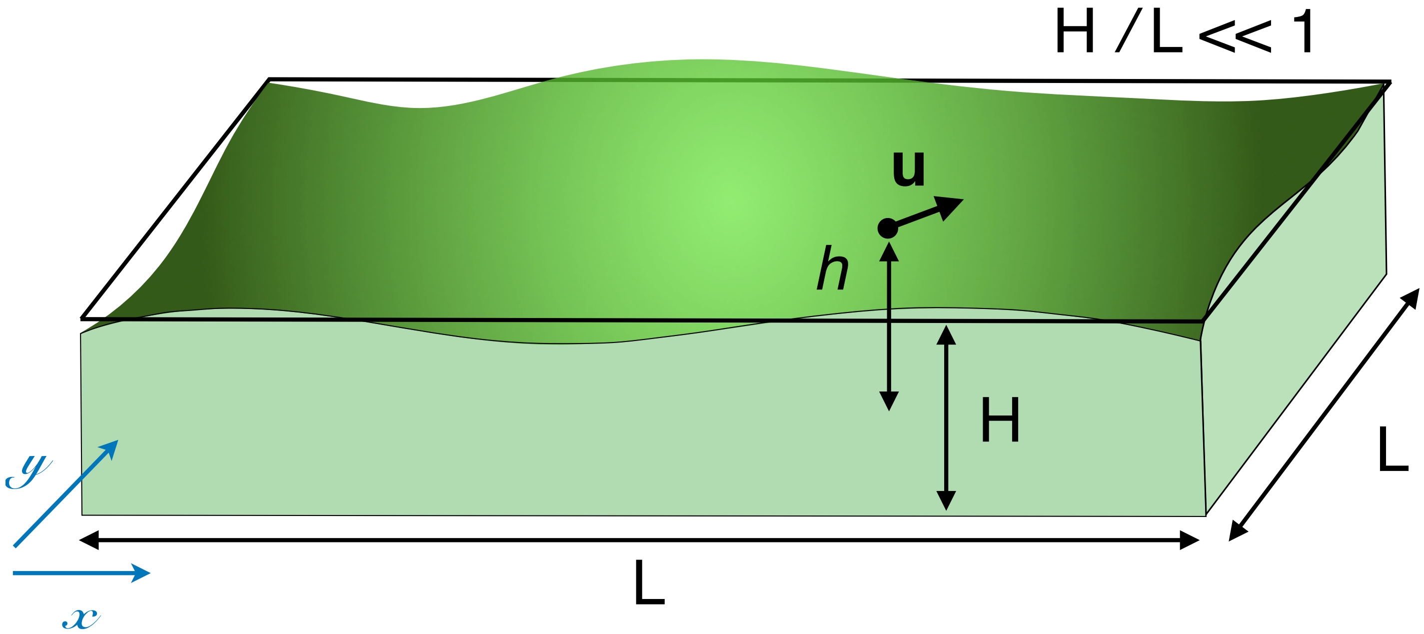

Equatorial waves

EQUATORIAL WAVES

✓ Incompressible

✓ Shallow





Mass conservation

$$\partial_t h + \nabla.(h\mathbf{u}) = 0$$

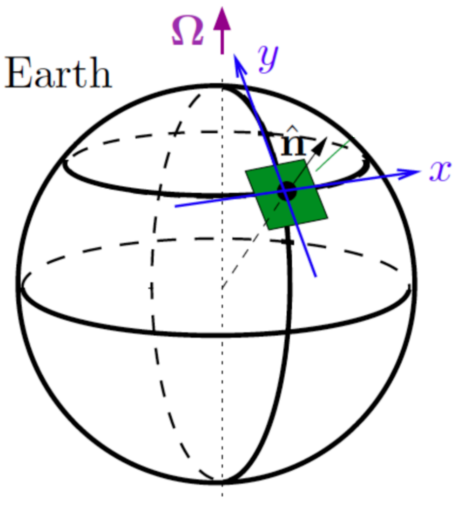
Momenta conservation

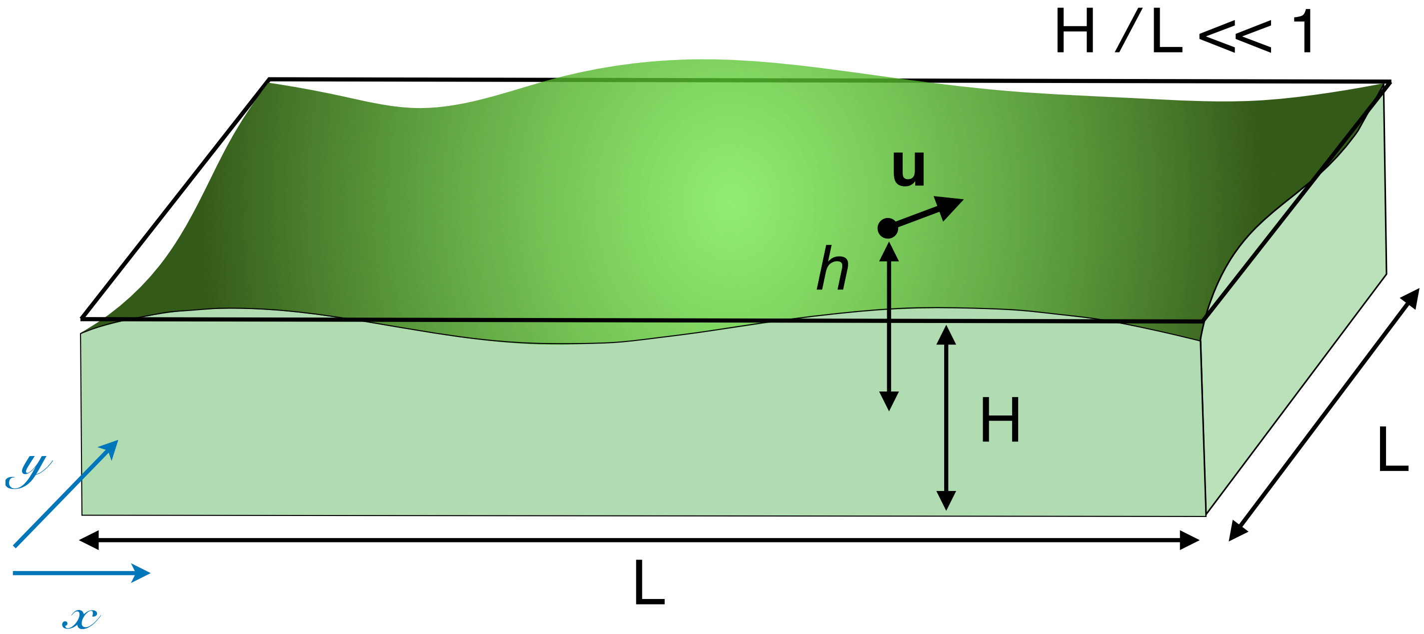
$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -g\nabla h - f\hat{\mathbf{n}} \times \mathbf{u}$$

Equatorial waves

✓ Incompressible

✓ Shallow





Mass conservation	$$\partial_t h + \nabla.(h\mathbf{u}) = 0$$
Momenta conservation	$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -g\nabla h - f\hat{\mathbf{n}} \times \mathbf{u}$$

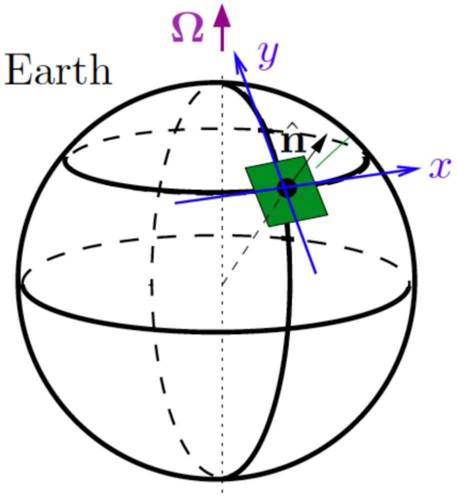
✓ Breaks time-reversal symmetry

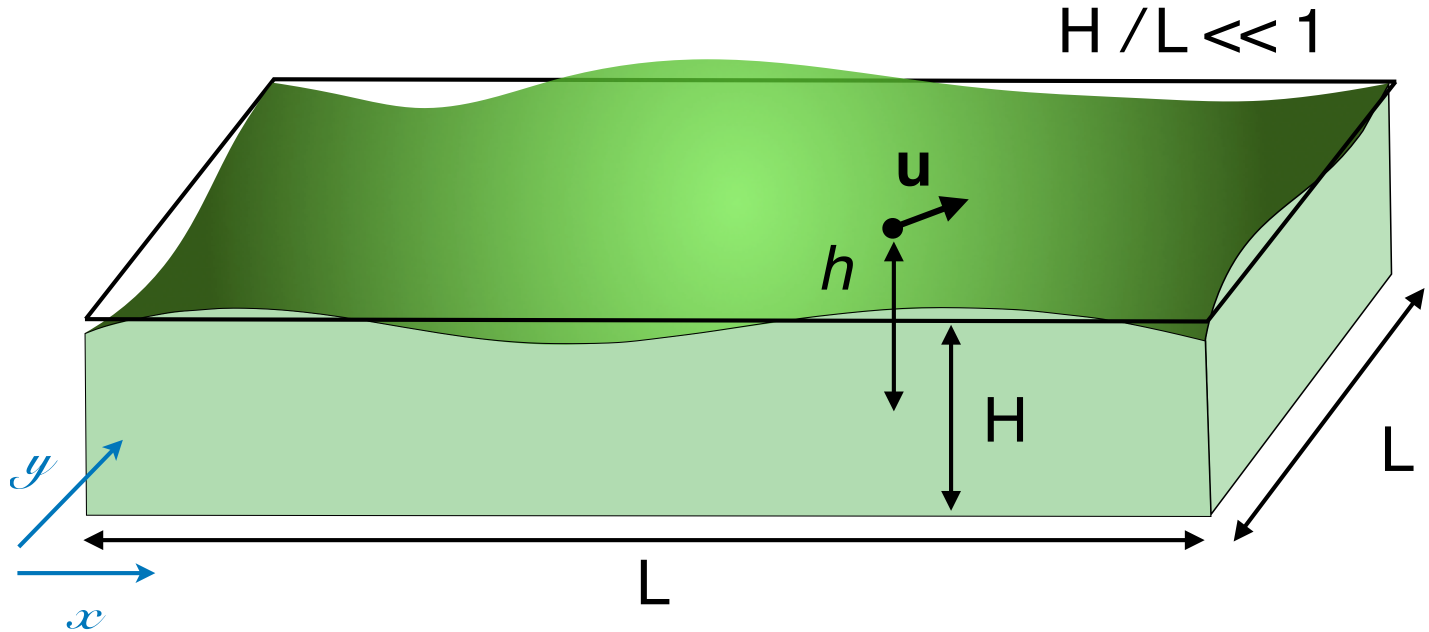
✓ Changes sign at the equator

Equatorial waves

✓ Incompressible

✓ Shallow





$$H = \begin{pmatrix} 0 & -if(y) & i\partial_x \\ if(y) & 0 & i\partial_y \\ i\partial_x & i\partial_y & 0 \end{pmatrix}$$

$$\omega \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\eta} \end{pmatrix} = \mathscr{H} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\eta} \end{pmatrix}$$

f = external parameter

$$\mathscr{H} = H(k_x, k_y, f)$$

$f(y)$ changes sign

$$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_y, y)$$

$$H = \begin{pmatrix} 0 & -if(y) & i\partial_x \\ if(y) & 0 & i\partial_y \\ i\partial_x & i\partial_y & 0 \end{pmatrix}$$

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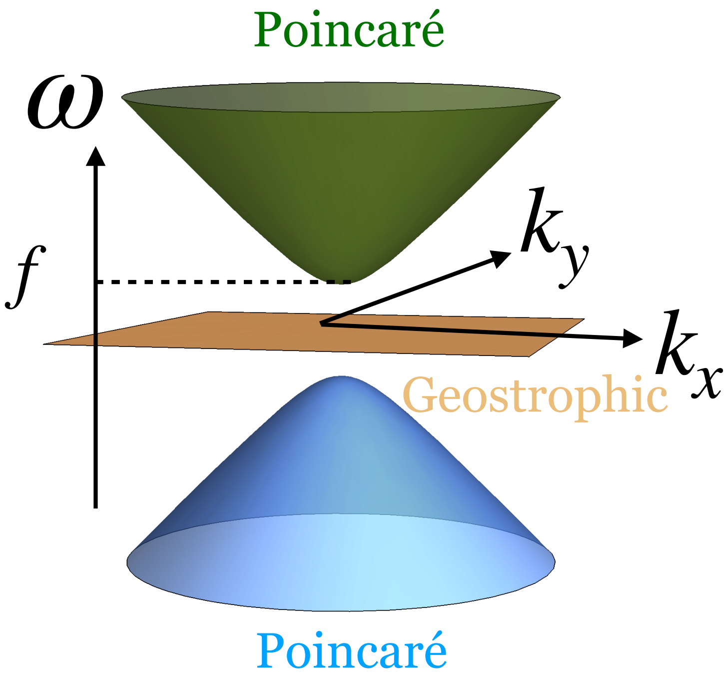

$f(y)$ changes sign

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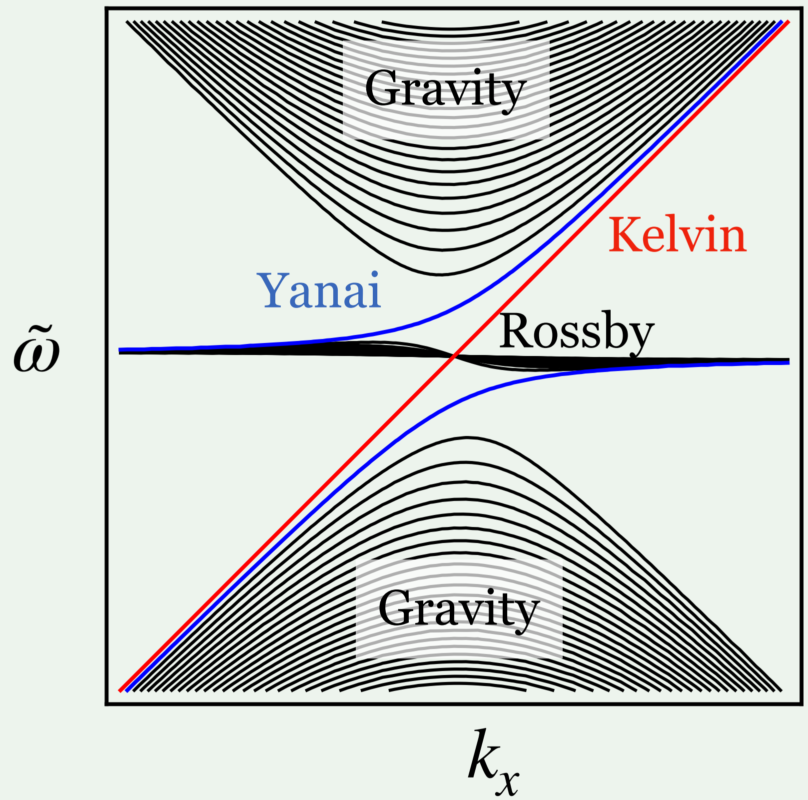
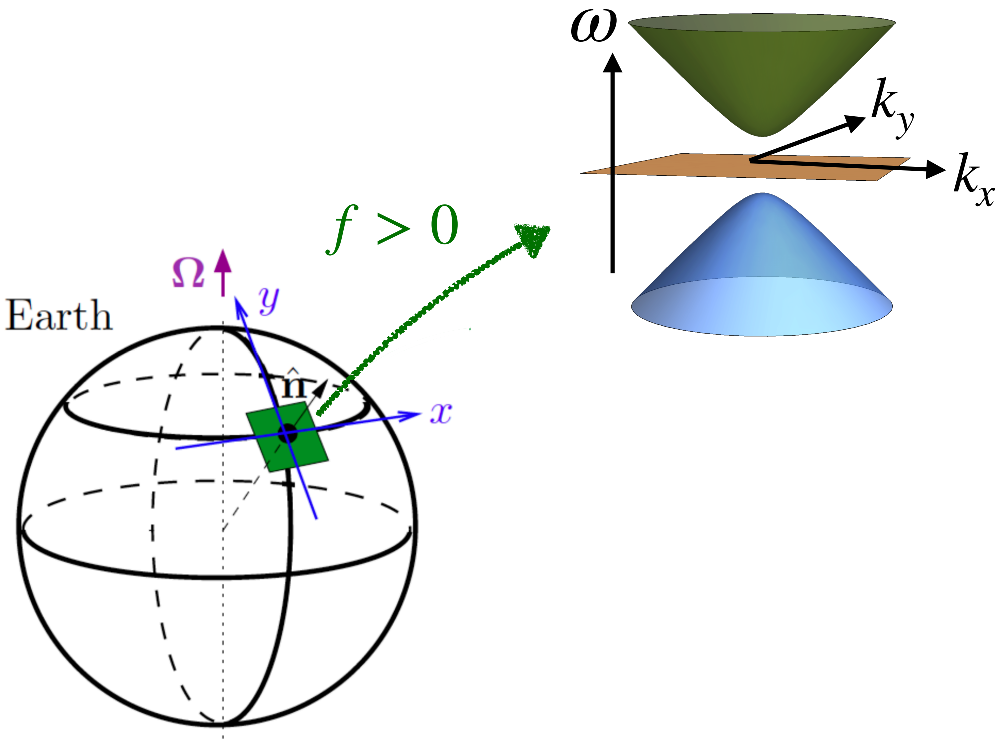


Matsuno '66

f = external parameter

$$\mathscr{H} = H(k_x, k_y, f)$$



$f(y)$ changes sign

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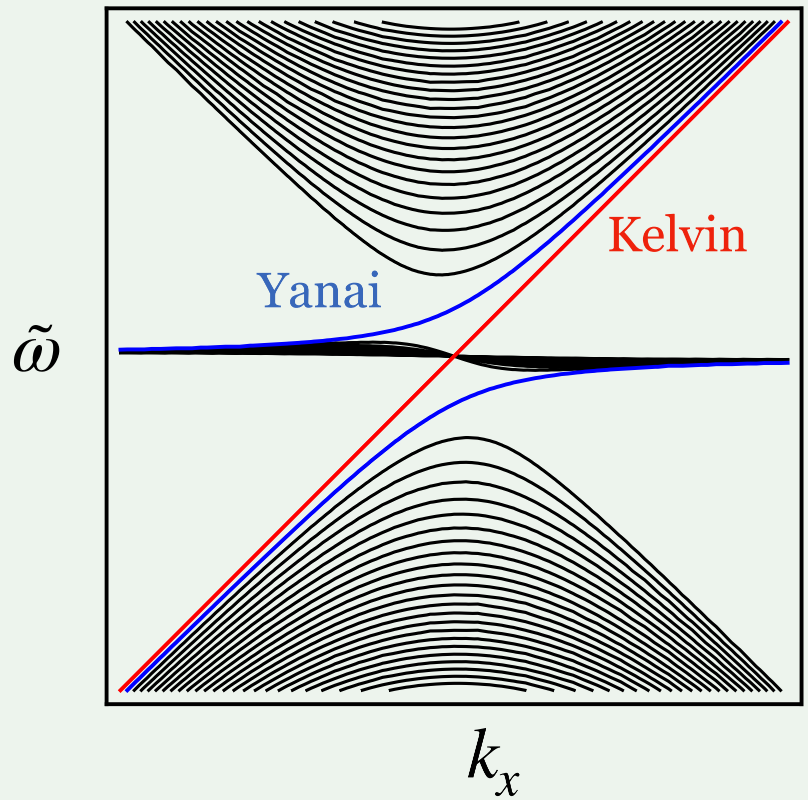

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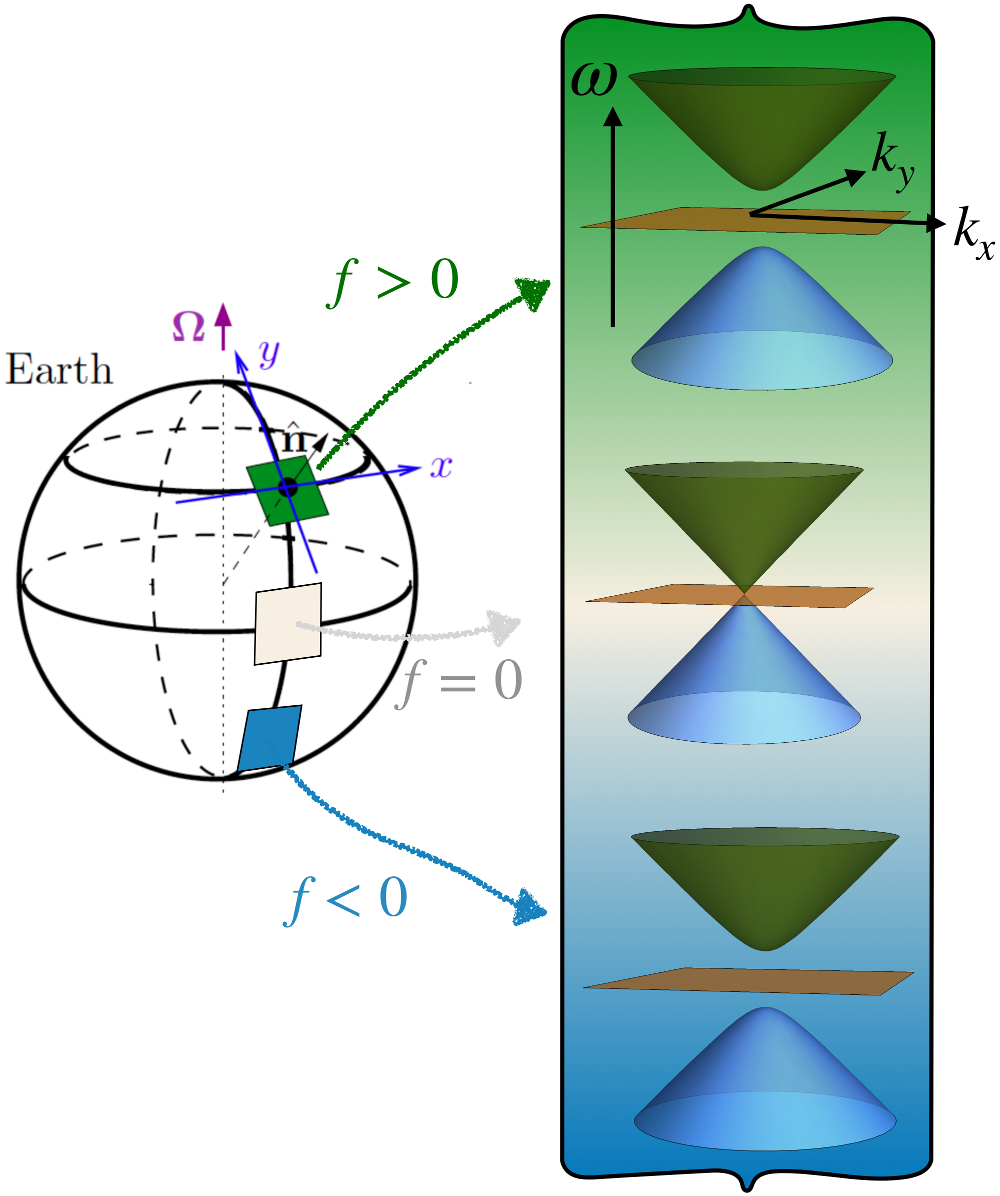

