Some differences between extratropical and equatorial eddy-driven jets...

… or how to spinup a jet without a vorticity source

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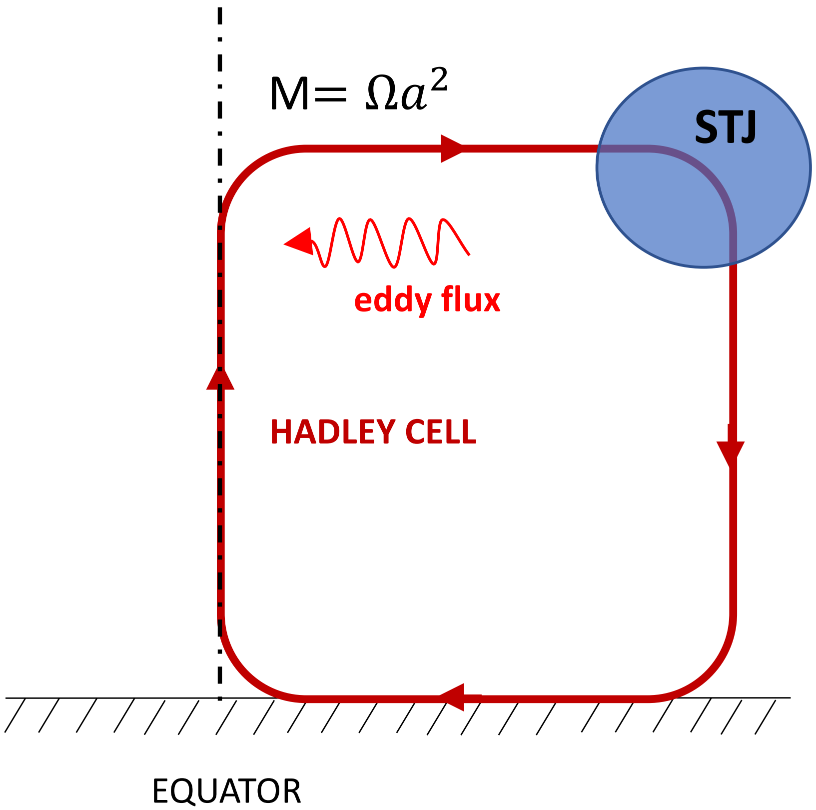
Physics at the equator: from the lab to the stars. ENS Lyon, October 17th

SUPERROTATION

- Faster rotation than the equatorial surface
- Angular momentum exceeds Ωa^2 somewhere
- Equatorial westerlies

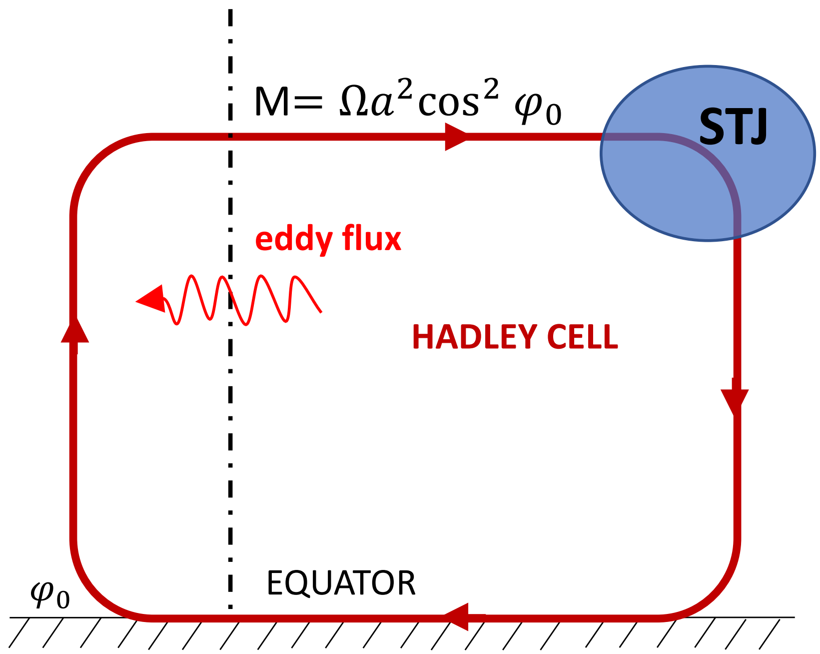


Hide's theorem

Without friction, a *symmetric* circulation conserves angular momentum $\mathrm{M} = \Omega a^2$ (with friction, it can only lose it)

Superrotation requires ***eddy momentum forcing***

Superrotating jets **must** be eddy-driven



Why Earth does not superrotate?

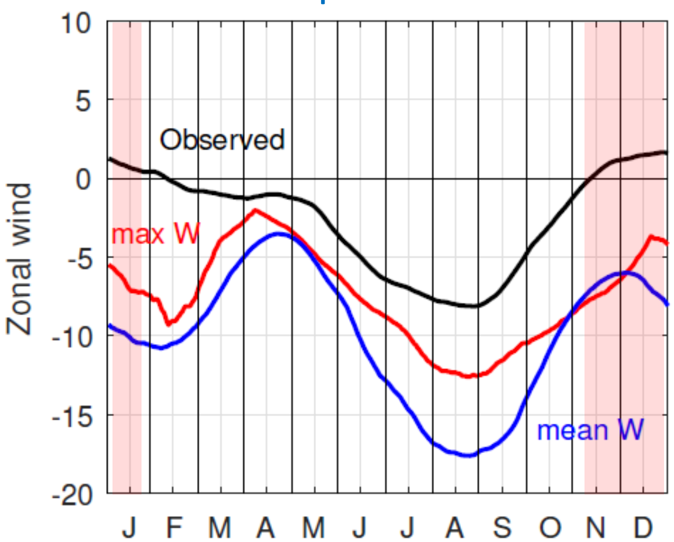
With non-zero obliquity, a *symmetric* circulation would conserve the ITCZ angular momentum: $M = \Omega a^2 \cos^2 \varphi_0$