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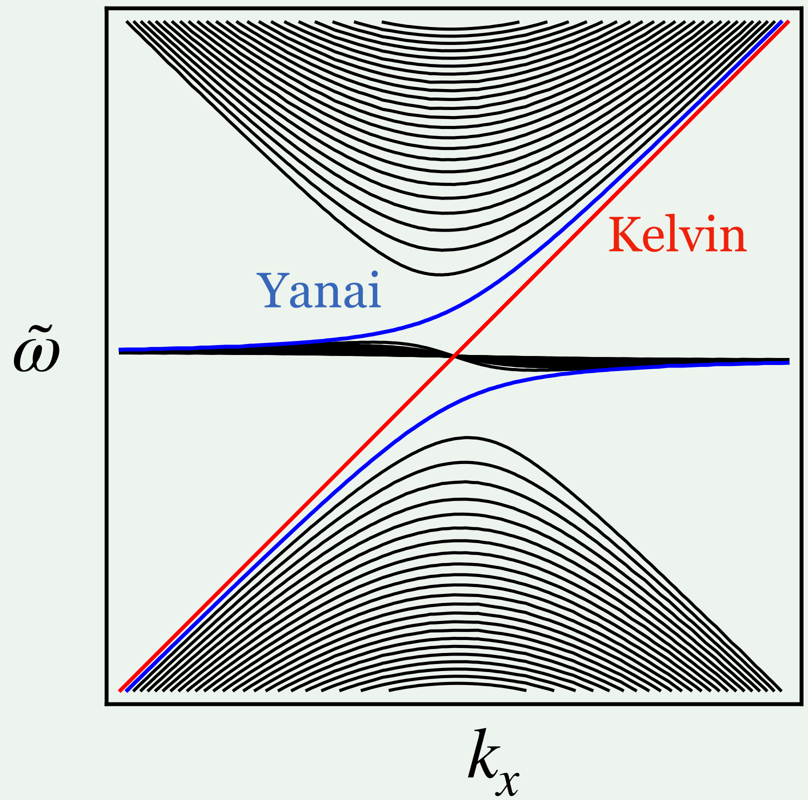

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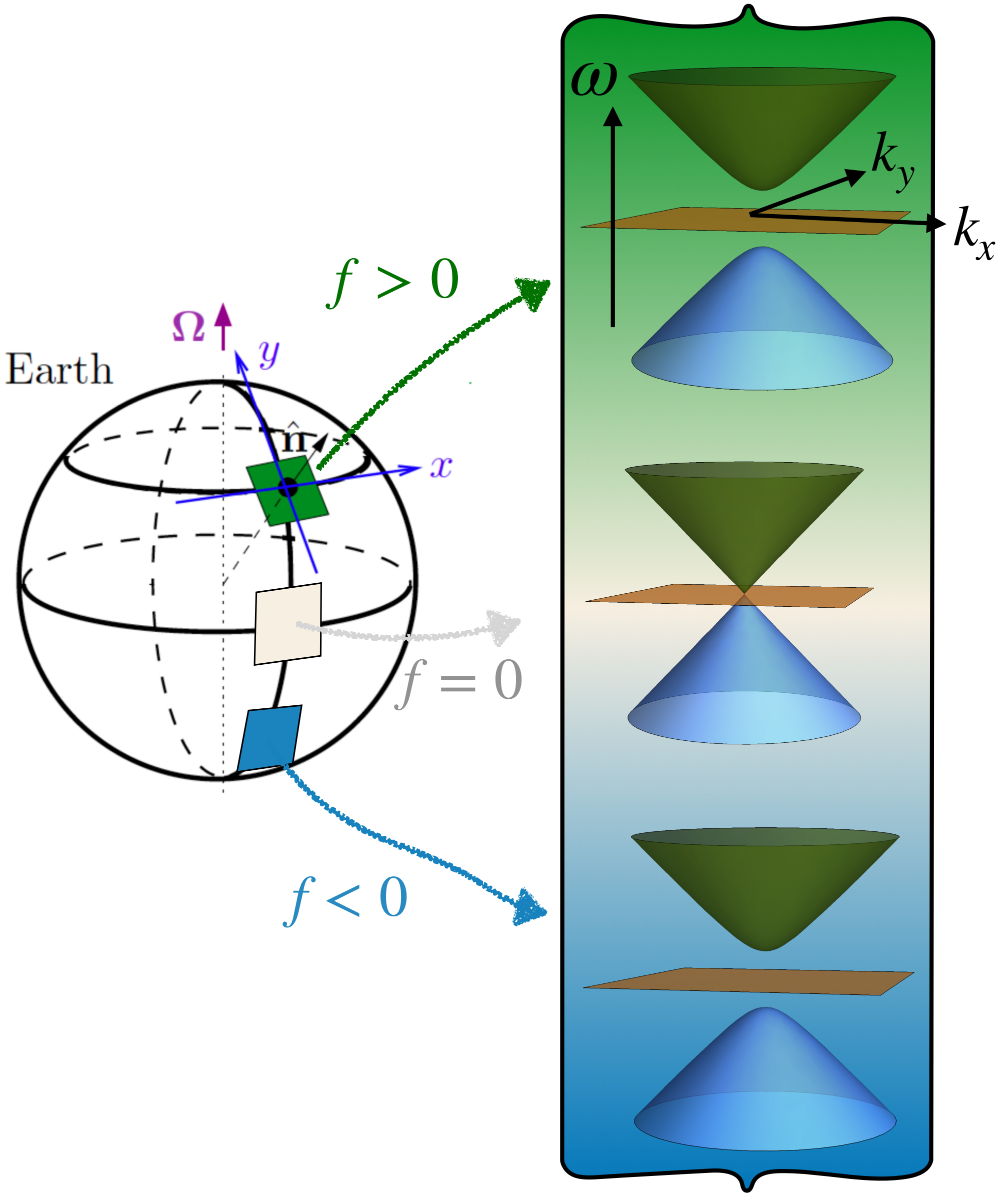

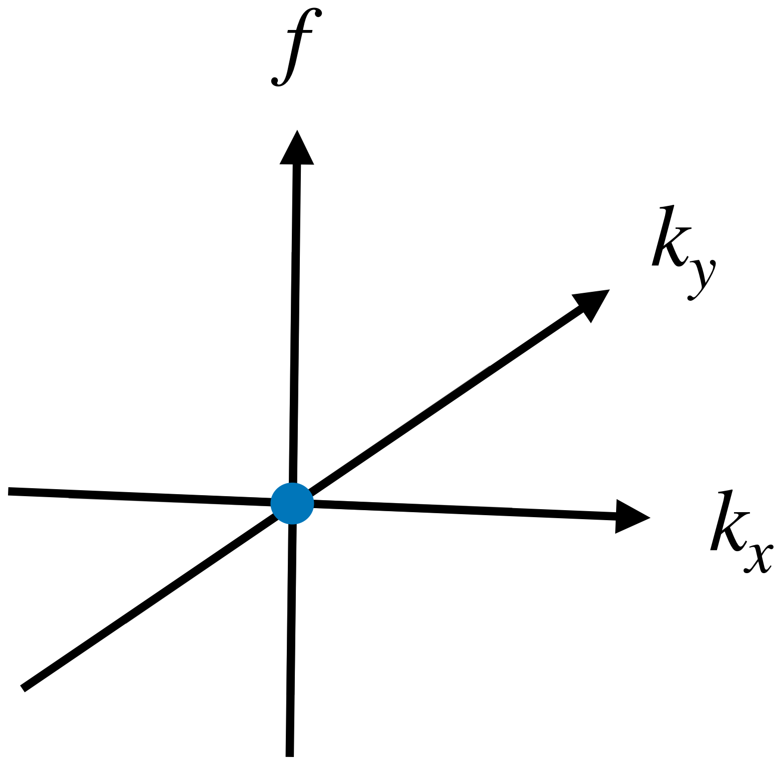

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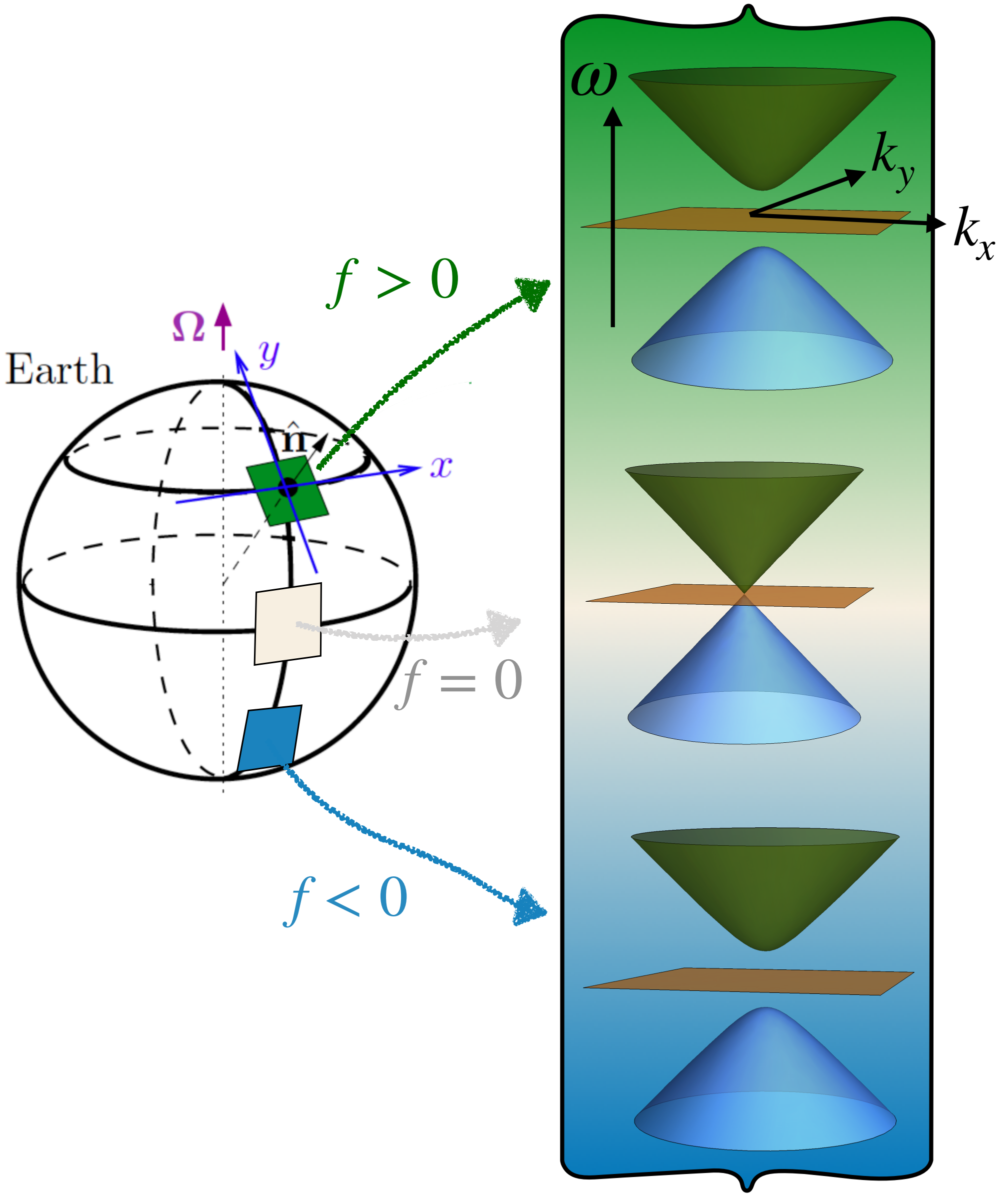

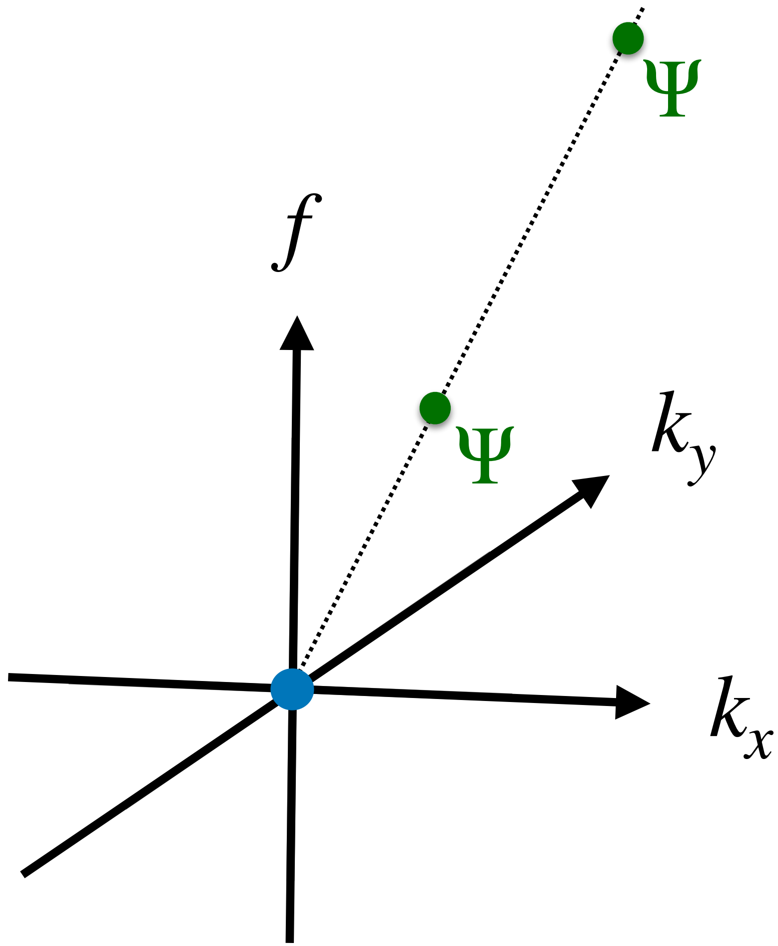

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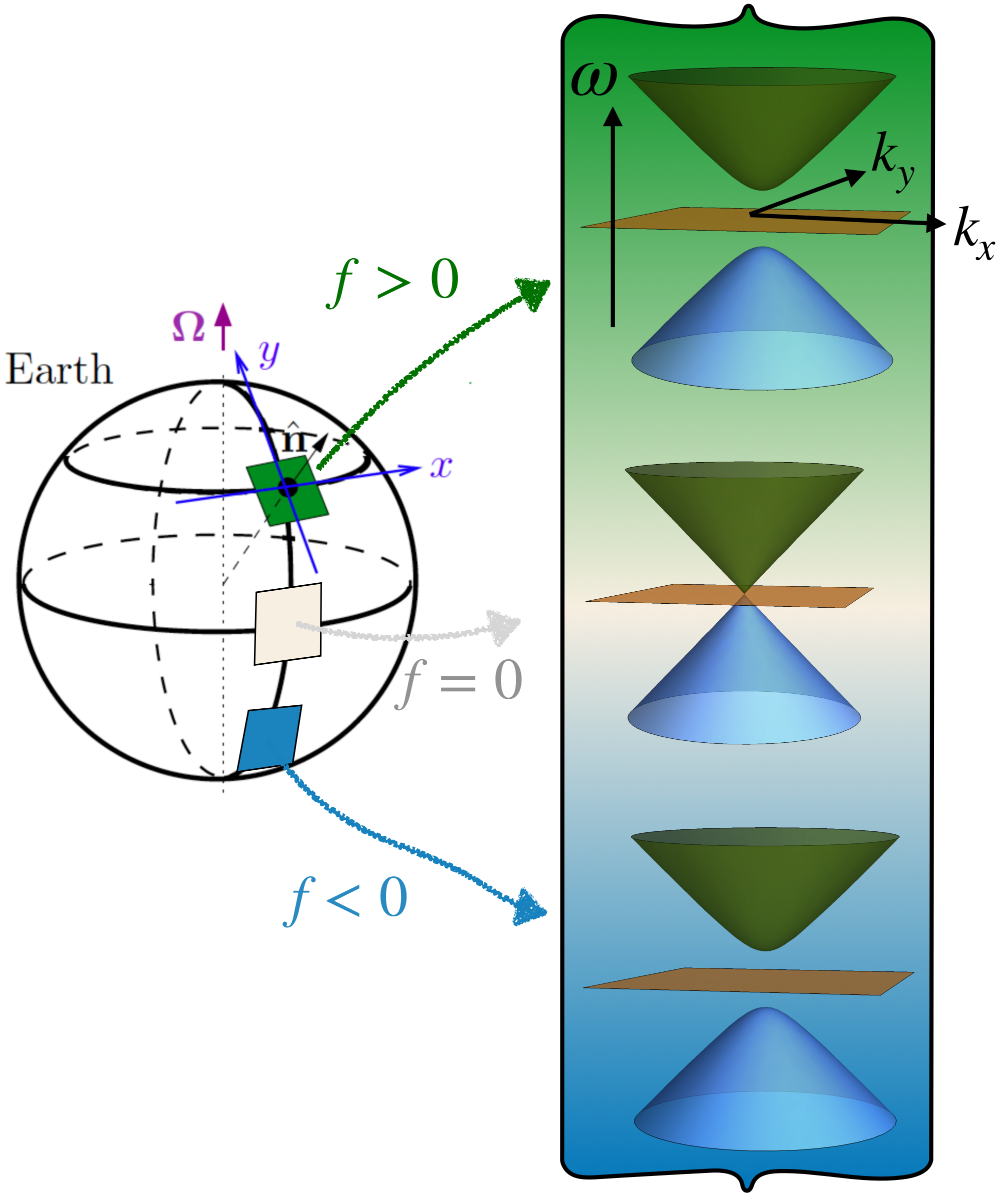

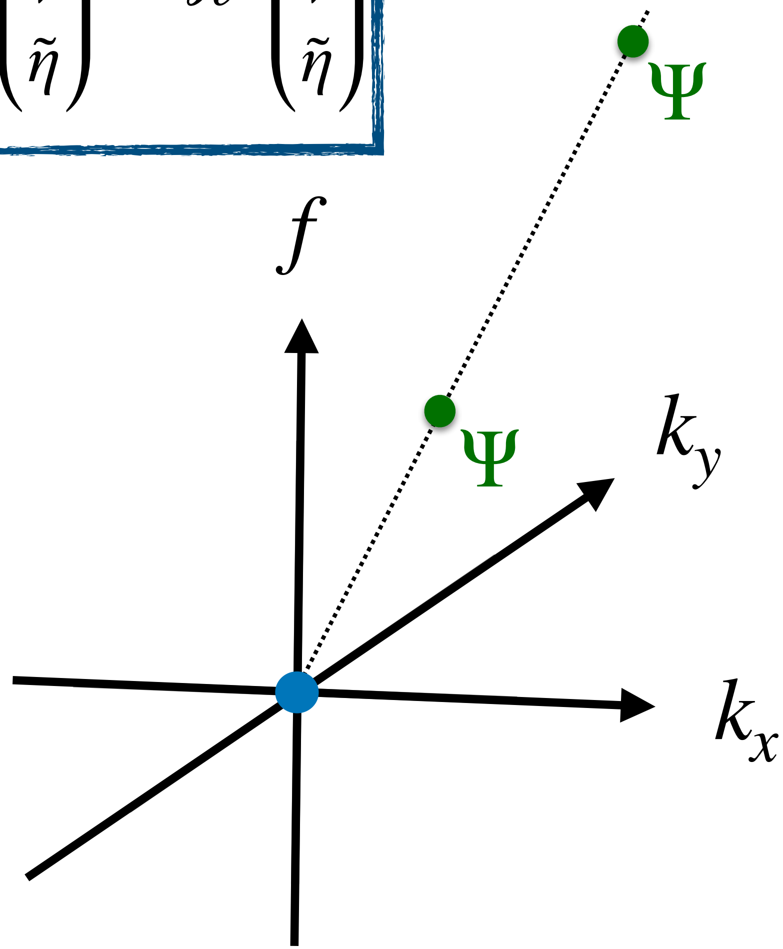

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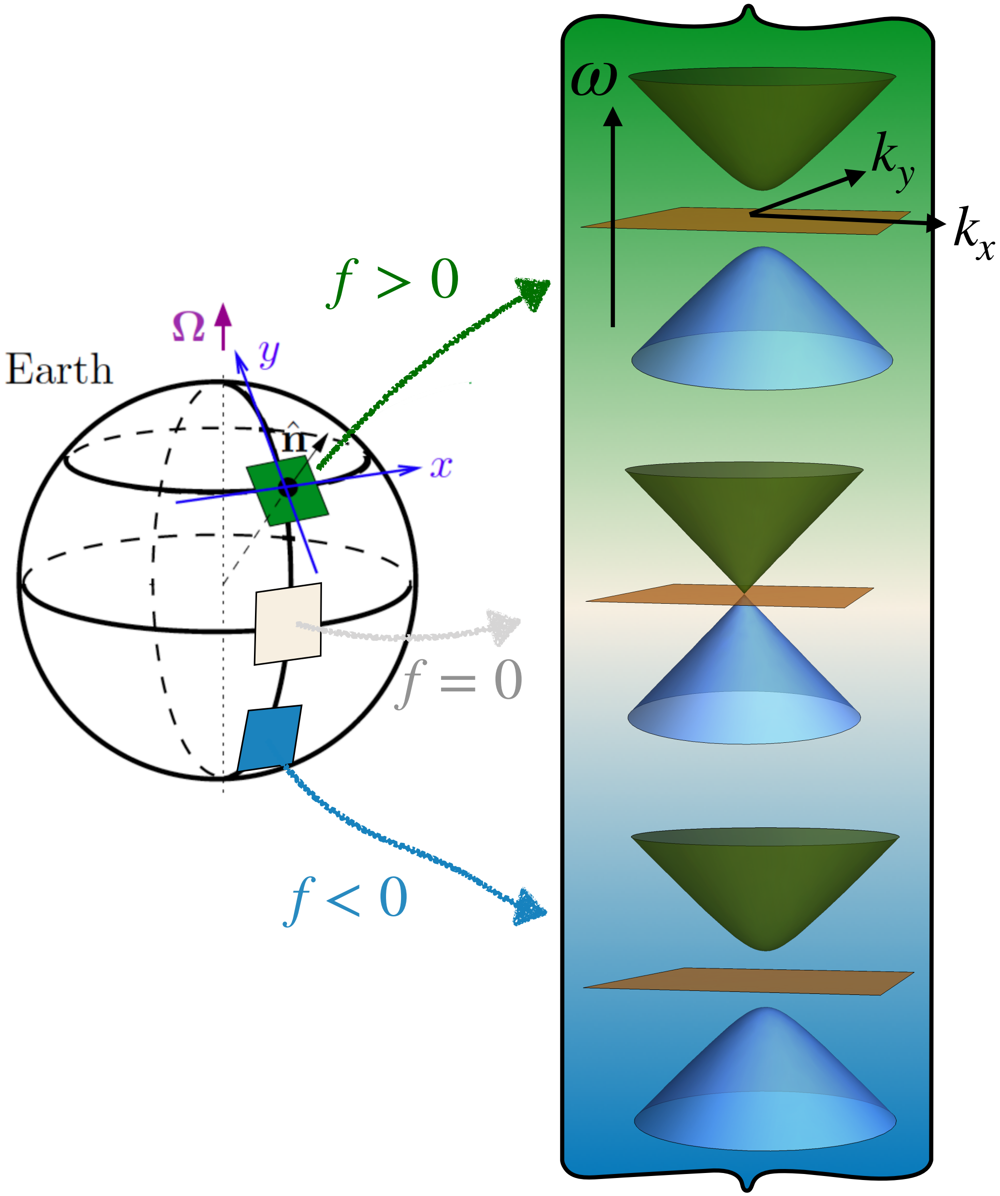

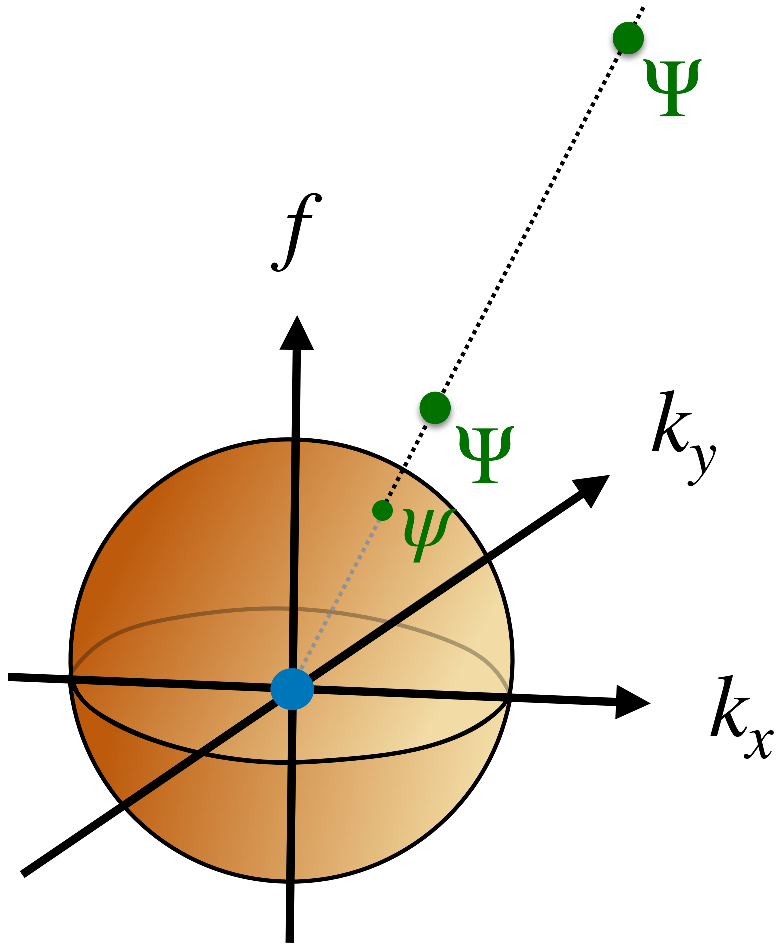

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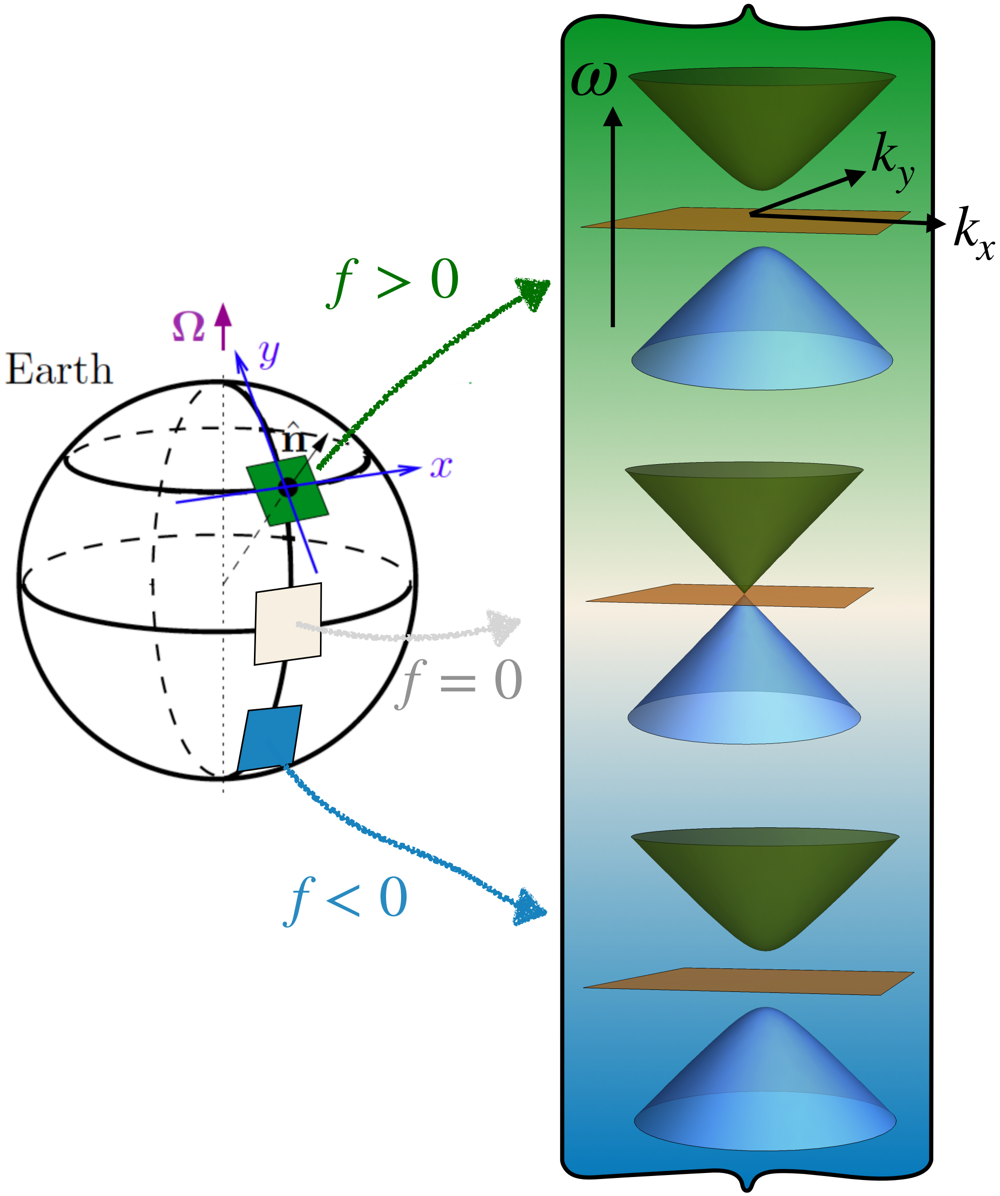

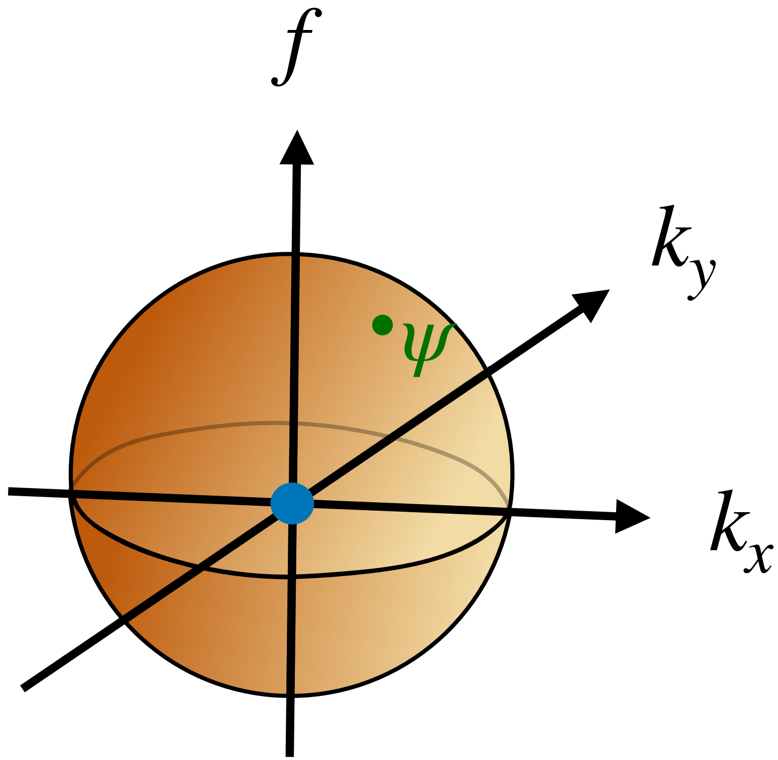

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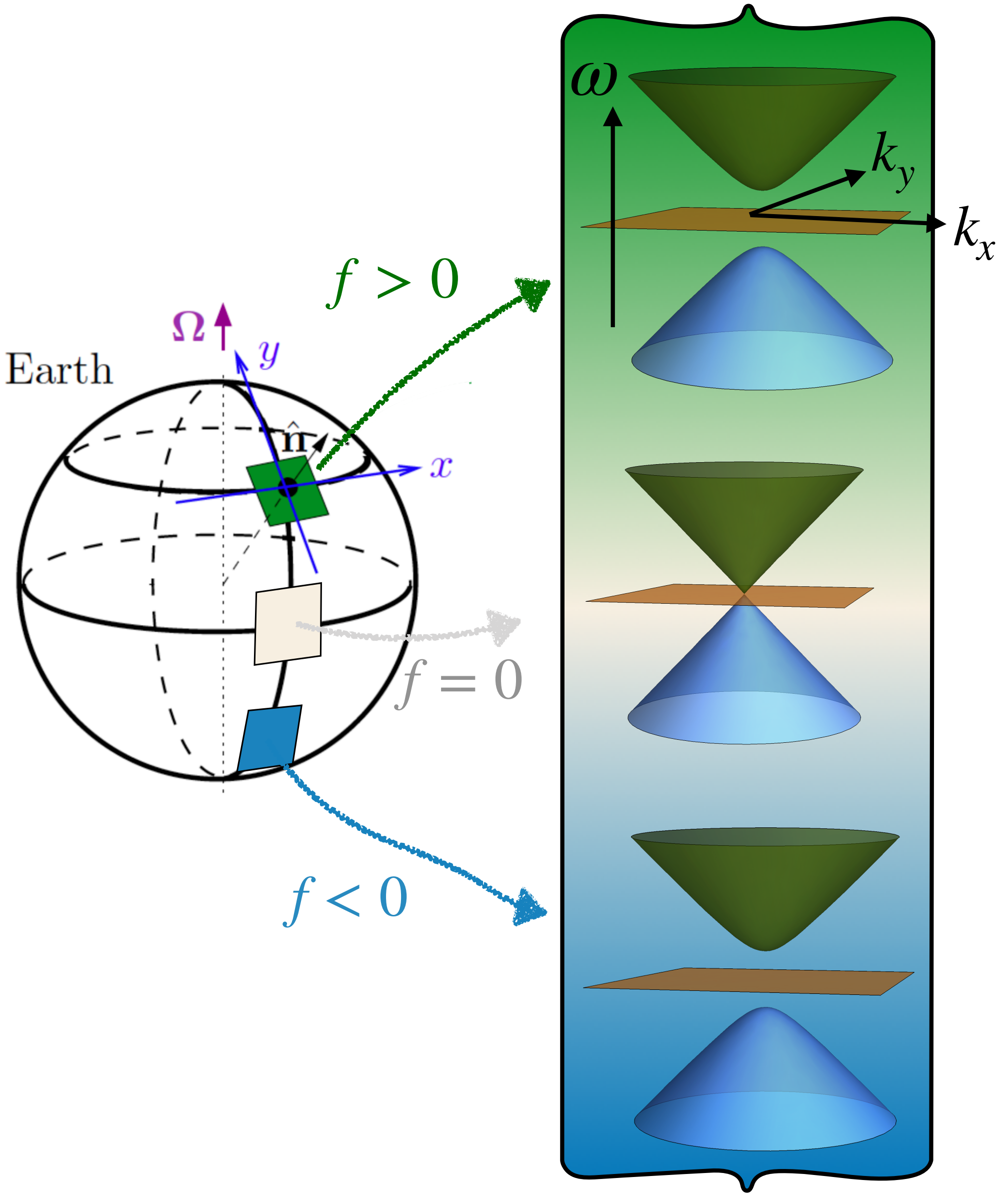

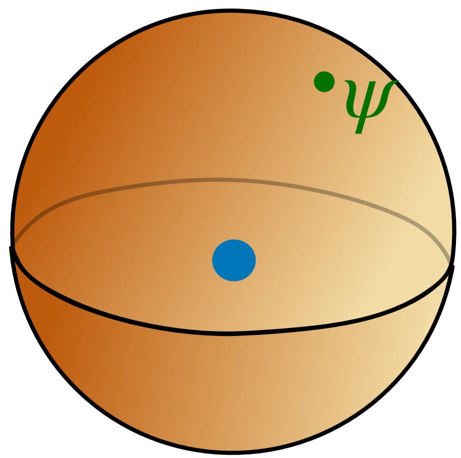

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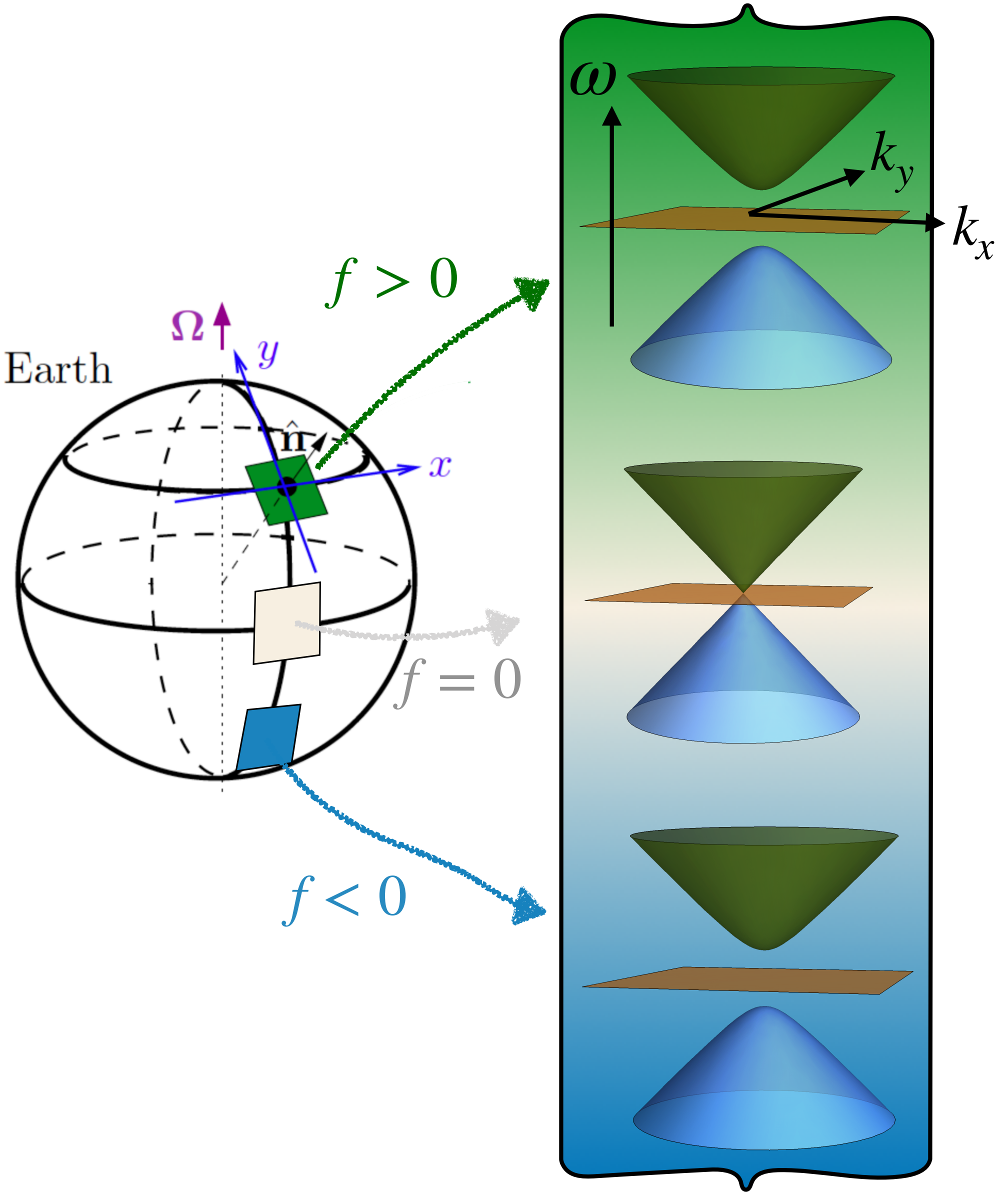