Westerly eddy forcing increases M on Earth, but not enough

$$\Omega a^2 \cos^2 \varphi_0 < M \lesssim \Omega a^2$$

Weak superrotation on Earth (faster rotation than the surface at the ITCZ)

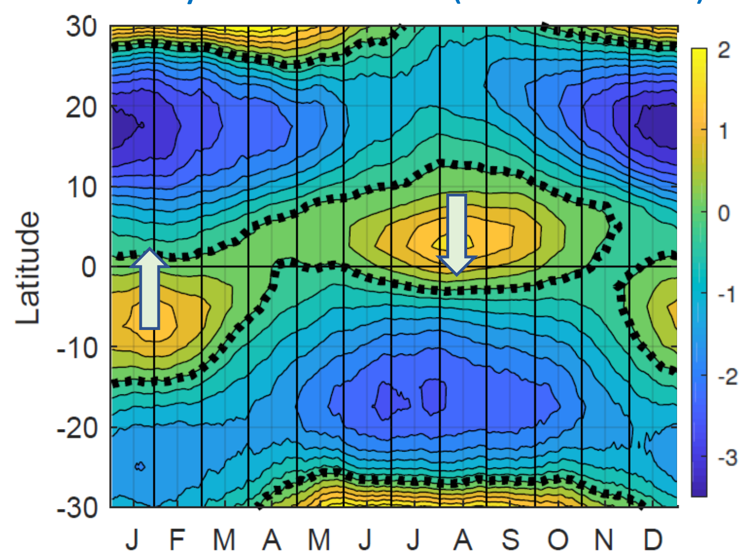
Mean equatorial wind



Impact of eddy forcing

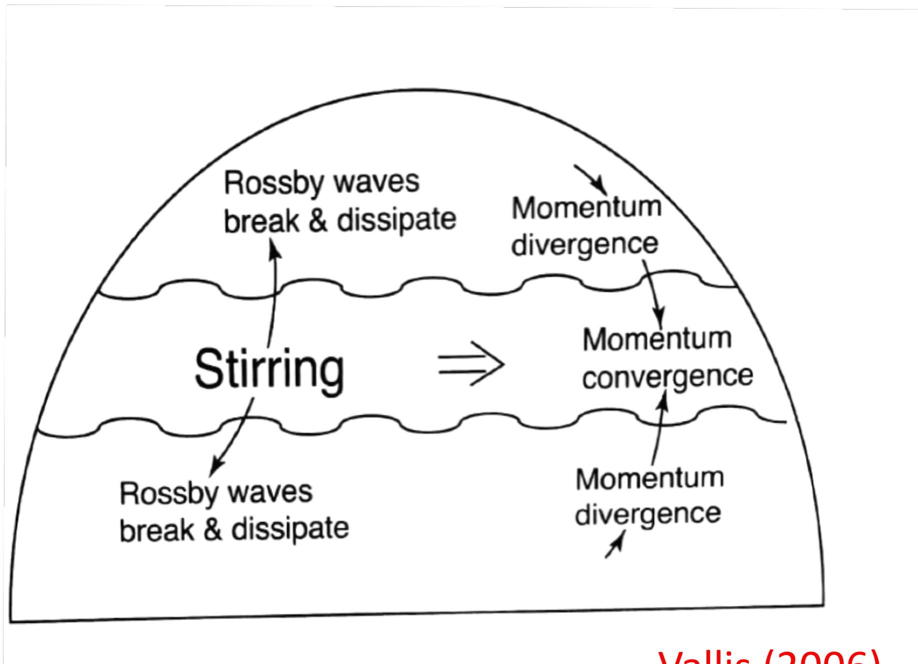
constant-M predictions $M = \Omega a^2 \cos^2 \varphi_0$

Eddy acceleration (300-150 hPa)



Superrotating jets **must** be eddy-driven

Large body of theory on eddy-driven jets (e.g., the terrestrial extratropical jet)



Vallis (2006)

$$\left.\frac{\partial \bar{u}}{\partial t}\right|_{eddy} \approx -\frac{\partial \overline{u'v'}}{\partial y} \approx \overline{v'\xi'} > 0$$

- Acceleration by ***upgradient*** vorticity fluxes (need a vorticity source)
- Mechanism: meridional Rossby wave propagation
- Dissipation (Andrews & McIntyre): breaking vorticity waves (mechanical)

Classical paradigm for superrotation based on these ideas (Held 1999).

Vorticity source due to eddy heating:

- SST structure -day/night contrast -MJO

Main problem with this idea (Showman and Polvani 2011):

Acceleration by ***upgradient*** vorticity fluxes (need a vorticity source)

VORTICITY SOURCE IS WEAK IN THE DEEP TROPICS

$$\mathcal{F}_{\xi} = -(f + \xi)Q'$$

Can superrotation occur without a vorticity source? What is the dynamics?