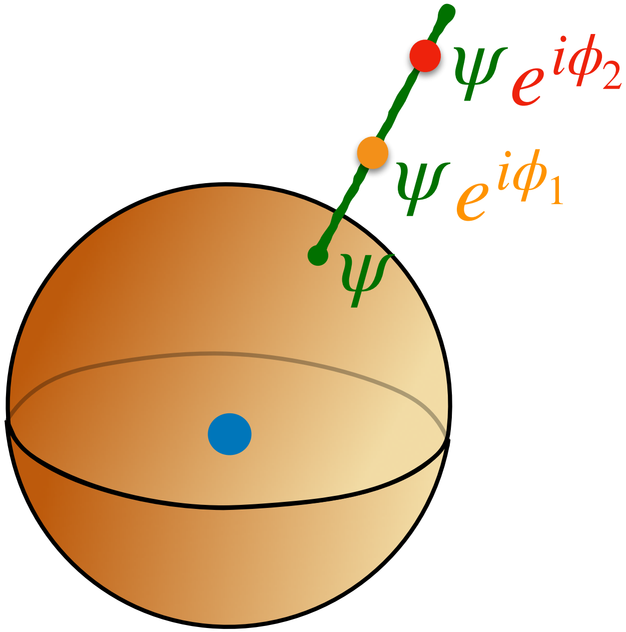

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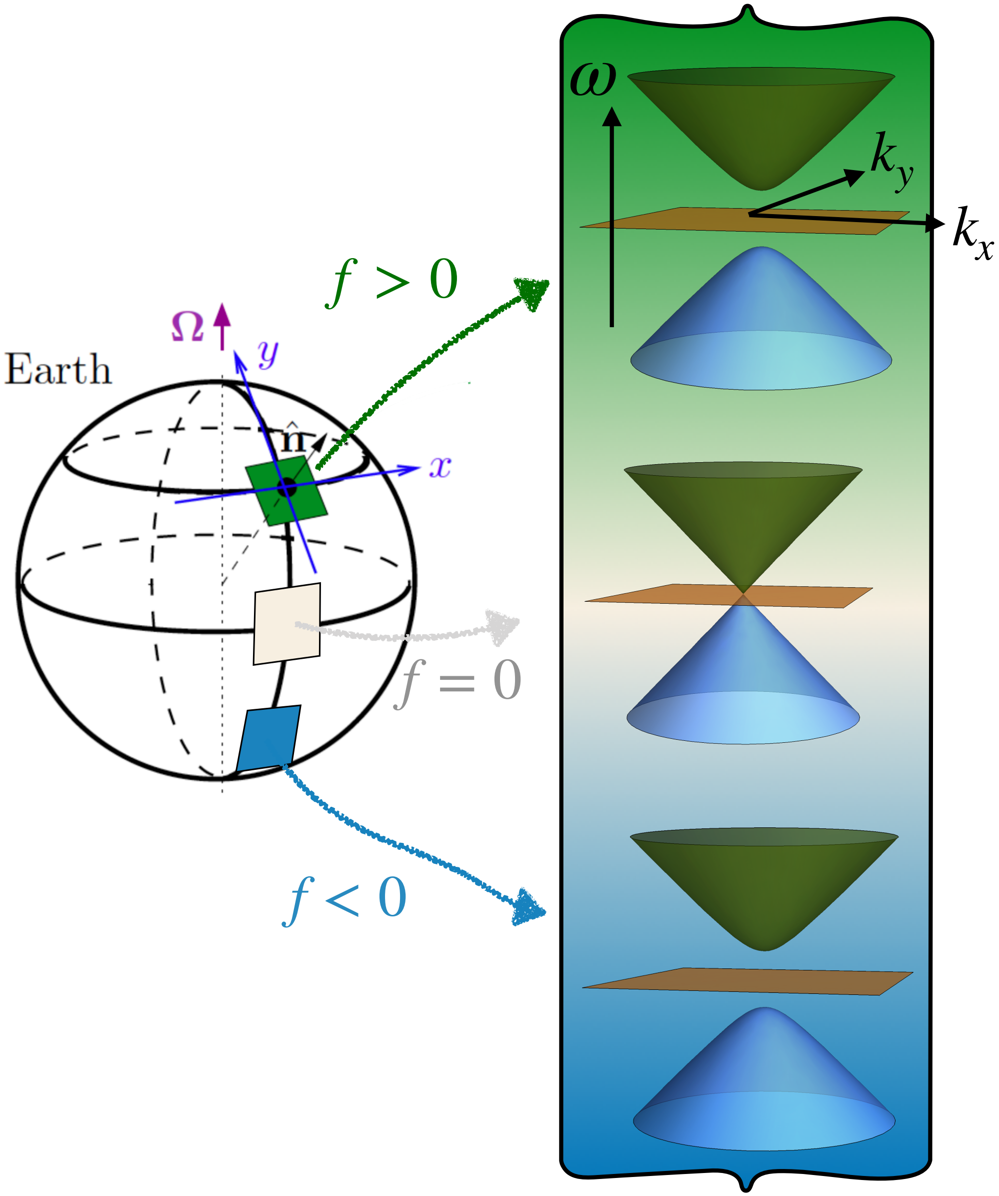
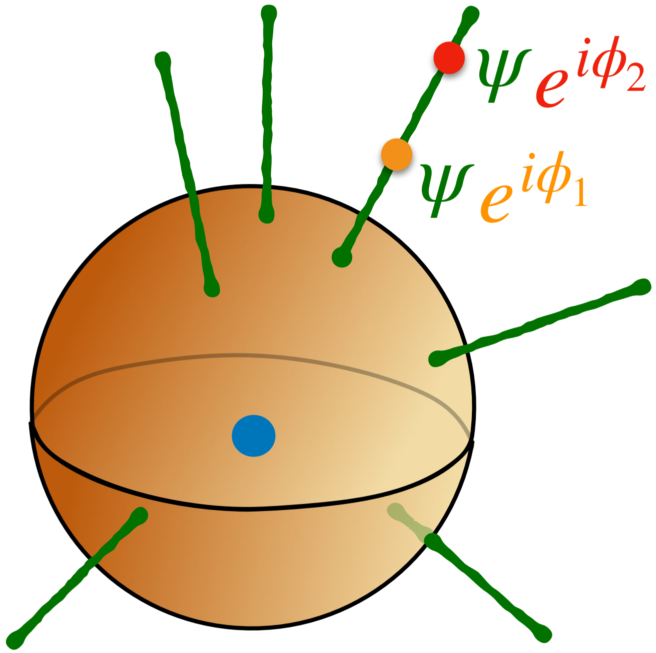
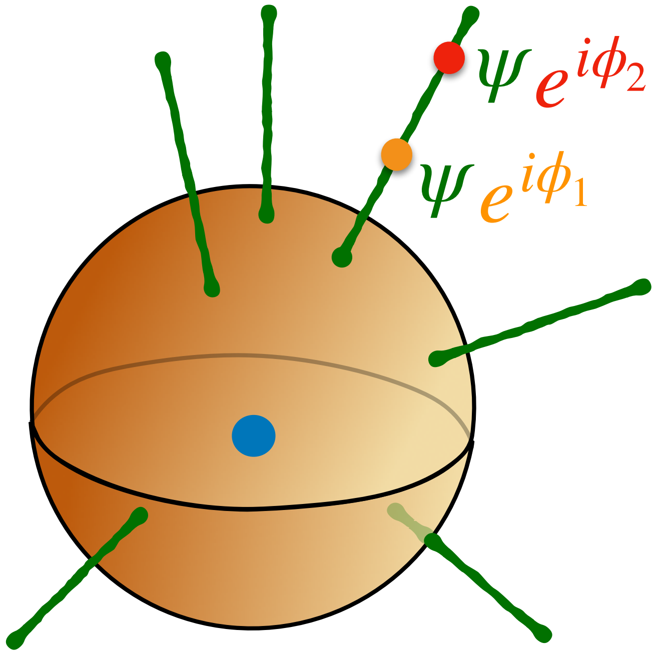




Vector Bundle

WHAT ABOUT TOPOLOGY ?



Vector Bundle

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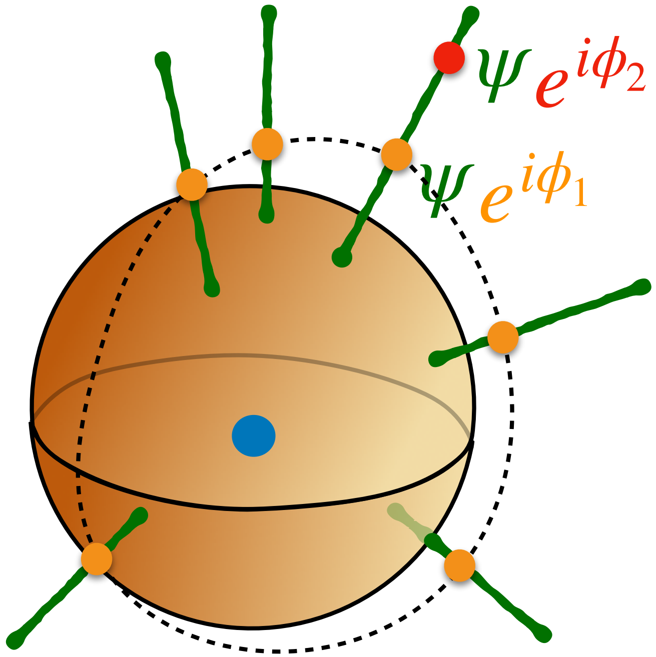

Vector Bundle

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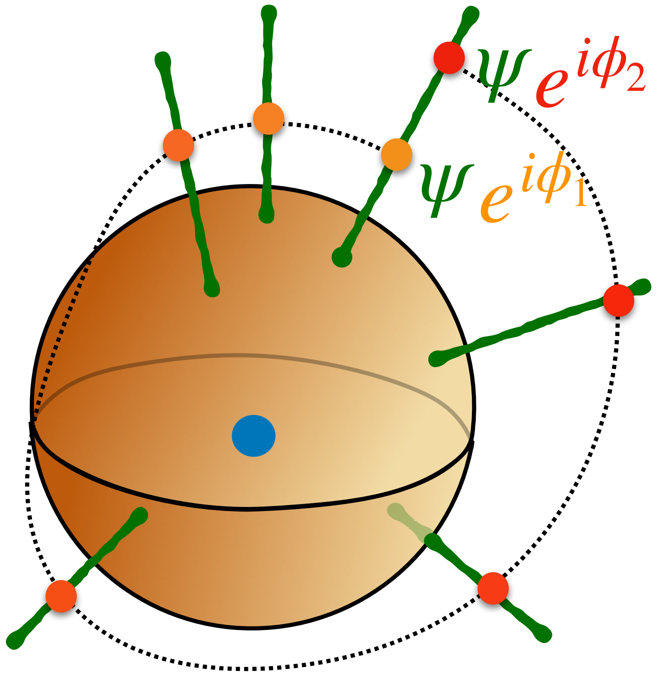

Vector Bundle

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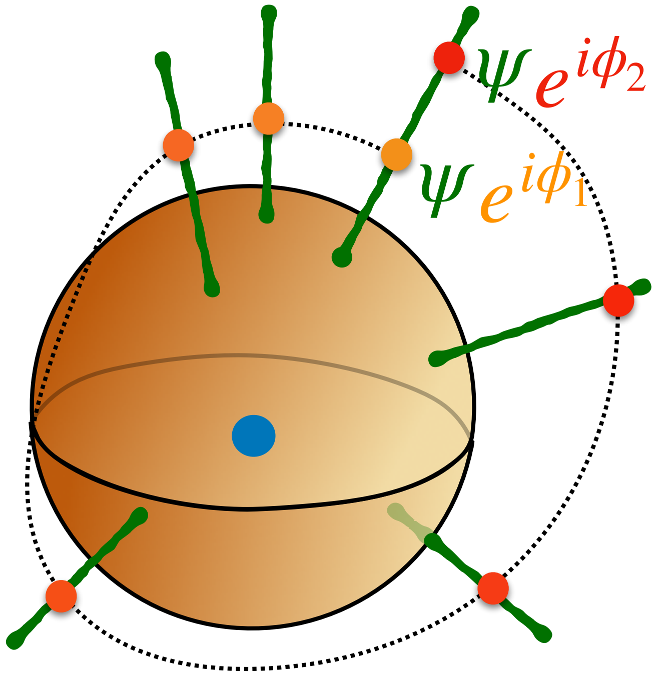
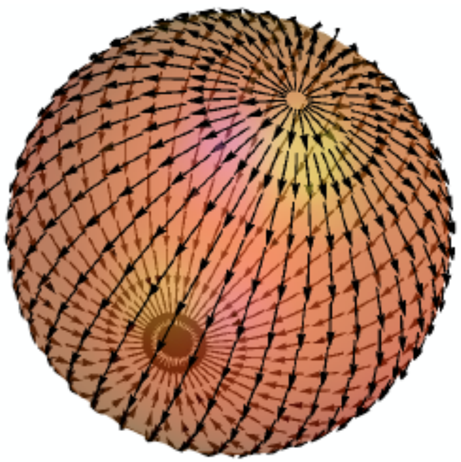


U(1) Vector Bundle

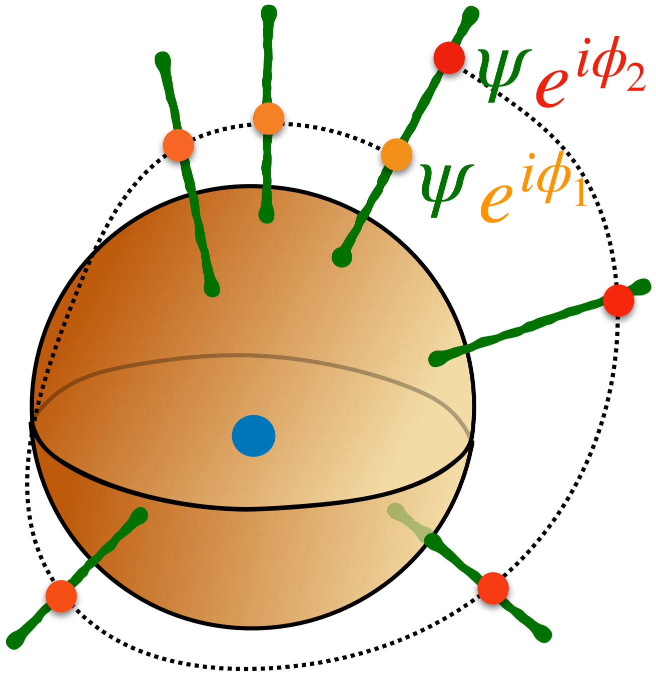
$$\mathscr{C} = \frac{1}{2\pi}\int_{S^2} F\, dS \in \mathbb{Z}$$

WHAT ABOUT TOPOLOGY?



tangent Vector Bundle

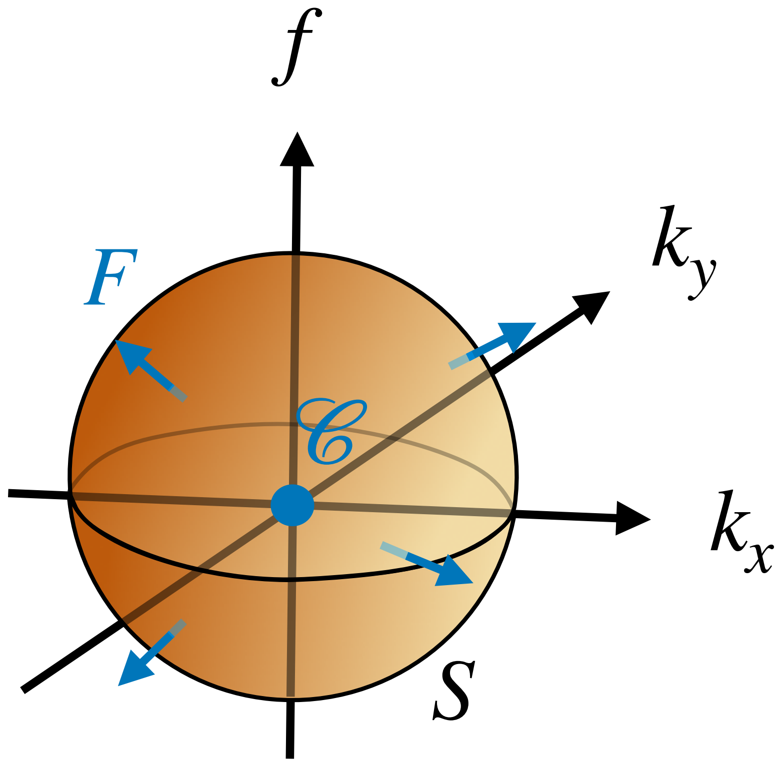
$$\chi = \frac{1}{2\pi}\int_{S^2} \kappa \, dS \in \mathbb{Z}$$



U(1) Vector Bundle

$$\mathscr{C} = \frac{1}{2\pi}\int_{S^2} F \, dS \in \mathbb{Z}$$

WHAT ABOUT TOPOLOGY?



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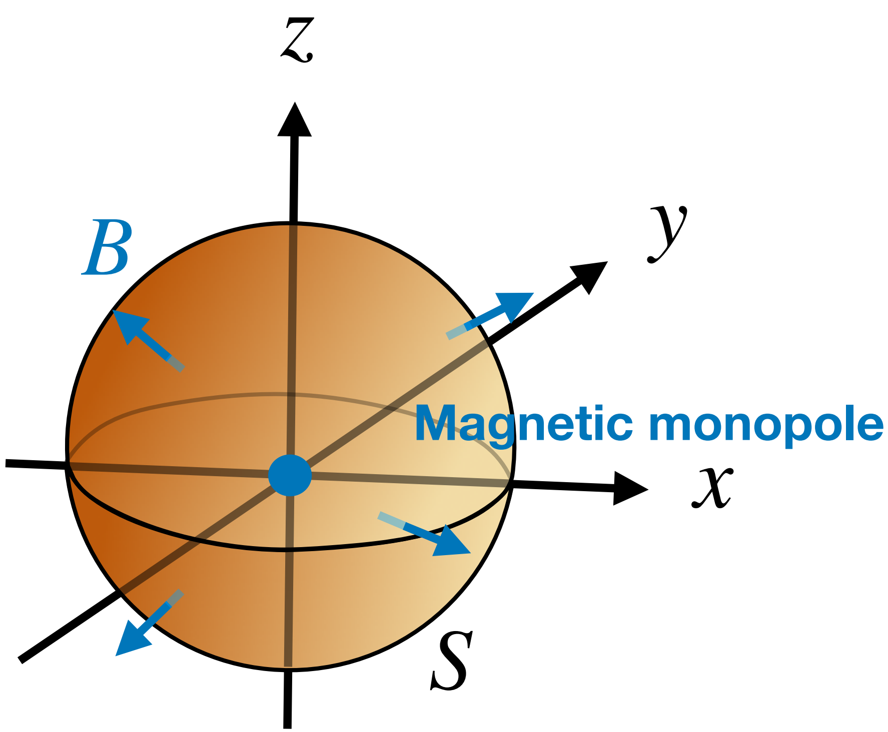
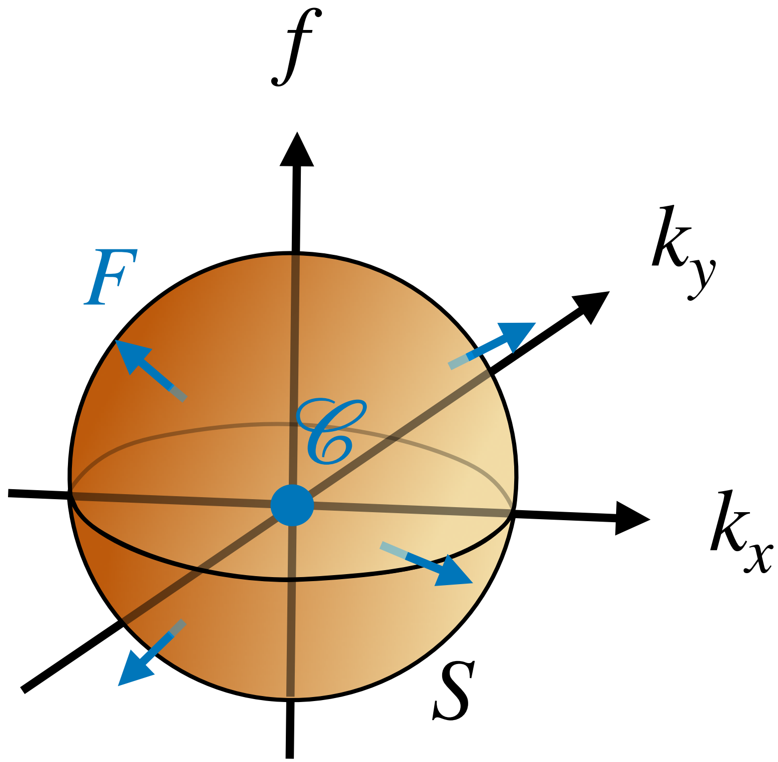
WHAT ABOUT TOPOLOGY?





U(1) Vector Bundle

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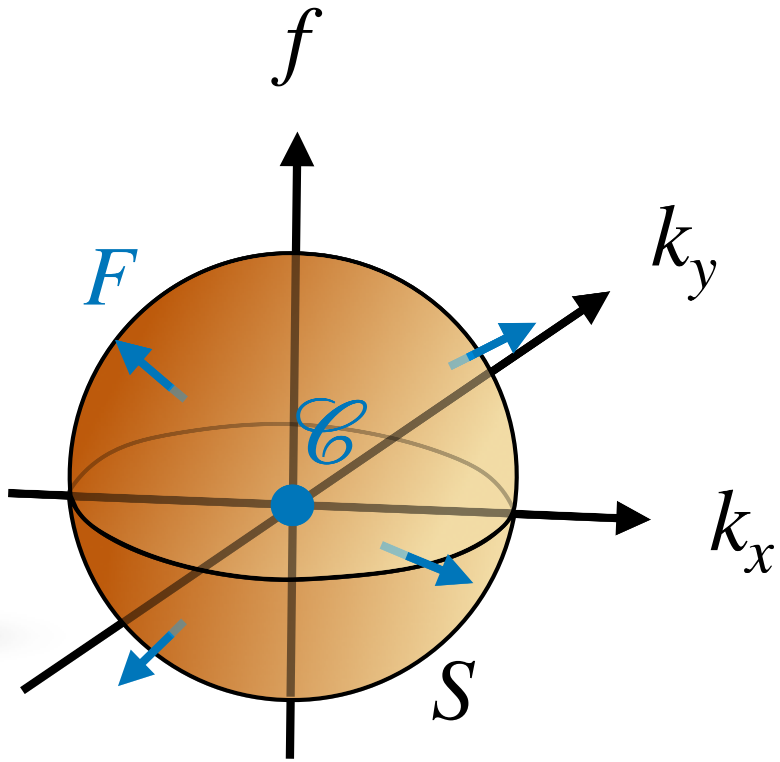
WHAT ABOUT TOPOLOGY?

Berry Curvature

$$F(\theta,\phi) = i\,\frac{\partial \psi^\dagger}{\partial \theta}\cdot\frac{\partial \psi}{\partial \phi} - i\,\frac{\partial \psi^\dagger}{\partial \phi}\cdot\frac{\partial \psi}{\partial \theta}$$

Gauge invariant, observable quantity

Enters the dynamics of a wave packet



U(1) Vector Bundle

$$\mathscr{C} = \frac{1}{2\pi}\int_{S^2} F\, dS \in \mathbb{Z}$$

WHAT ABOUT TOPOLOGY ?

Berry Curvature

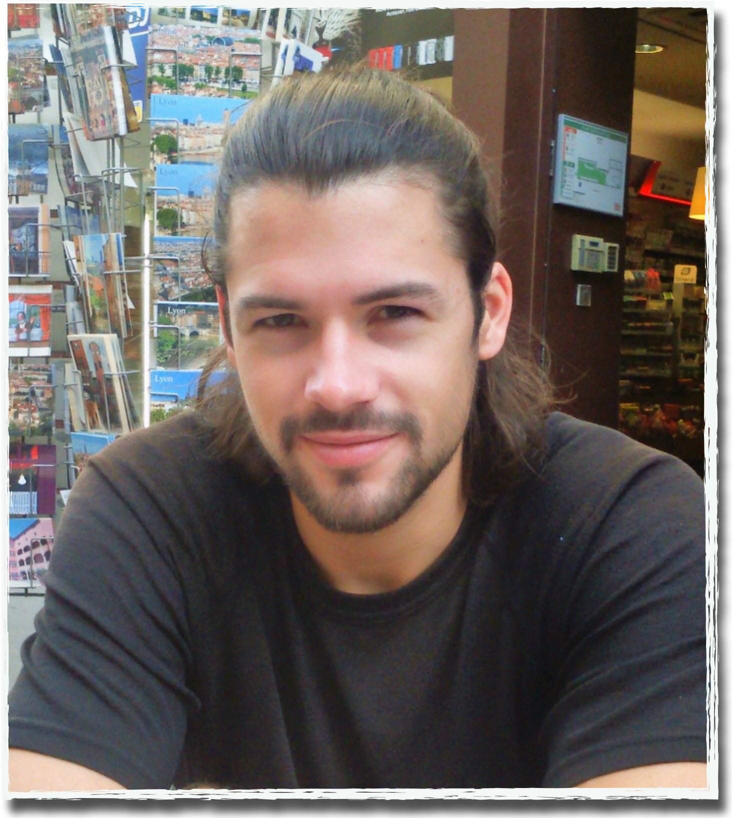
$$F(\theta,\phi) = i\,\frac{\partial \psi^\dagger}{\partial \theta}\cdot\frac{\partial \psi}{\partial \phi} - i\,\frac{\partial \psi^\dagger}{\partial \phi}\cdot\frac{\partial \psi}{\partial \theta}$$

Gauge invariant, observable quantity

Enters the dynamics of a wave packet

↓

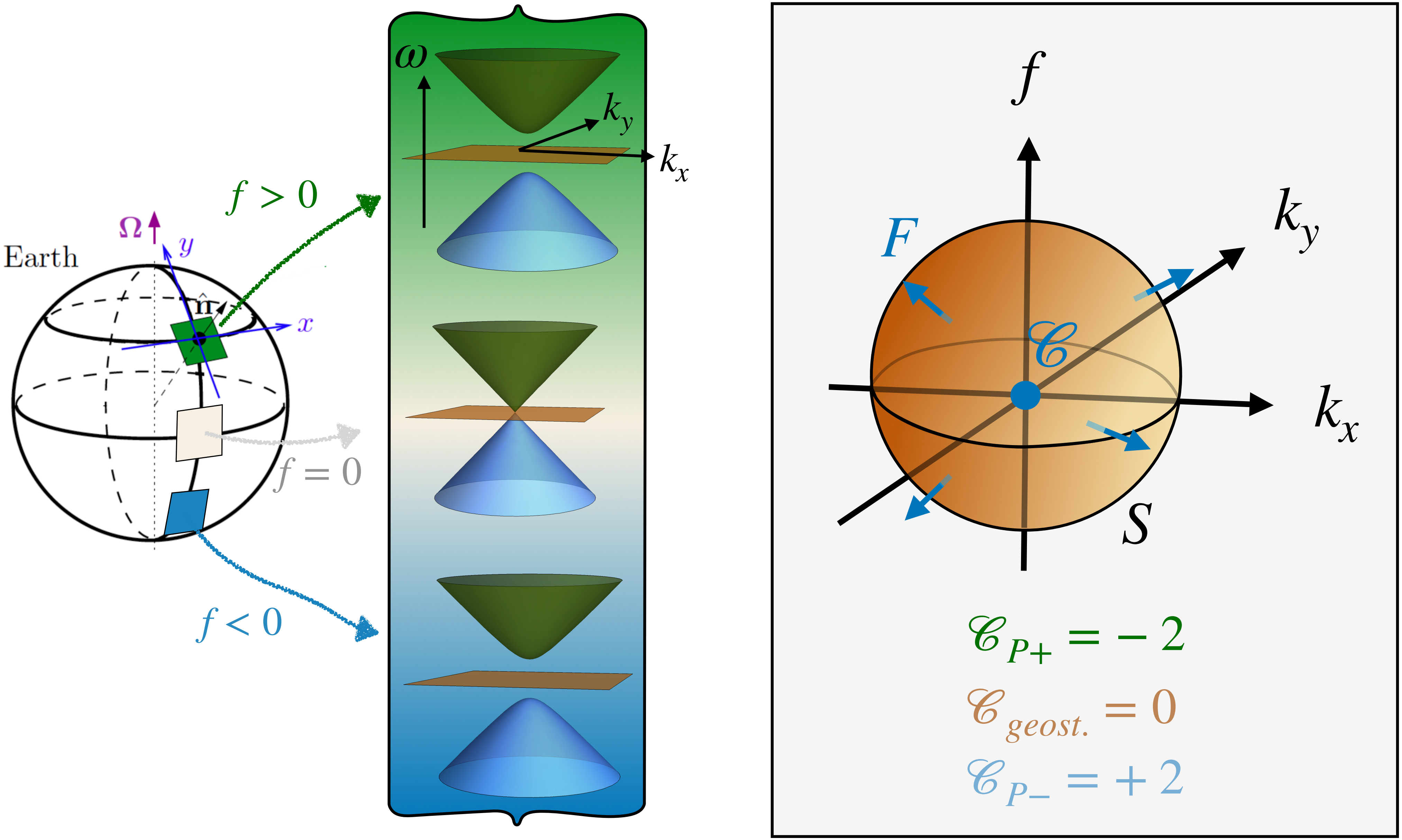
manifestation in ray theory



See Nicolas Perez Poster!

f = external parameter

$$\mathscr{H} = H(k_x, k_y, f)$$



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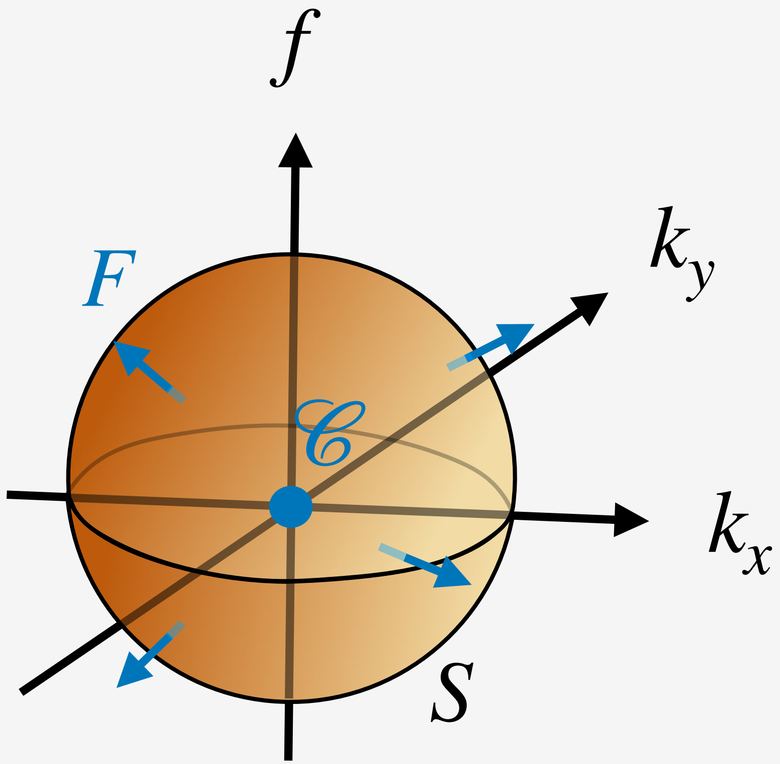


$$\mathscr{C}_{P+} = -2$$

$$\mathscr{C}_{geost.} = 0$$

Topology

from degeneracy points

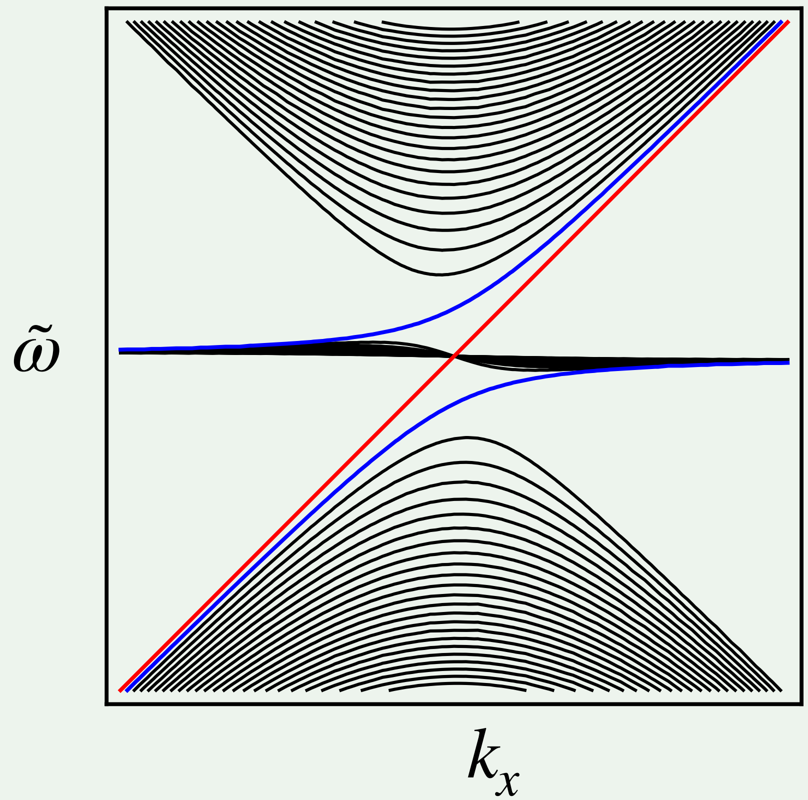
$$\mathscr{C}_{P-} = +2$$

$f(y)$ changes sign

$$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_y, y)$$

(Math) see F. Faure arxiv:1901.10592

Topological spectral flow



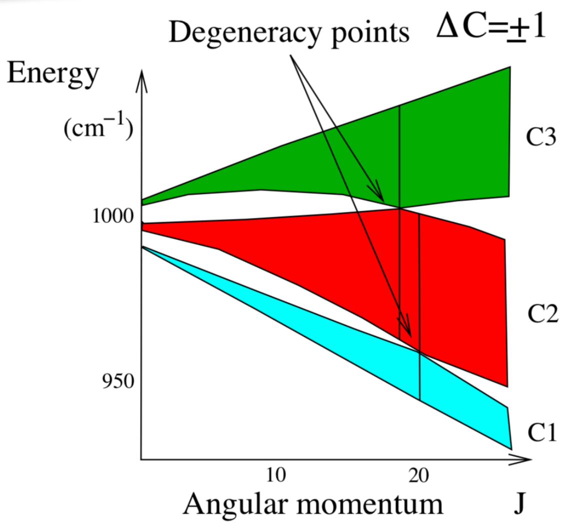
Uni-directional

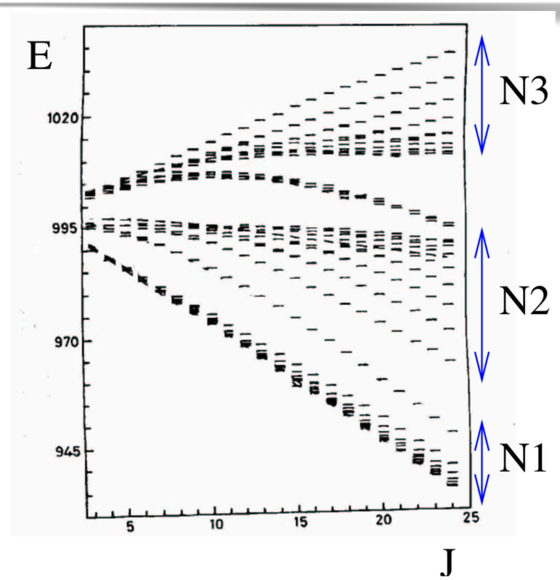
confined waves

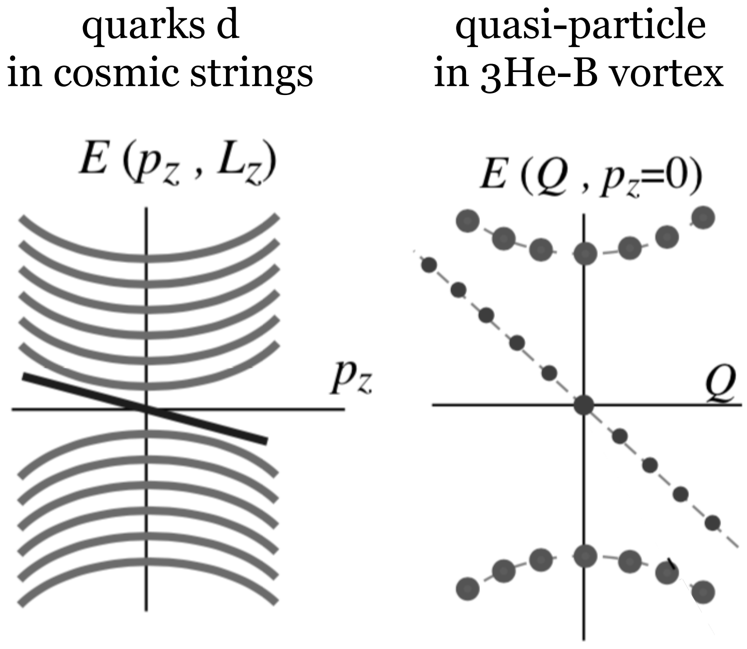
VOLUME 85, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JULY 2000

Topological Chern Indices in Molecular Spectra

F. Faure[1,*] and B. Zhilinskii[2,†]





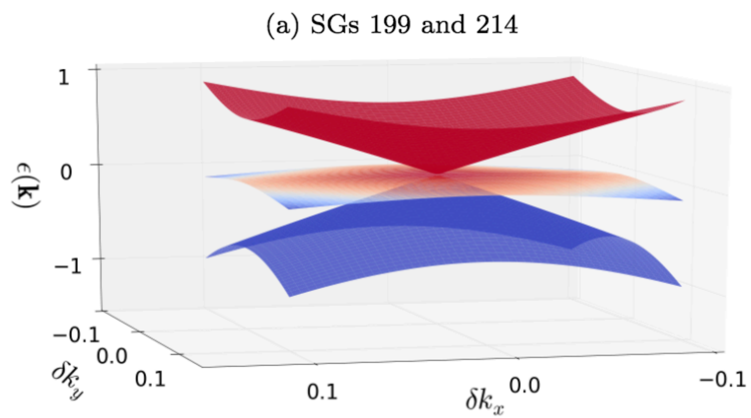


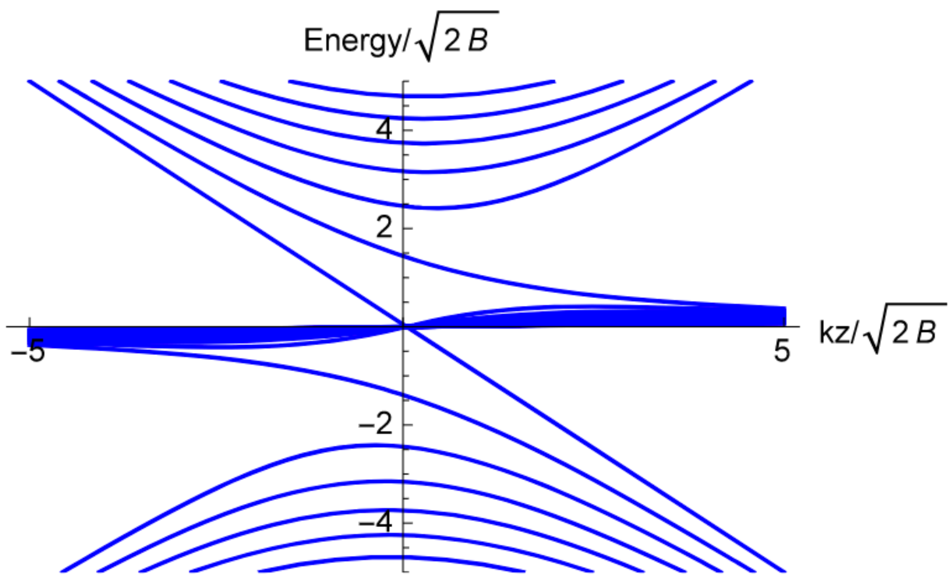
The Universe in Helium droplet,
Volovik (2003)

RESEARCH ARTICLE

Beyond Dirac and Weyl fermions: Unconventional quasiparticles in conventional crystals

Barry Bradlyn[1,*], Jennifer Cano[1,*], Zhijun Wang[2,*], M. G. Vergniory[3], C. Felser[4], R. J. Cava[5], B. Andrei Bernevig[2,†]

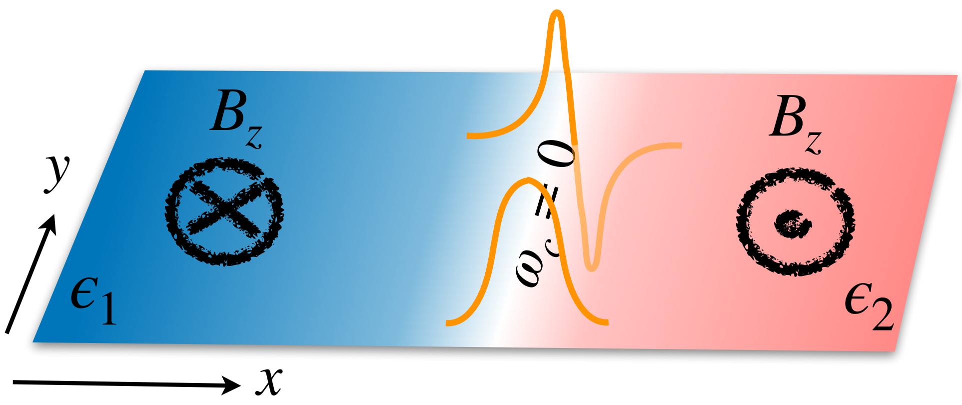


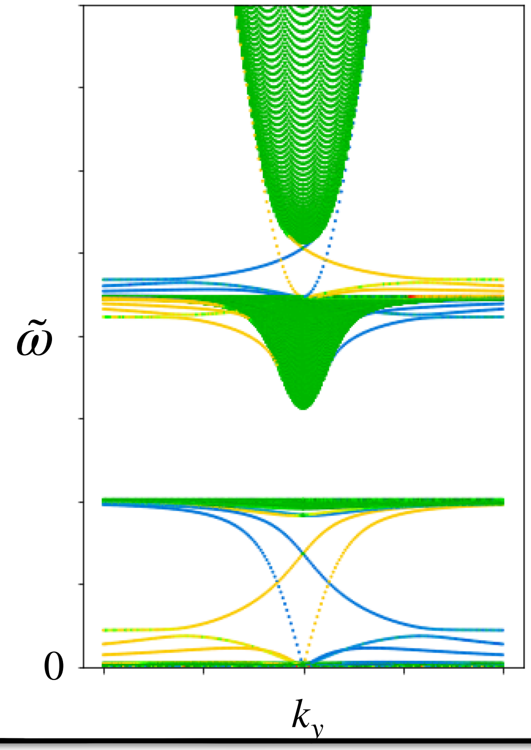


Chiral Maxwell waves in continuous media from Berry monopoles

Marco Marciani, Pierre Delplace

(Submitted on 21 Jun 2019 (v1), last revised 19 Jul 2019 (this version, v2))



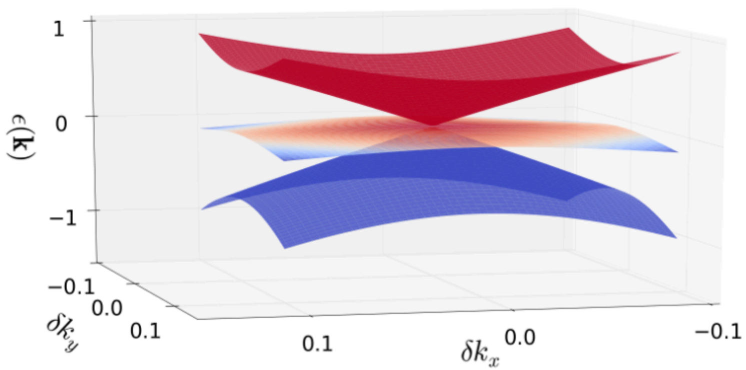


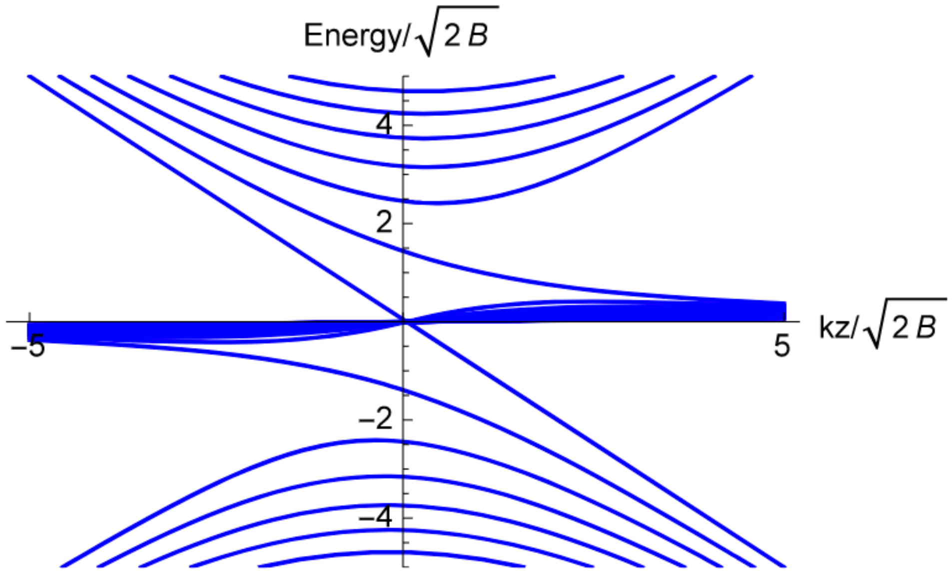
RESEARCH ARTICLE

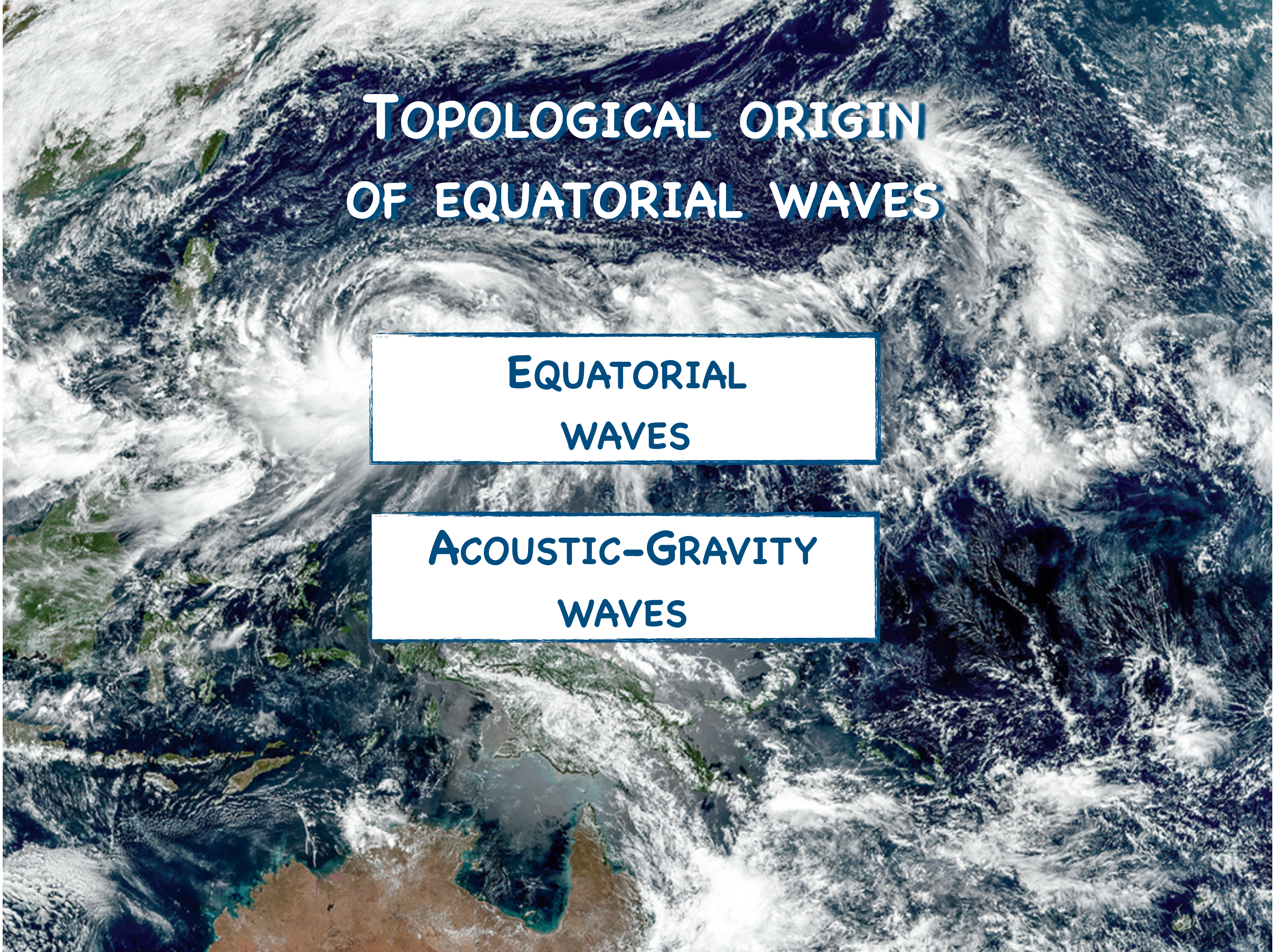
Beyond Dirac and Weyl fermions: Unconventional quasiparticles in conventional crystals