1. ***An intriguing limit: small/slowly rotating planets***

   Non-small thermal Ro, wide-tropics

   $\xi_a = \xi + f$ small over broad tropical region => expect $\overline{v'\xi'} \to 0$

   Prone to superrotation (Kelvin-Rossby instability)

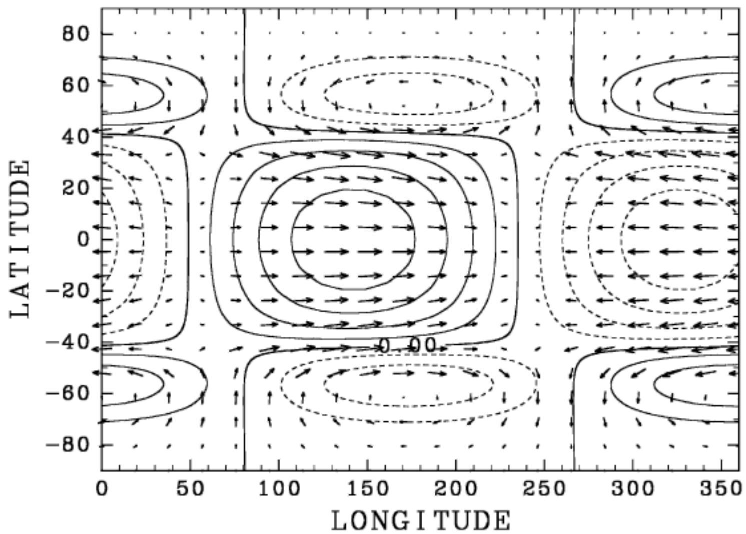
2. ***Rapidly rotating planets*** (Earth)

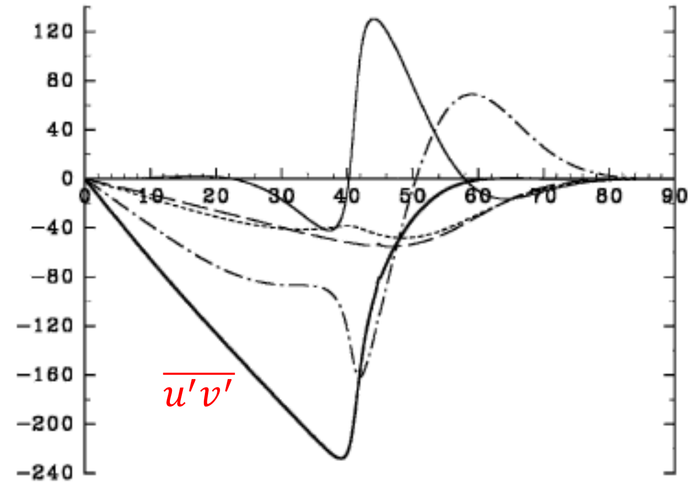
   ξ_a small near equator only $\rightarrow$ $\overline{v'\xi'}$ may not be small

   But with <u>weak vorticity source,</u> expect downgradient $\overline{v'\xi'} < 0$

Kelvin-Rossby instability and small-planet superrotation

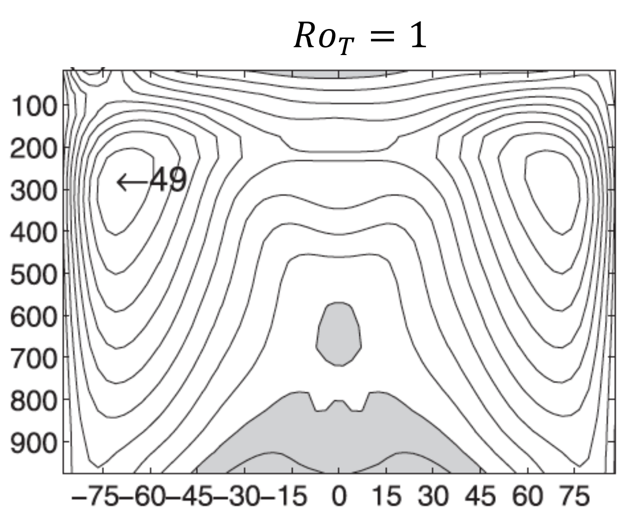
- First suggested by Iga and Matsuda (2005) as a possible driver for Venus superrotation





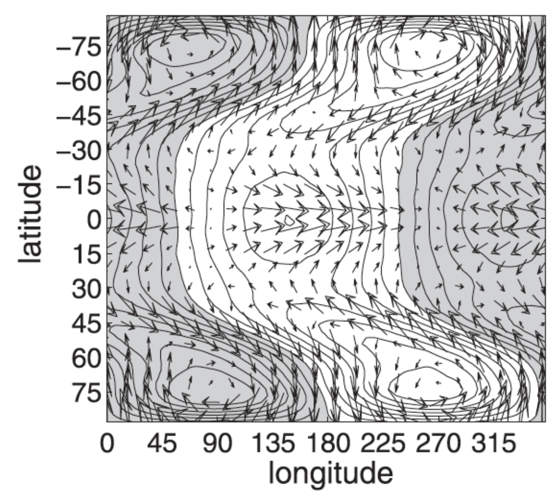
Iga and Matsuda (2005)

- Mitchell and Vallis (2010) and Potter et al (2014) find spontaneous transition to superrotation at ***large thermal Rossby number*** in idealized dry GCM



Acceleration imparted by mode with K-R structure

Potter et al (2014)