Barry Bradlyn[1,*], Jennifer Cano[1,*], Zhijun Wang[2,*], M. G. Vergniory[3], C. Felser[4], R. J. Cava[5], B. Andrei Bernevig[2,†]

(a) SGs 199 and 214







Topological origin of equatorial waves

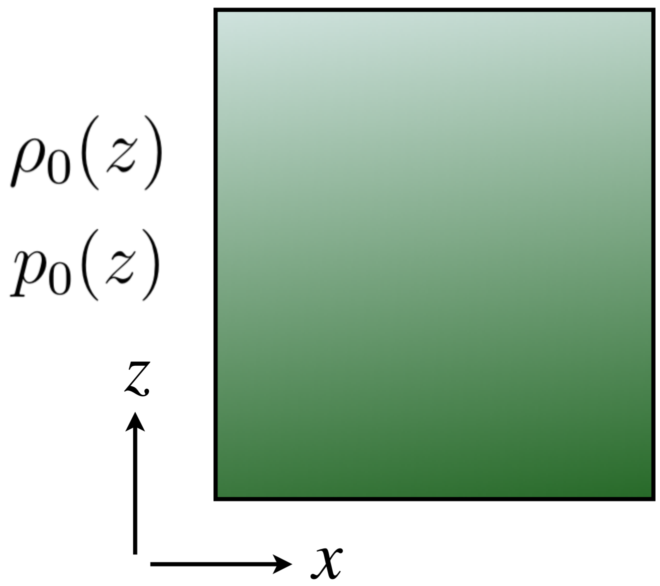
Equatorial waves

Acoustic-Gravity waves

Acoustic-Gravity Waves

✓ Compressible

✓ Stratified in density



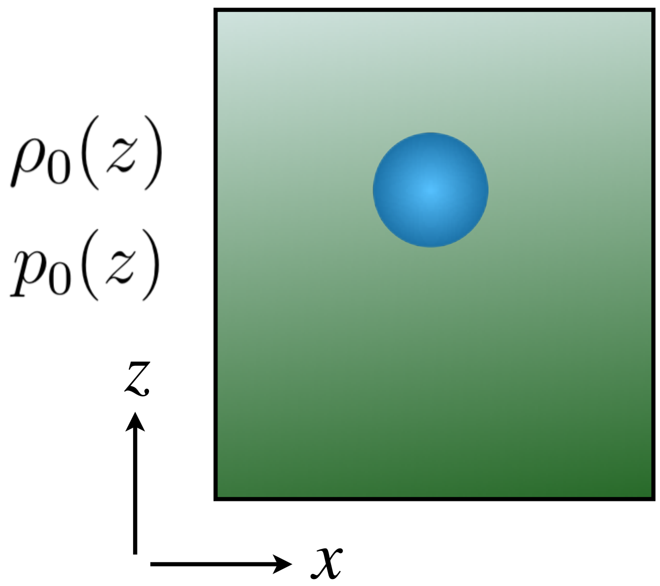
Acoustic-Gravity Waves

✓ Compressible

✓ Stratified in density

$$N = \sqrt{-g\frac{\partial_z \rho_0}{\rho_0} - \frac{g^2}{c_s^2}}$$

Buoyancy frequency *(~ 10 mHz)*



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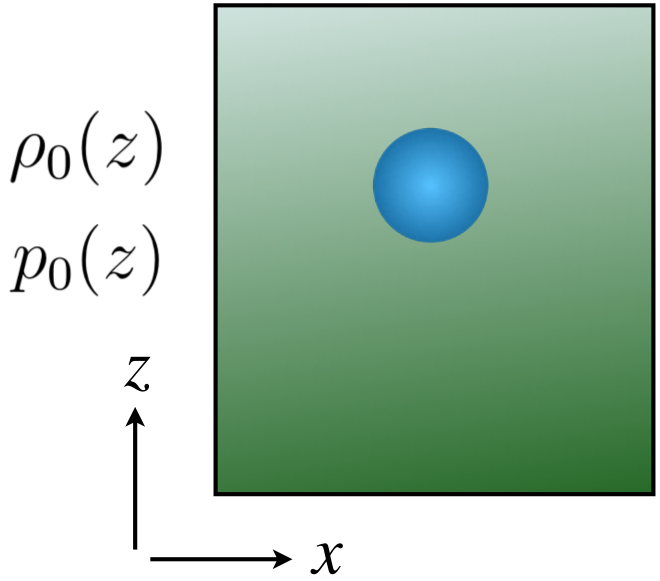


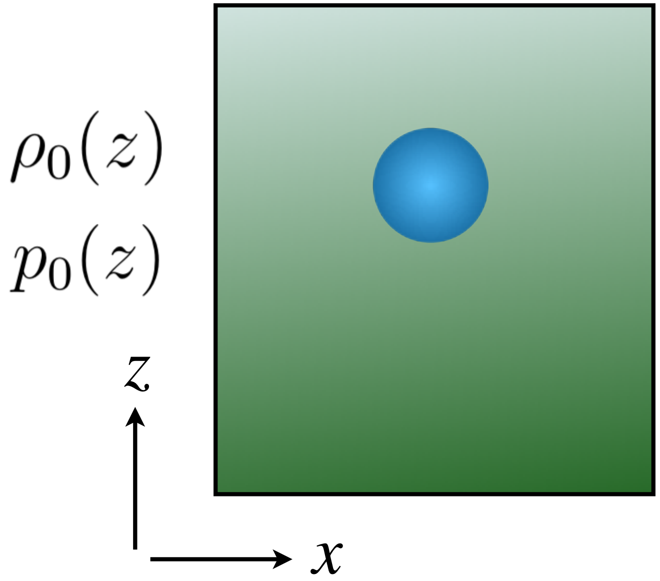
Mass conservation	$\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0$
Momenta conservation	$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\rho g \mathbf{e}_z - \nabla p$
Entropy conservation	$\mathrm{d}s = 0$

Acoustic-Gravity Waves

- ✓ Compressible
- ✓ Stratified in density

$$N = \sqrt{-g\frac{\partial_z \rho_0}{\rho_0} - \frac{g^2}{c_s^2}}$$

Buoyancy frequency *(~ 10 mHz)*



$$H = \begin{pmatrix} 0 & 0 & 0 & i\partial_x \\ 0 & 0 & iN(z) & -iS(z) + i\partial_z \\ 0 & -iN(z) & 0 & 0 \\ i\partial_x & iS(z) + i\partial_z & 0 & 0 \end{pmatrix}$$

$$\omega \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix} = \mathscr{H} \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix}$$

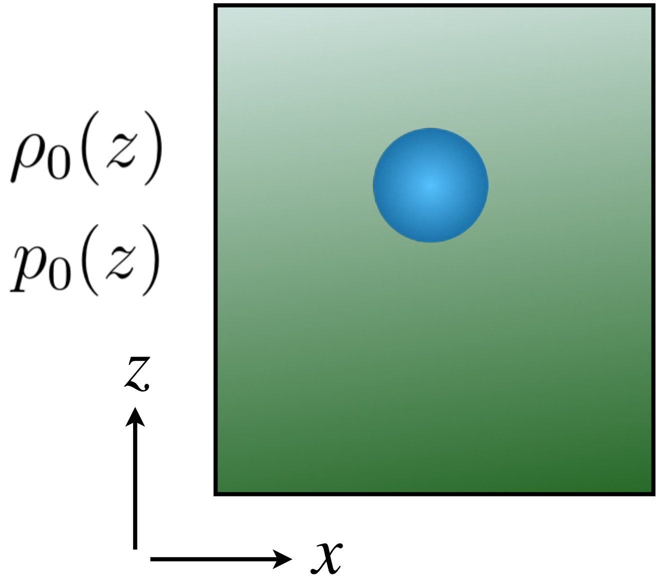
Acoustic-Gravity Waves

✓ Compressible

✓ Stratified in density

$$N = \sqrt{-g\frac{\partial_z \rho_0}{\rho_0} - \frac{g^2}{c_s^2}}$$

Buoyancy frequency (~ 10 mHz)



$$S = \frac{1}{2}\left(\frac{N^2 c_s}{g} - \frac{g}{c_s}\right)$$

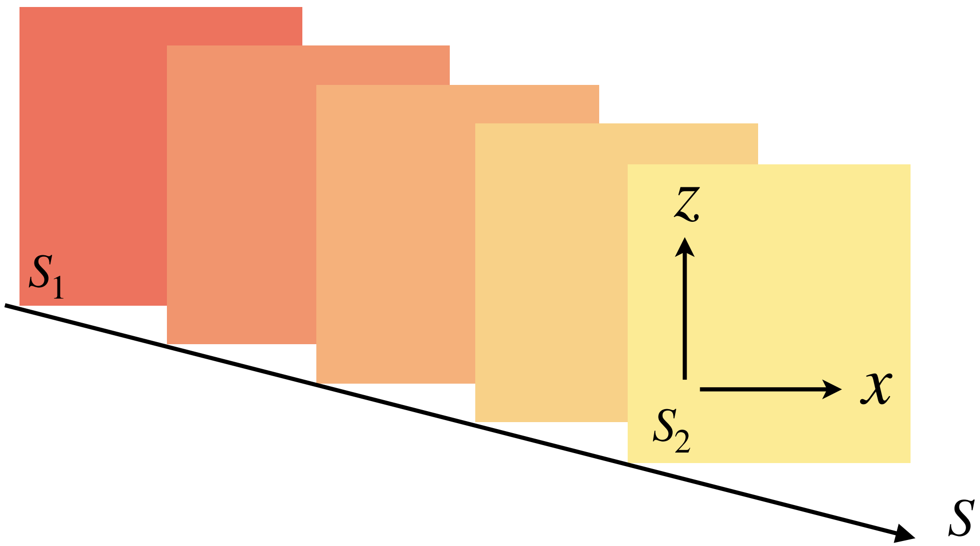
✓ Breaks vertical mirror symmetry

$$H = \begin{pmatrix} 0 & 0 & 0 & i\partial_x \\ 0 & 0 & iN(z) & -iS(z) + i\partial_z \\ 0 & -iN(z) & 0 & 0 \\ i\partial_x & iS(z) + i\partial_z & 0 & 0 \end{pmatrix}$$

$$\omega \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix} = \mathscr{H} \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix}$$

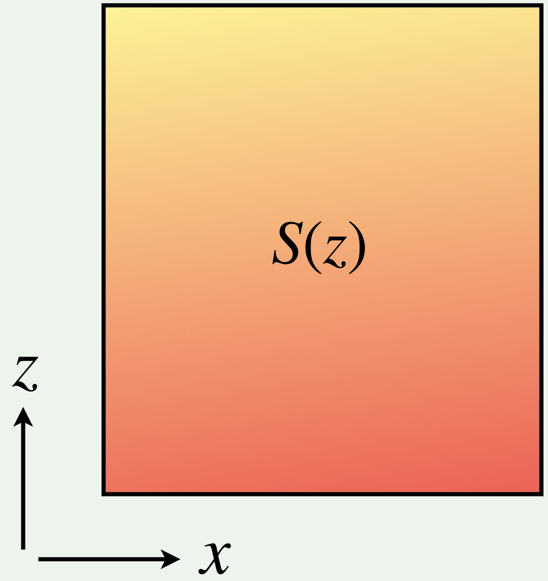
S is a parameter

$$\mathscr{H} = H(k_x, k_z, S)$$



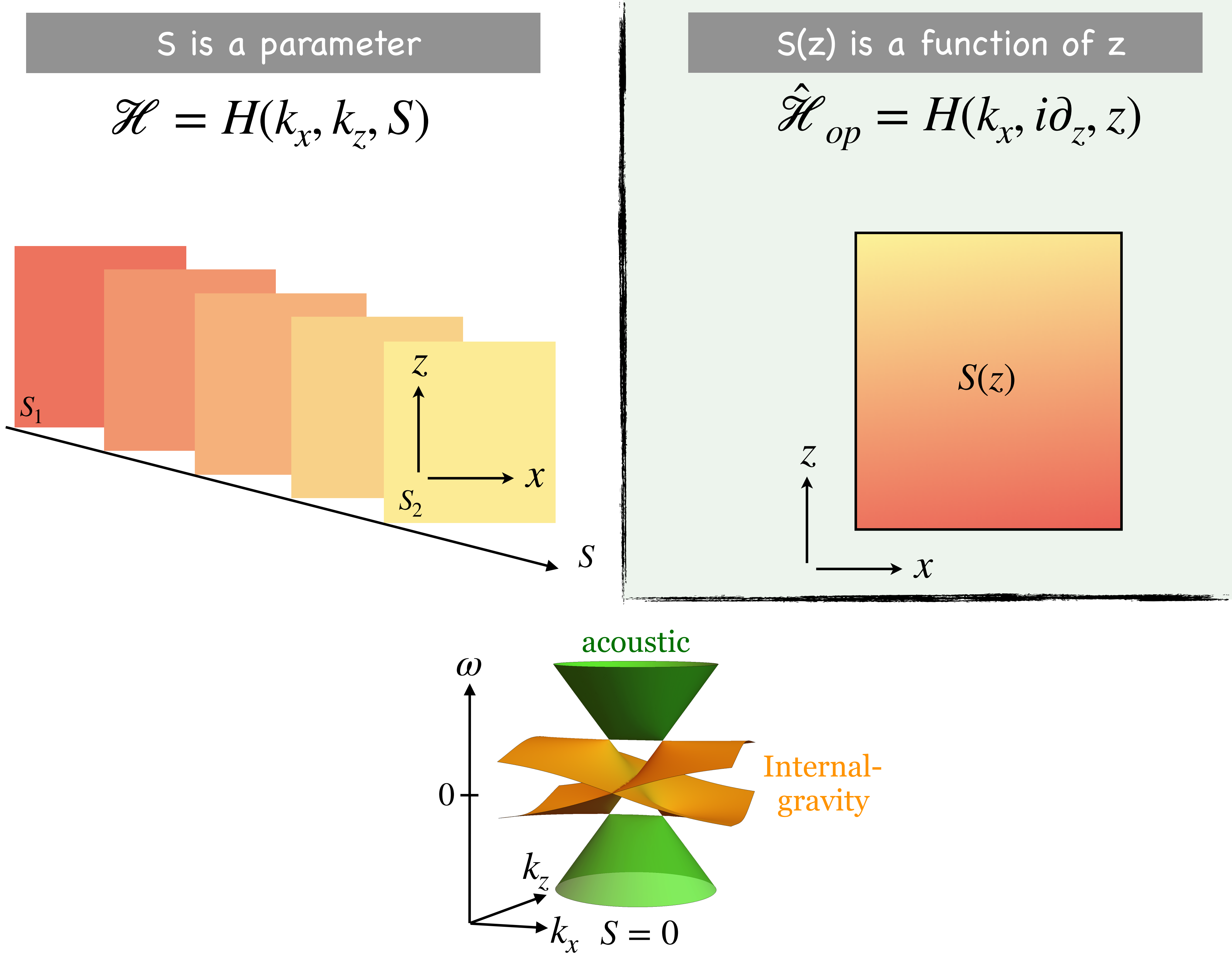
S(z) is a function of z

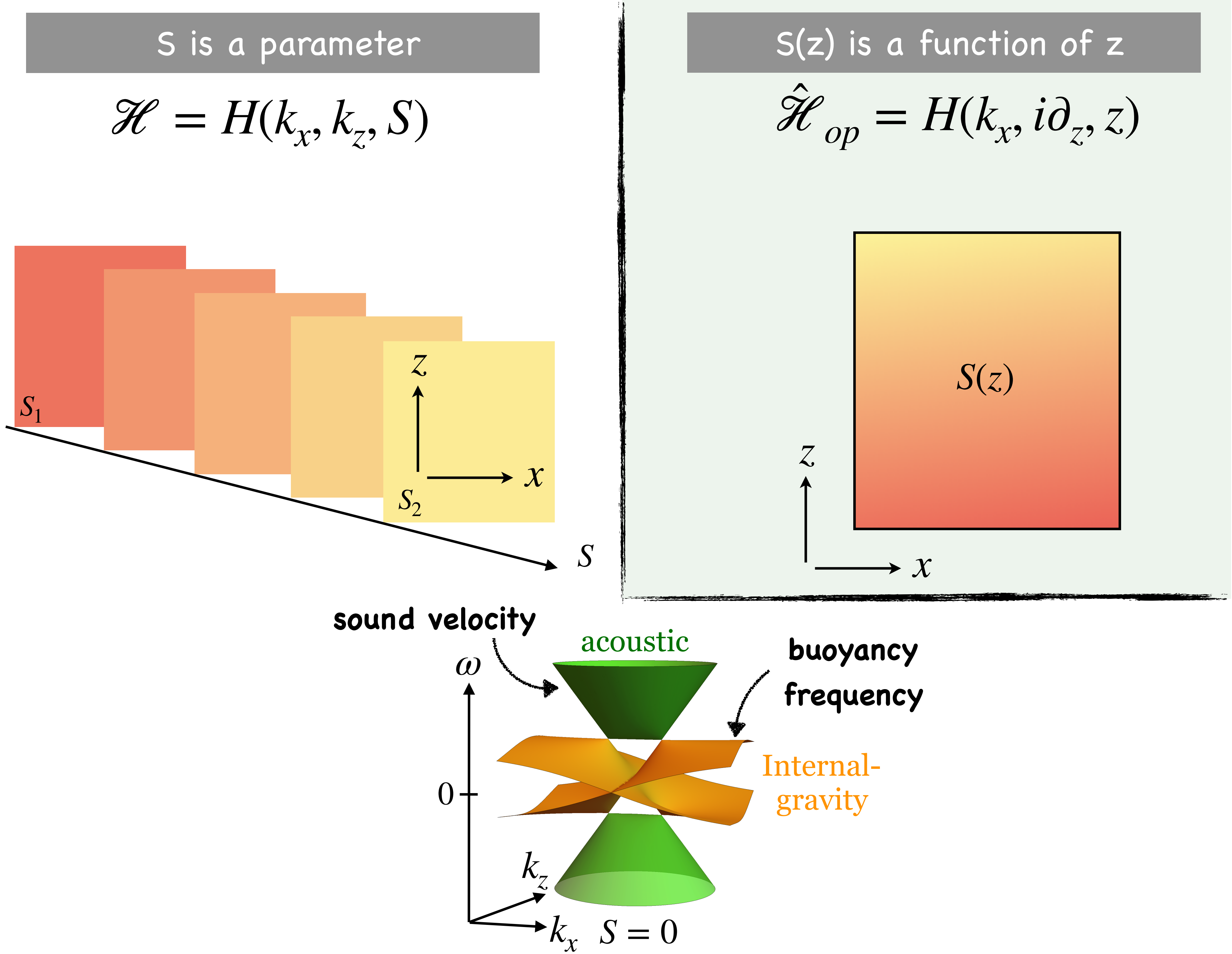
$$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$$