Equilibration of Kelvin-Rossby instability: shallow water

Barotropic but divergent

Small absolute vorticity $\xi_a = \xi + f \ll f$

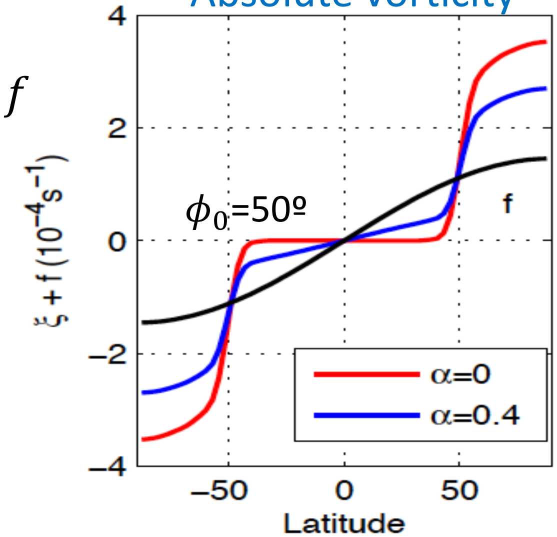
Phase-locking requires Fr ~ O(1)

$c_K \sim \sqrt{gh} \sim c_R \sim U_{jet}$ $\quad gh = 1000\ m^2 s^{-2}$

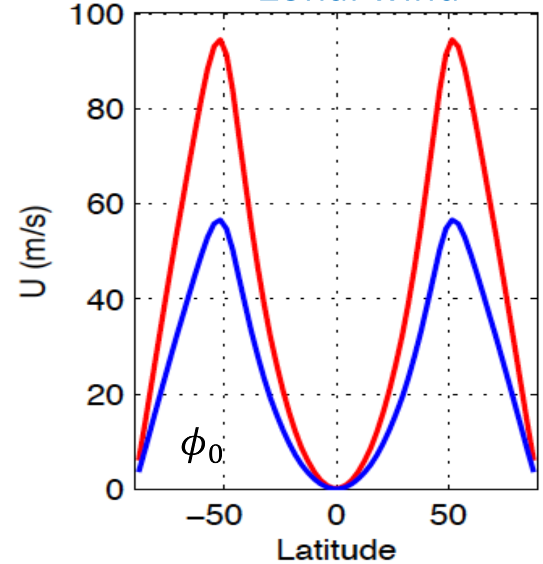
$\Omega = \Omega_{EARTH}$ $\quad R = \frac{1}{4} R_{EARTH}$

Zurita-Gotor & Held (2018)

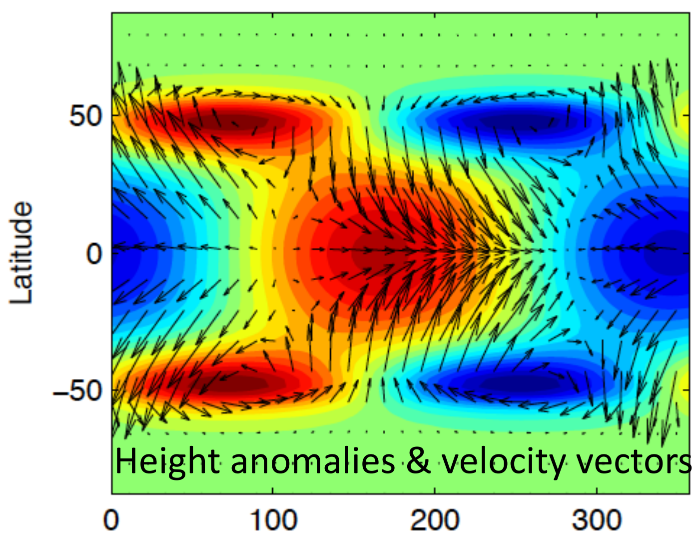
Absolute vorticity



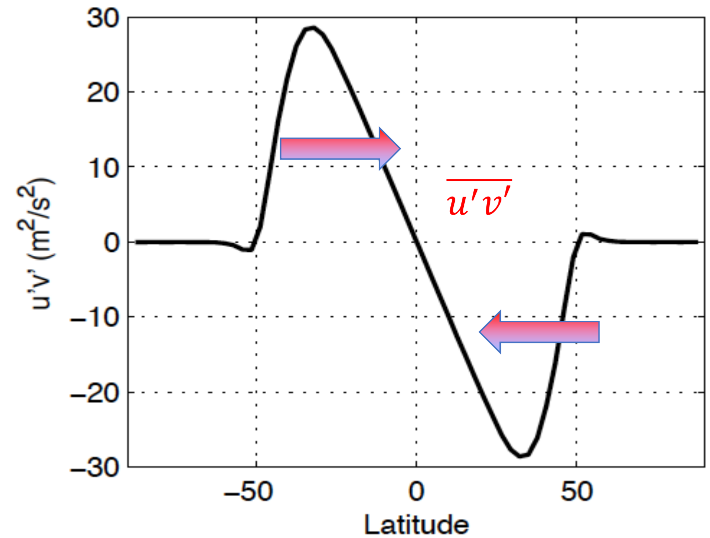
Zonal wind



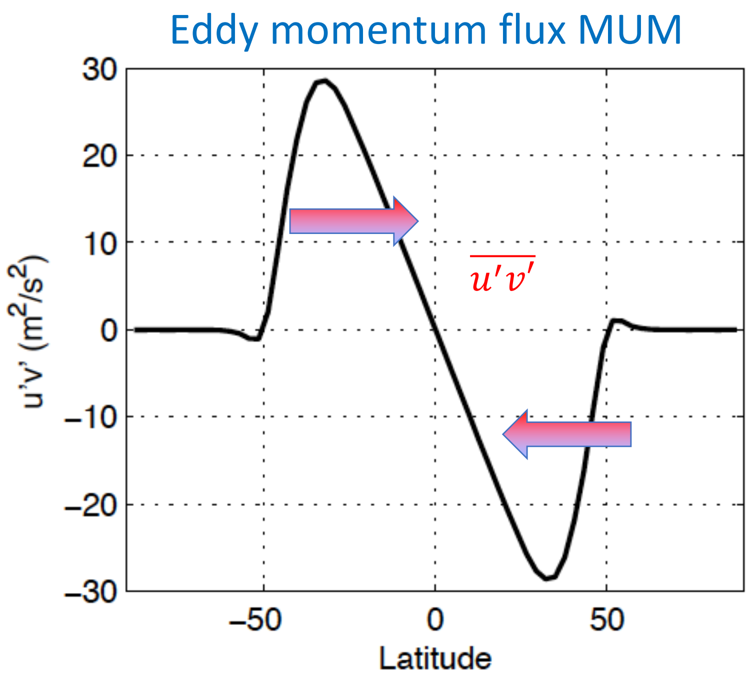
Most Unstable Mode (α=0)

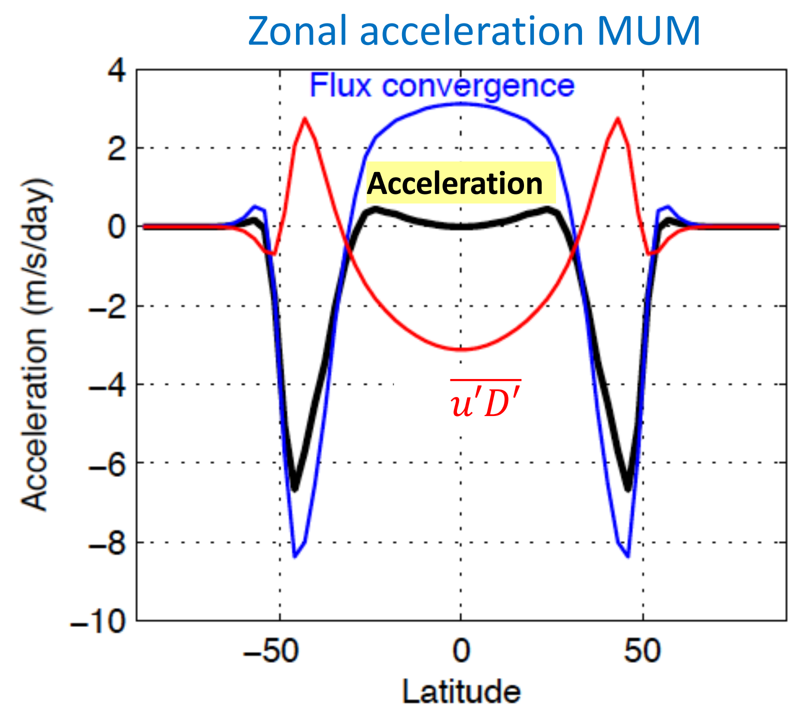


Eddy momentum flux MUM



Mode produces equatorward momentum flux but no Eulerian-mean acceleration!!!





Vector-invariant momentum equation:

$$\frac{\partial \bar{u}}{\partial t} = \overline{v'\xi'} = -\frac{\partial \overline{u'v'}}{\partial y} + \overline{u'D'} \approx 0 \qquad D' = \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}$$

Expect small vorticity fluxes because of the small vorticity gradient for this problem

This requires cancellation between the eddy momentum convergence and the $\overline{u'D'}$ term

Mysterious perfect cancellation $\overline{v'\xi'} = -\frac{\partial \overline{u'v'}}{\partial y} + \overline{u'D'} \approx 0$

Better perspective: two distinct contributions to meridional eddy momentum convergence:

$$-\frac{\partial \overline{u'v'}}{\partial y} = \overline{v'\xi'} - \overline{u'D'}$$

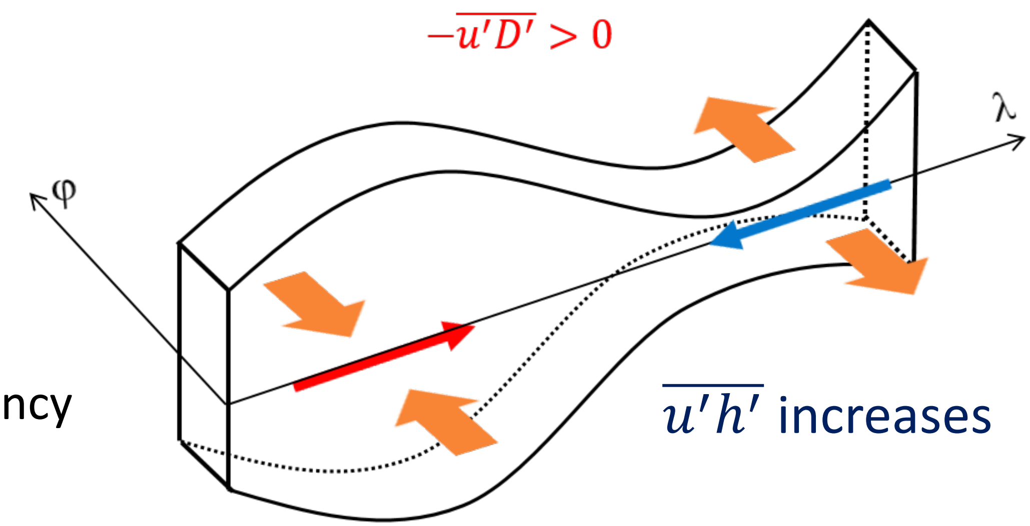
$\overline{v'\xi'}$ dominates in the extratropics but is very small near the equator, where $\overline{u'D'}$ dominates