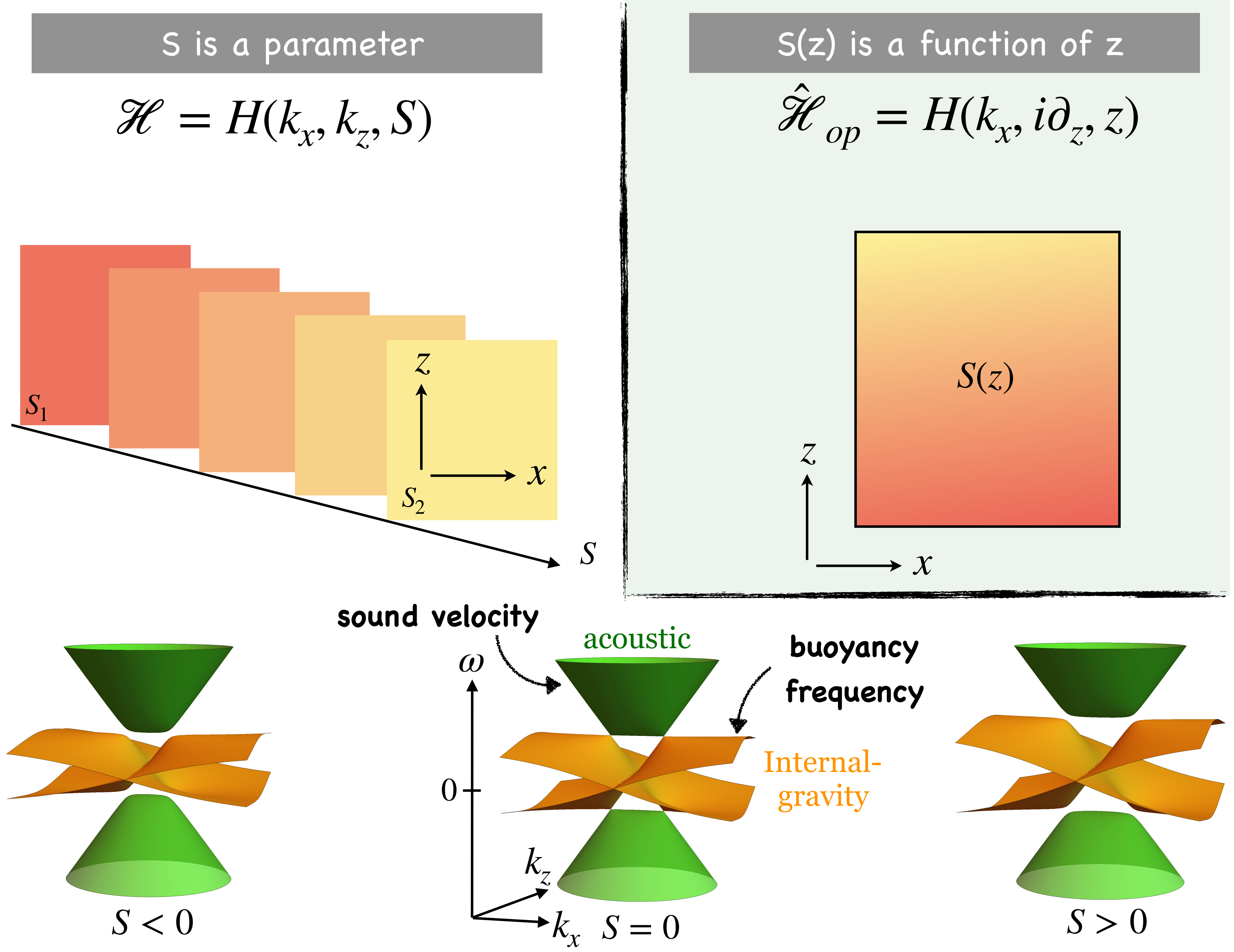


$$H = \begin{pmatrix} 0 & 0 & 0 & i\partial_x \\ 0 & 0 & iN(z) & -iS(z) + i\partial_z \\ 0 & -iN(z) & 0 & 0 \\ i\partial_x & iS(z) + i\partial_z & 0 & 0 \end{pmatrix}$$

$$\omega \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix} = \mathscr{H} \begin{pmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\rho} \\ \tilde{p} \end{pmatrix}$$


S is a parameter
$\mathscr{H} = H(k_x, k_z, S)$
S_1
z
x
S_2
S
S(z) is a function of z
$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$
$S(z)$
z
x
acoustic
ω
0
Internal-
gravity
k_z
k_x
$S = 0$


S is a parameter
$\mathscr{H} = H(k_x, k_z, S)$
S_1
S_2
z
x
S
S(z) is a function of z
$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$
$S(z)$
z
x
sound velocity
acoustic
buoyancy frequency
Internal-gravity
ω
0
k_z
k_x
$S = 0$



S is a parameter
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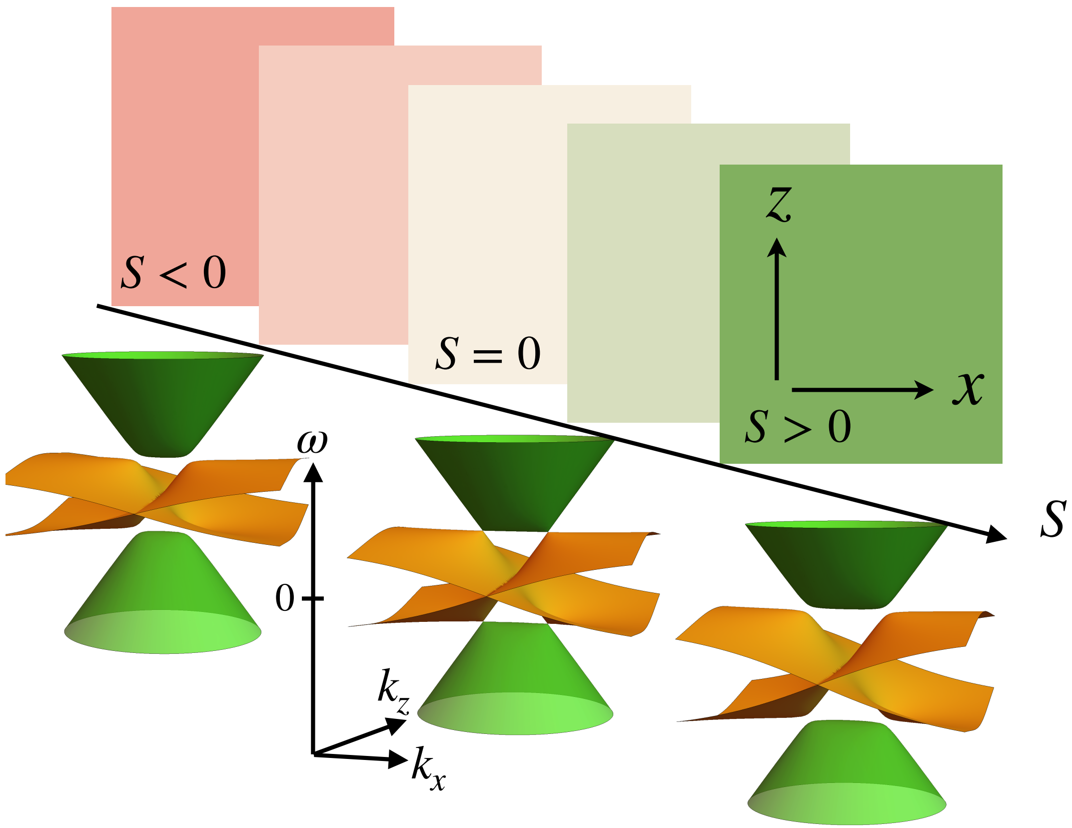
S_1
z
x
S_2
S
S(z) is a function of z
$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$
$S(z)$
z
x
sound velocity
acoustic
buoyancy
frequency
Internal-
gravity
ω
0
k_z
k_x
$S < 0$
$S = 0$
$S > 0$

S is a parameter

$$\mathscr{H} = H(k_x, k_z, S)$$



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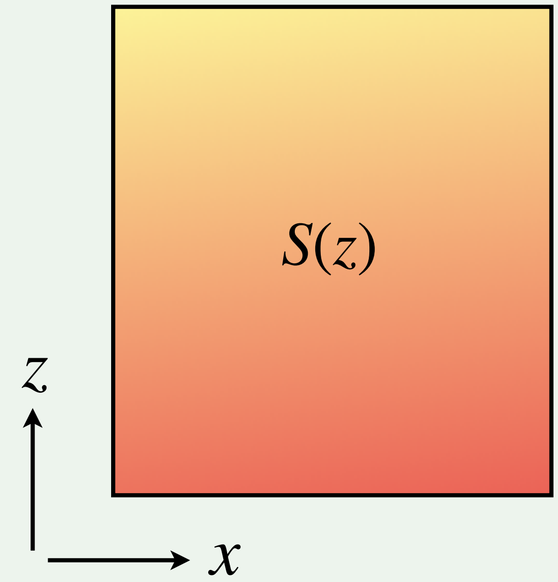
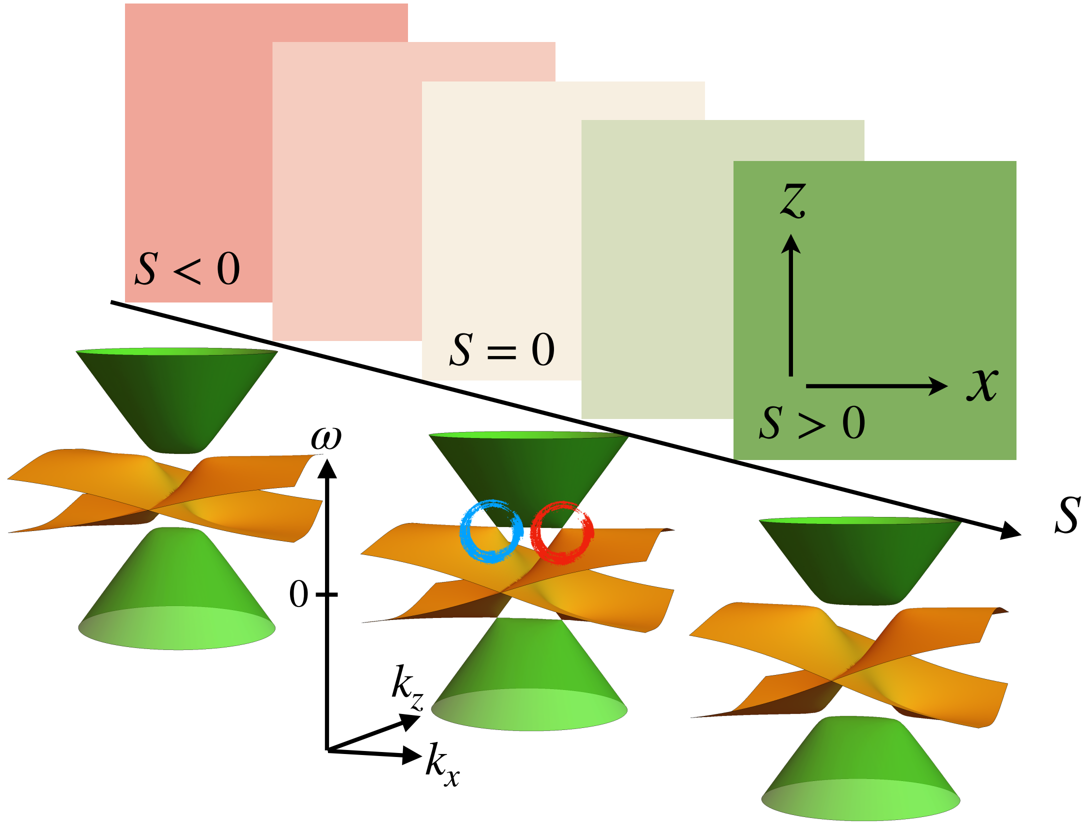
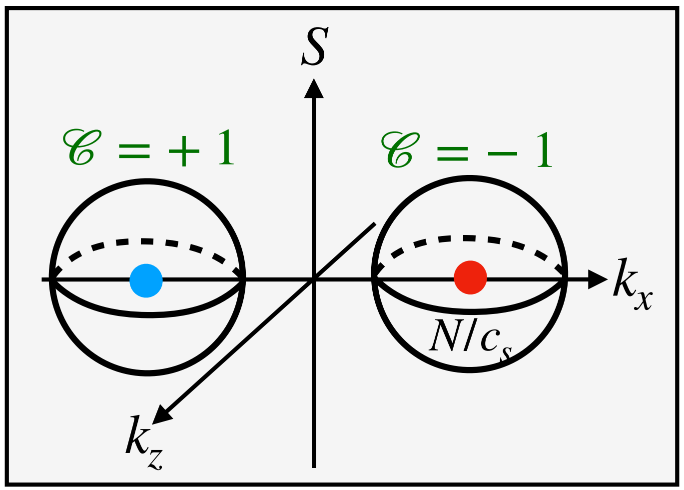


S is a parameter

$$\mathscr{H} = H(k_x, k_z, S)$$





S(z) is a function of z

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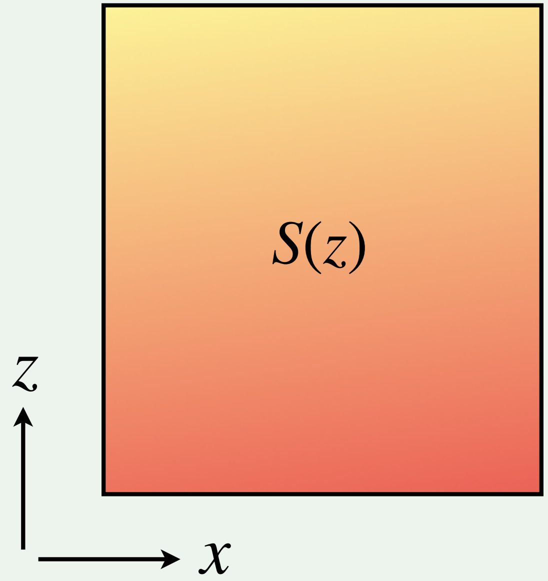
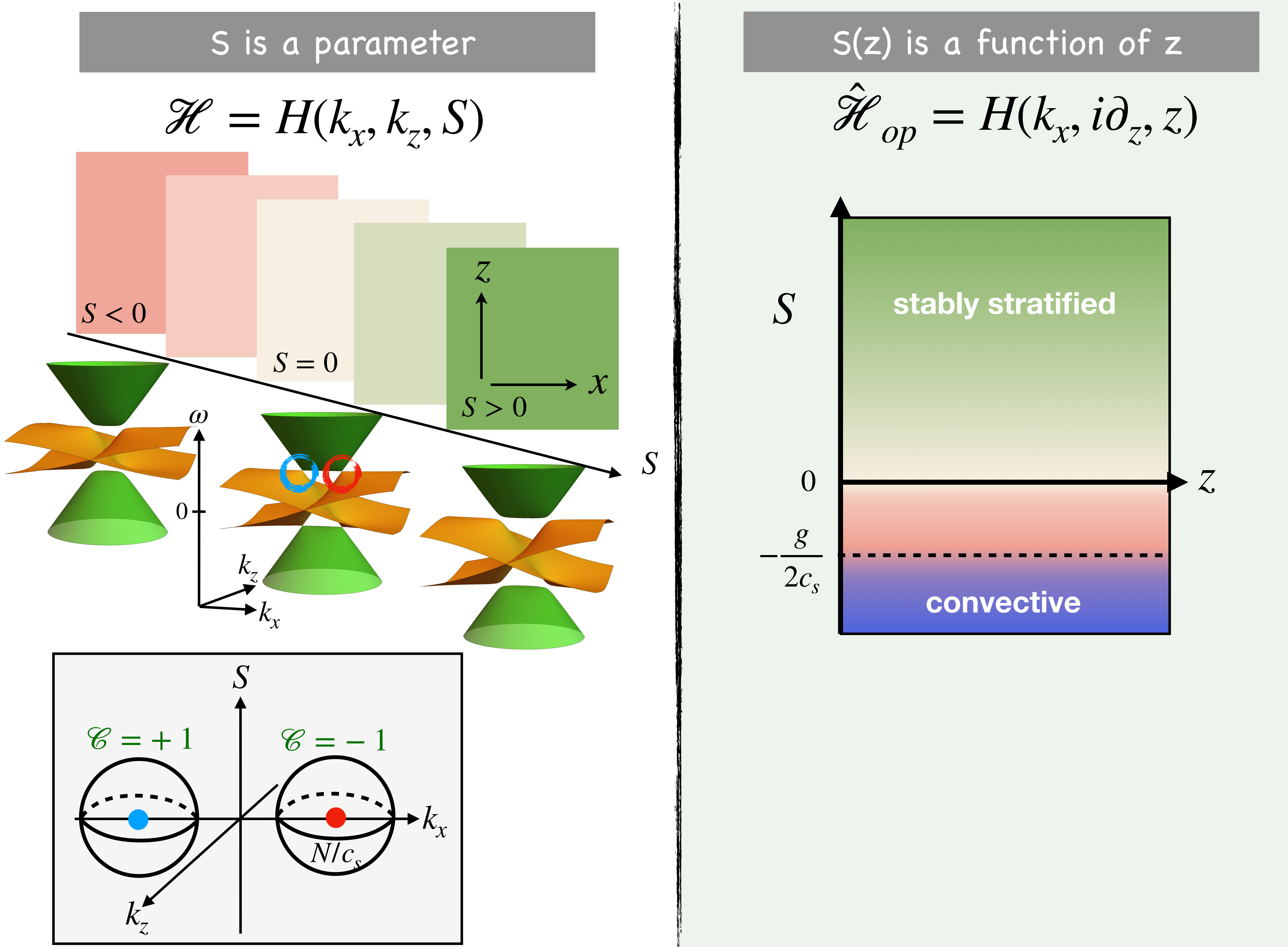
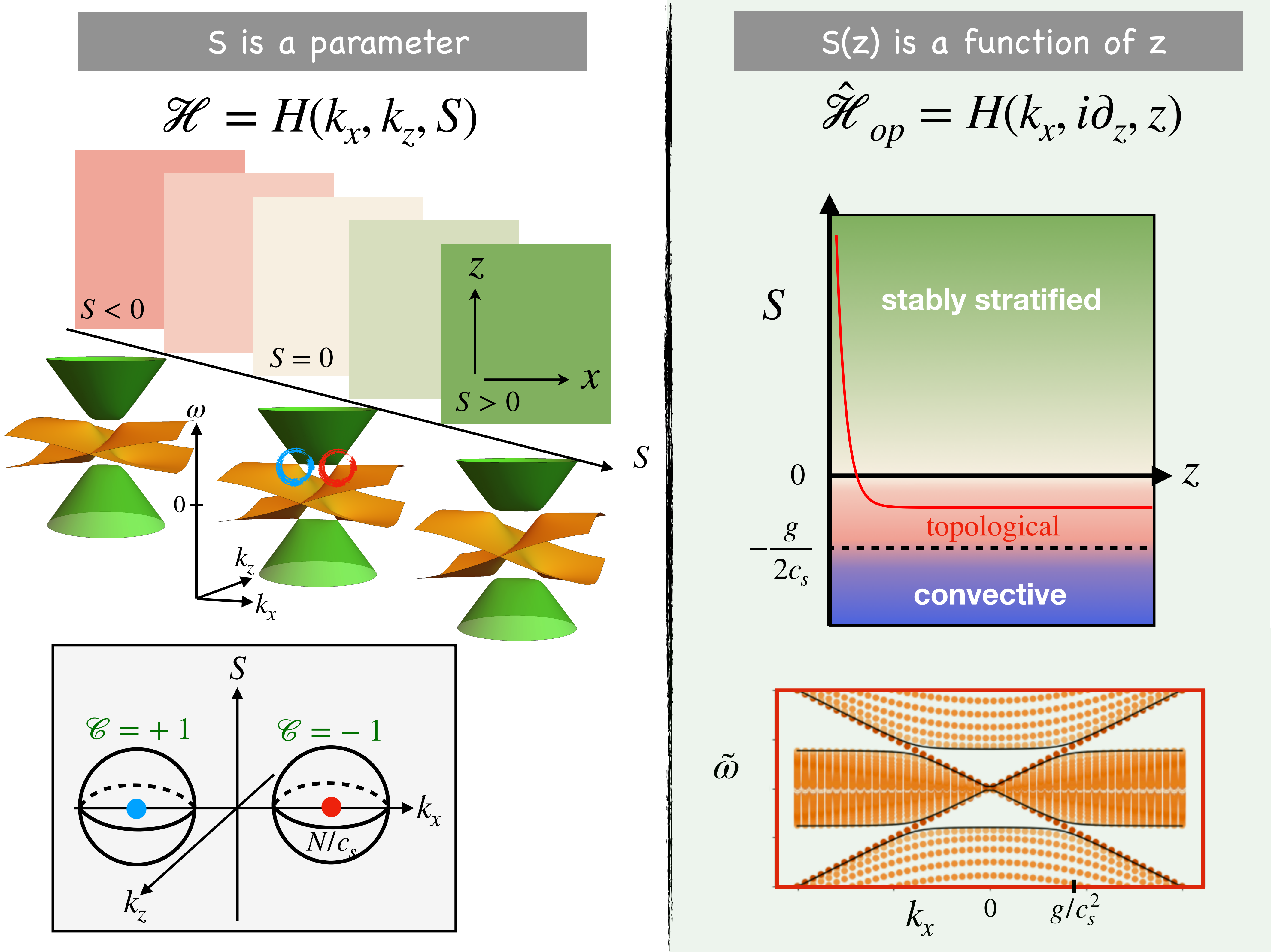
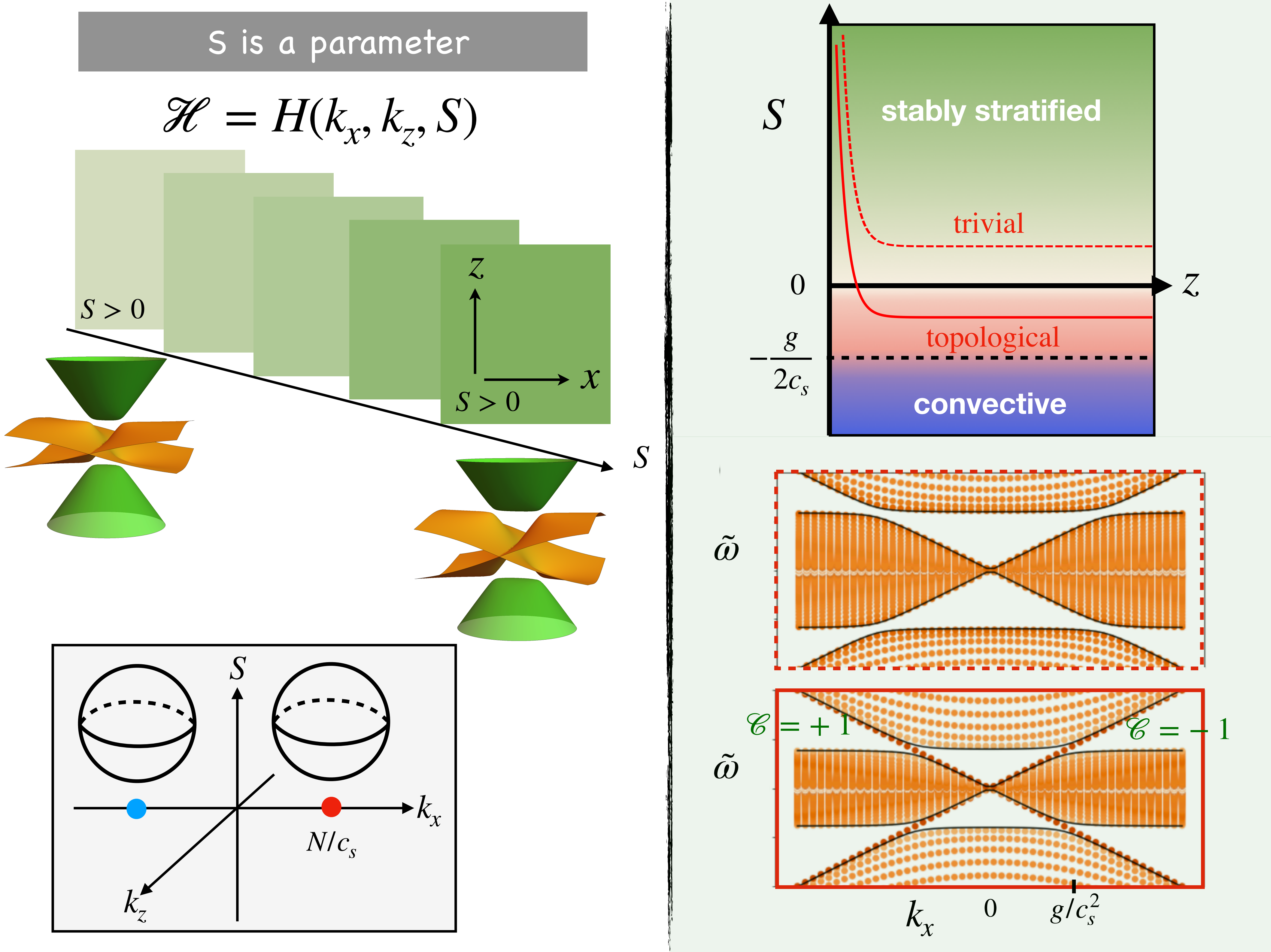