Where is the momentum flux going? $\frac{\partial \overline{u^*}}{\partial t} = -\frac{\partial \overline{u'v'}}{\partial y} = \overline{v'\xi'} - \overline{u'D'}$

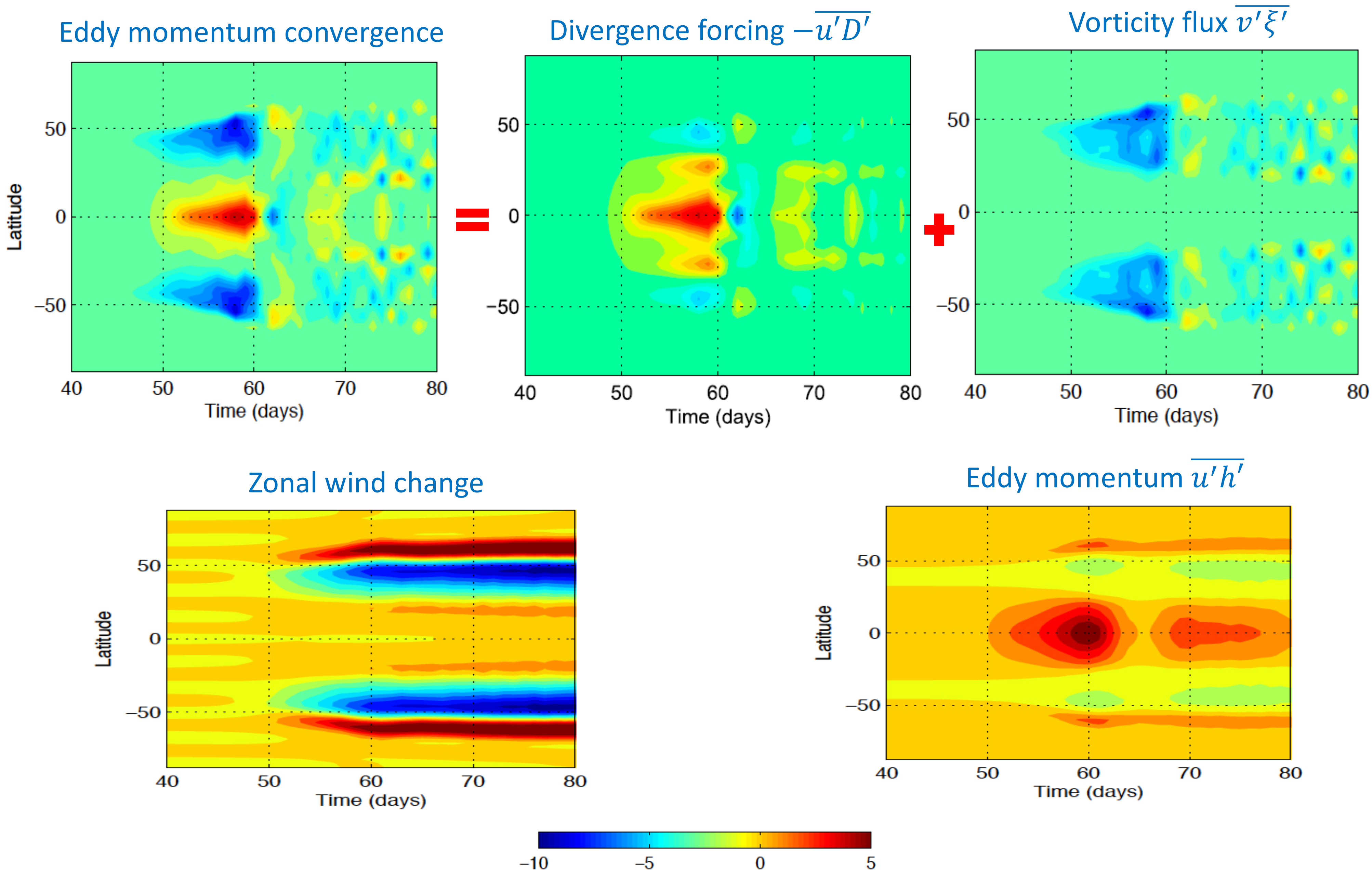
The momentum convergence does not force just $\bar{u}$ but the ***full mass-weighted momentum***

$$\overline{u^*} = \frac{\overline{hu}}{\bar{h}} = \bar{u} + \overline{u'h'} / \bar{h}$$

- Showed that $\bar{u}$ is only forced by $\overline{v'\xi'}$
- The $\overline{u'D'}$ term must then force $\overline{u'h'}$
- This term captures the full ***linear*** tendency

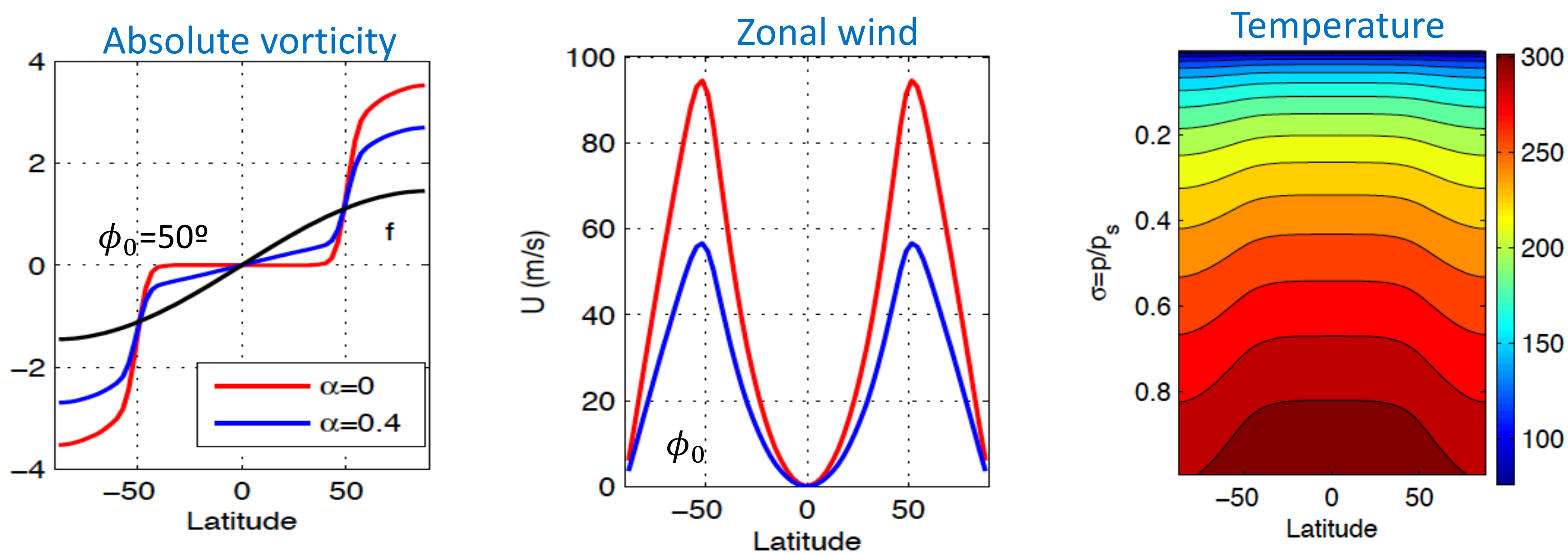


Weakly unstable/weakly nonlinear simulation



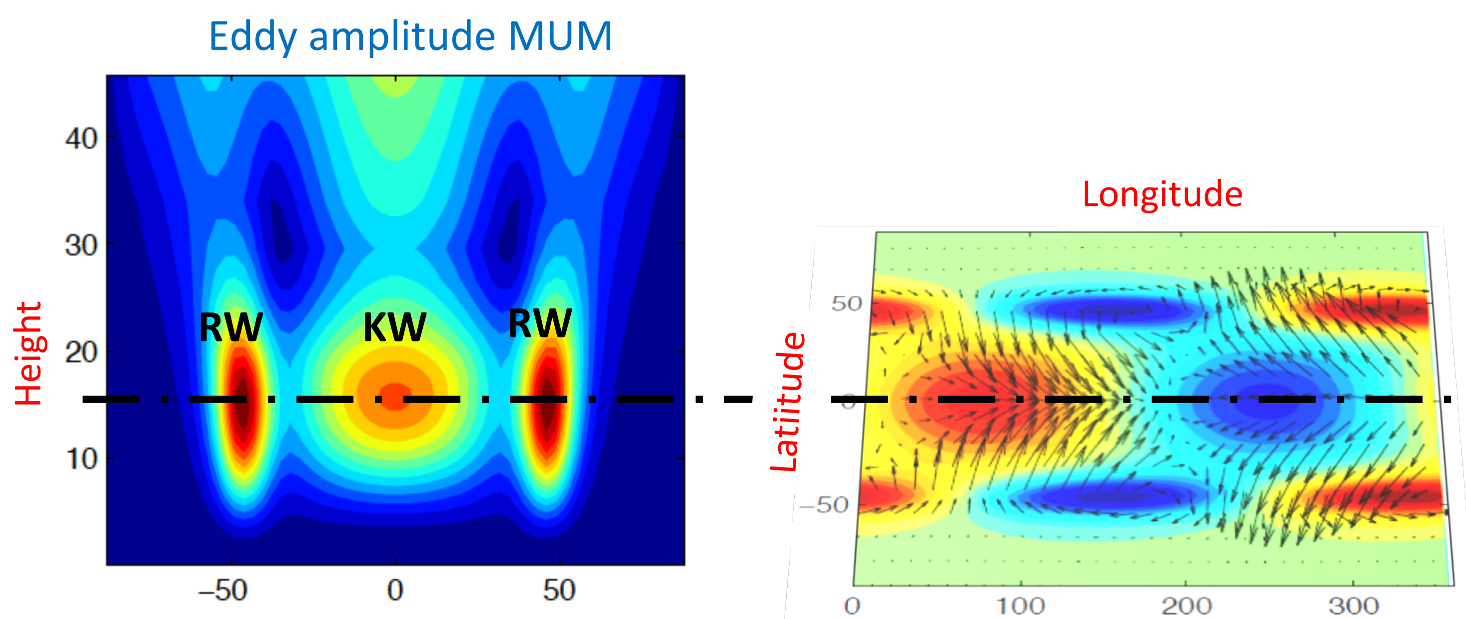
More unstable NL simulations crash. As the Kelvin wave steepens, $h' \rightarrow \bar{h}$ and $h \rightarrow 0$