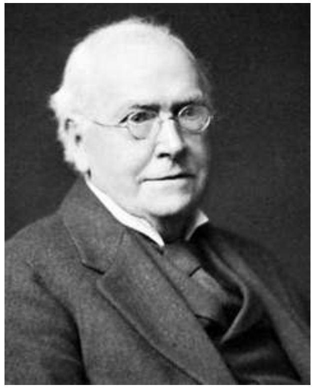


S is a parameter
$\mathscr{H} = H(k_x, k_z, S)$
$S < 0$
$S = 0$
$S > 0$
z
x
ω
0
k_z
k_x
S
$\mathscr{C} = +1$
$\mathscr{C} = -1$
N/c_s
S(z) is a function of z
$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$
S
stably stratified
0
z
$-\frac{g}{2c_s}$
convective

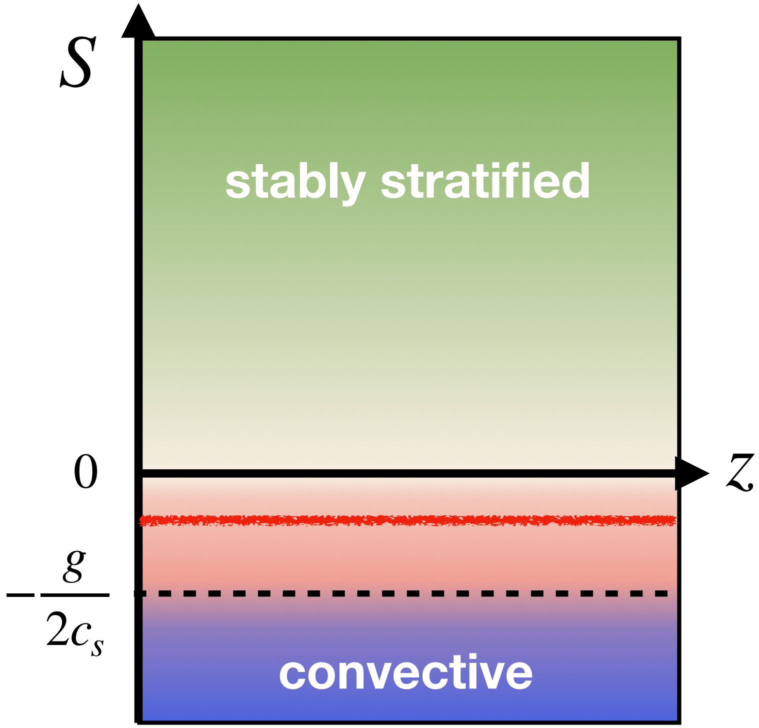

S is a parameter
$\mathscr{H} = H(k_x, k_z, S)$
$S < 0$
$S = 0$
$S > 0$
z
x
S
ω
0
k_z
k_x
S
$\mathscr{C} = +1$
$\mathscr{C} = -1$
k_x
N/c_s
k_z
S(z) is a function of z
$\hat{\mathscr{H}}_{op} = H(k_x, i\partial_z, z)$
S
stably stratified
0
z
topological
$-\frac{g}{2c_s}$
convective
$\tilde{\omega}$
k_x
0
g/c_s^2



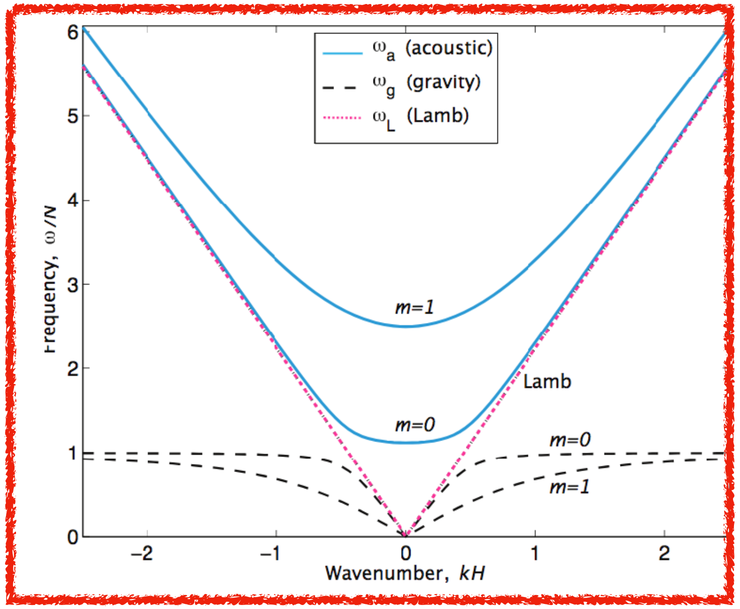
S is a parameter
$\mathscr{H} = H(k_x, k_z, S)$
$S > 0$
z
x
$S > 0$
S
S
k_x
N/c_s
k_z
S
stably stratified
trivial
0
z
topological
$-\frac{g}{2c_s}$
convective
$\tilde{\omega}$
$\tilde{\omega}$
$\mathscr{C} = +1$
$\mathscr{C} = -1$
k_x
0
g/c_s^2



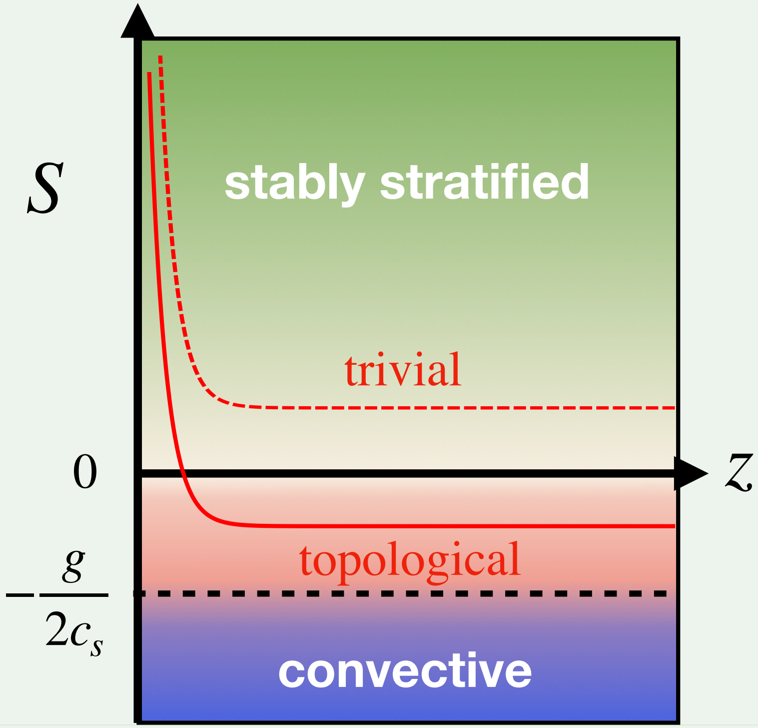
Horace LAMB (1911)

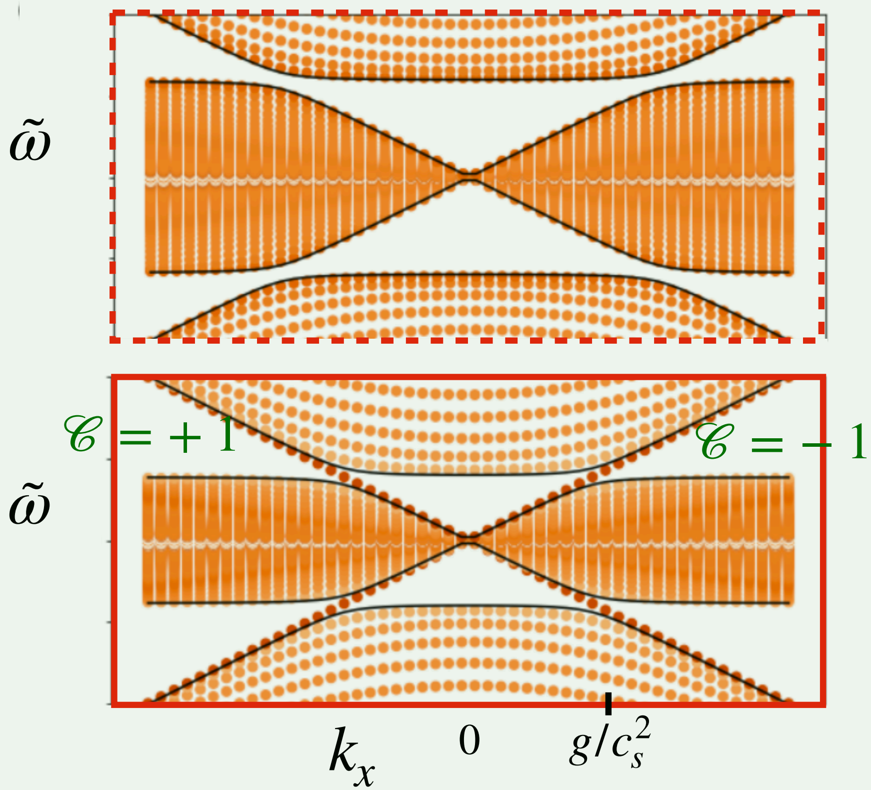


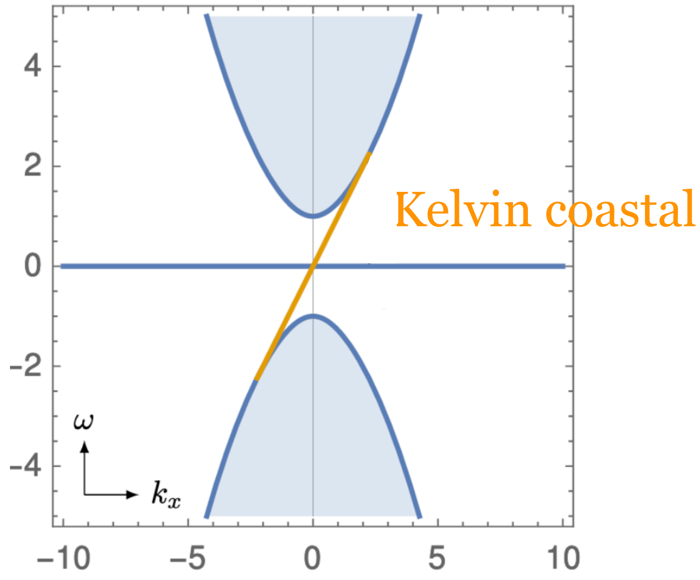
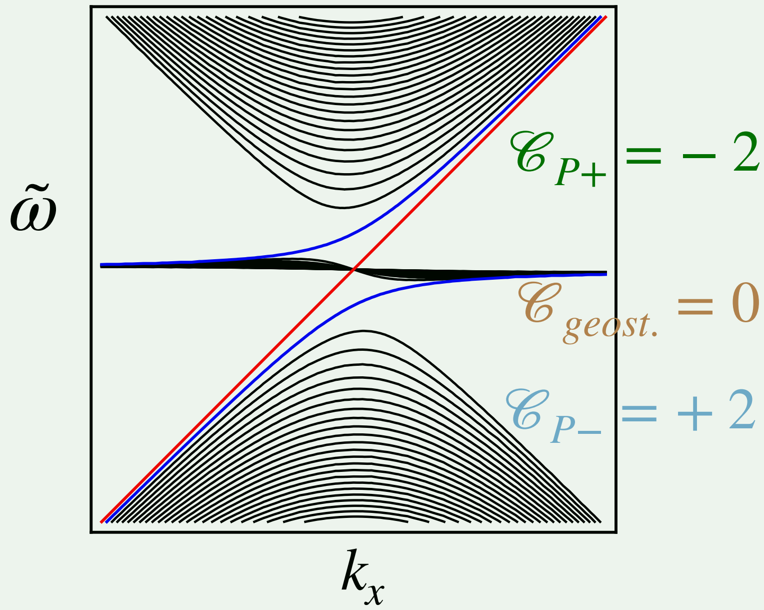
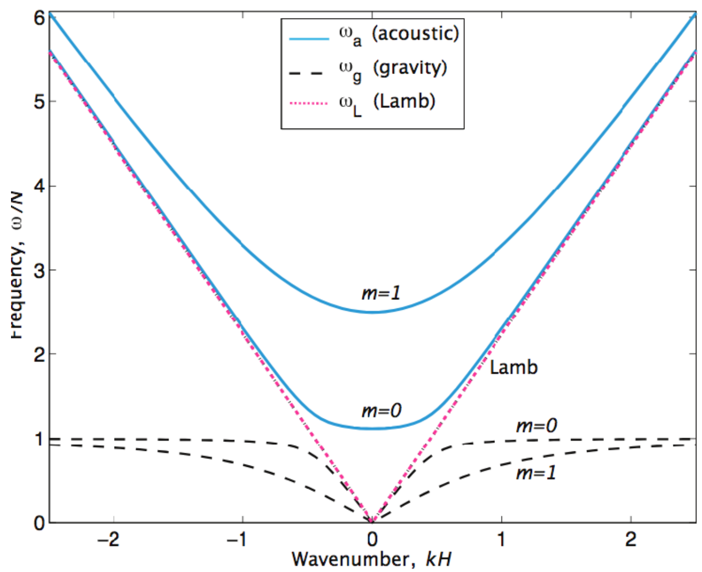
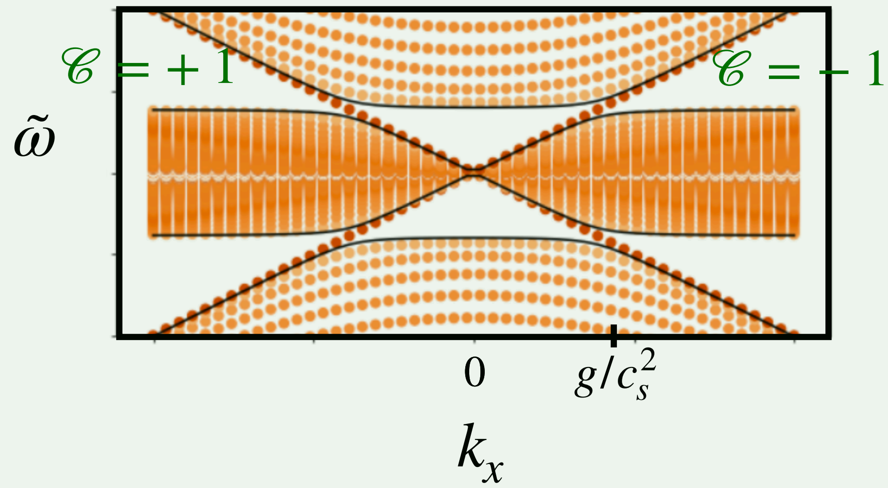
+ solid boundary



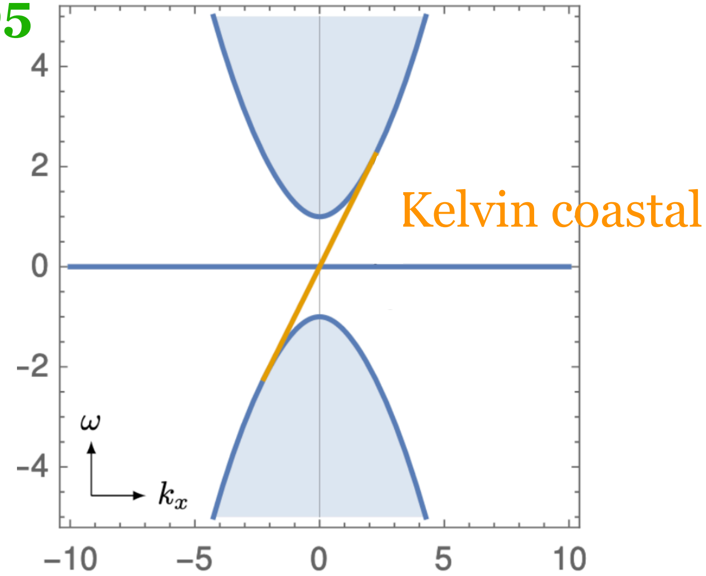
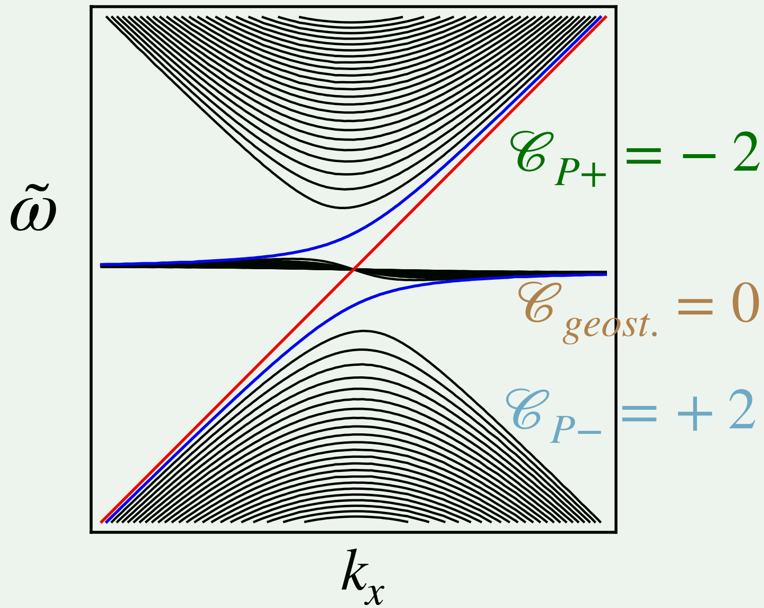
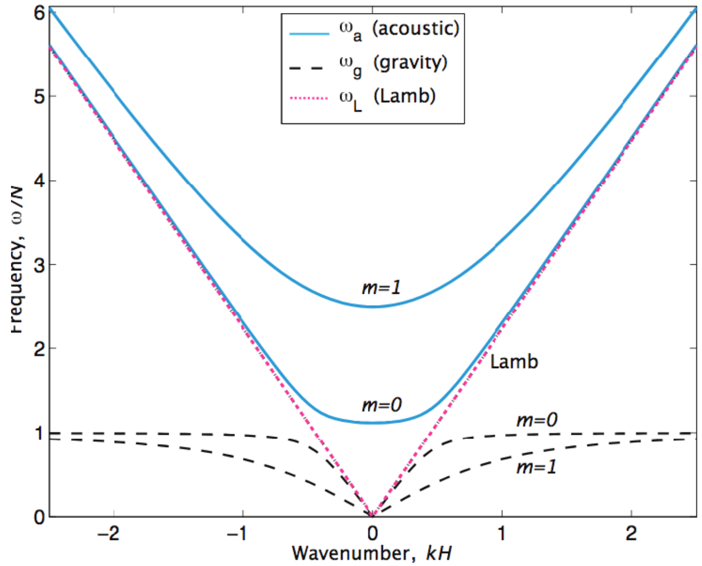
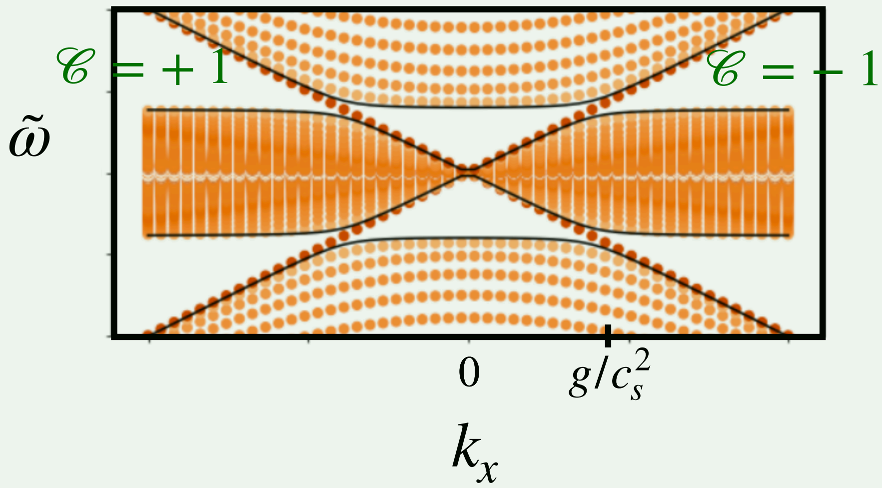
- from Atmospheric and oceanic fluid dynamics (G. K. Vallis)





	Boundary conditions (impermeability)	Interface/anysotropy - topological spectral flow
Shallow Water		
Compressible & stratified		

- from Atmospheric and oceanic fluid dynamics (G. K. Vallis)

	Boundary conditions (impermeability)	**Interface/anysotropy - topological spectral flow**
Shallow Water	**Iga '95** 	
Compressible & stratified	**Iga '2001** 	

- from Atmospheric and oceanic fluid dynamics (G. K. Vallis)

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Topological origin of equatorial waves

Pierre Delplace, J. B. Martson and Antoine Venaille

Science 358, 1075 (2017)

Topological transition in stratified atmospheres

Manolis Perrot, Pierre Delplace and Antoine Venaille

Nat. Phys. (2019)

Chiral Maxwell waves in continuous media from Berry monopoles

Marco Marciani and Pierre Delplace

arXiv:1906.09057

Brad Marston
Brown University

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