Equilibration of Kelvin-Rossby instability in 3D

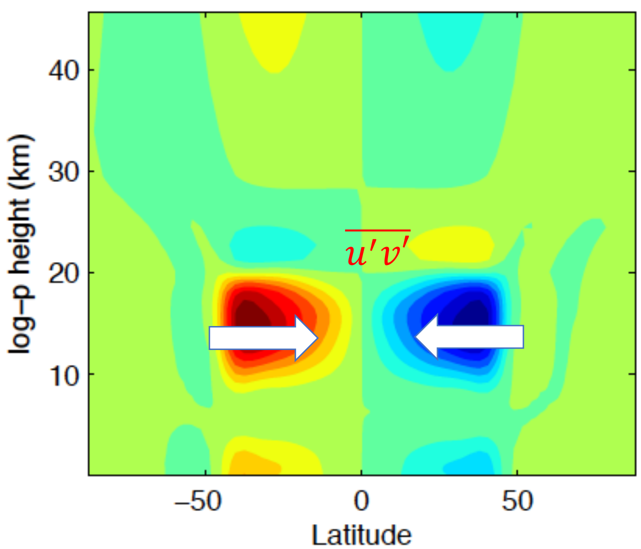


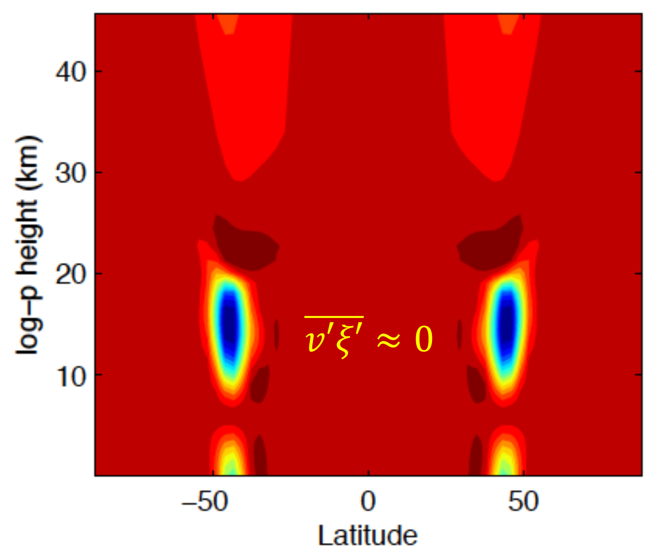
Barotropic basic state (same as before), uniformly stratified => INTERNAL Kelvin modes

$$gh_{eq} = 1000\, m^2 s^{-2}\ (MUM)$$
$$\Updownarrow$$
$$\lambda_z \approx 29km$$

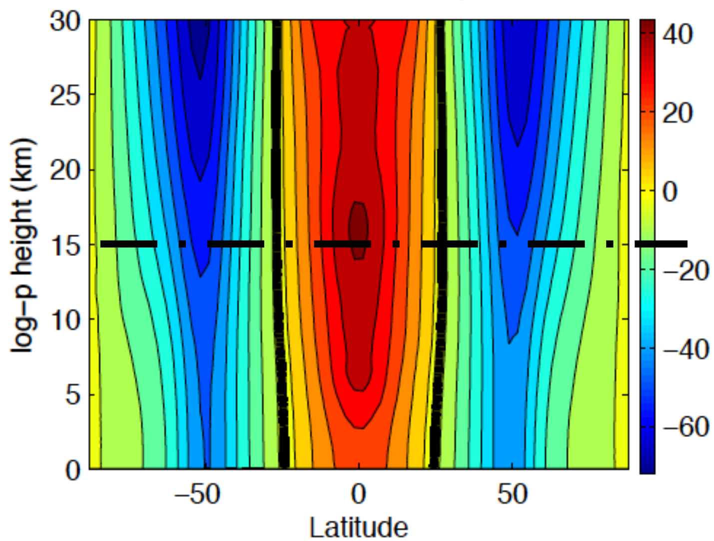


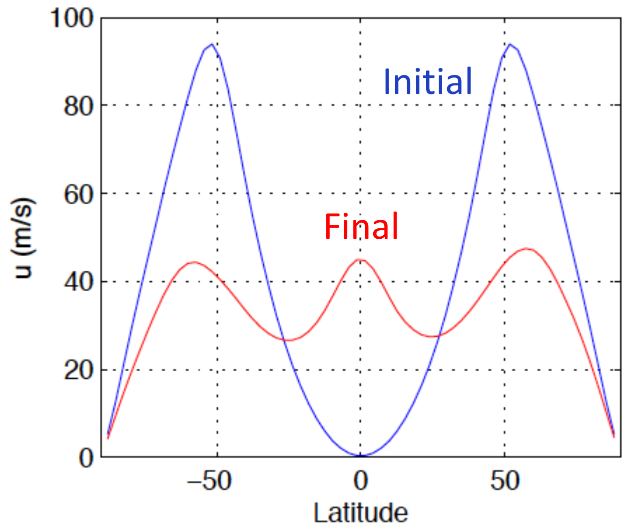
As before, equatorward momentum fluxes but weak vorticity fluxes in the tropics



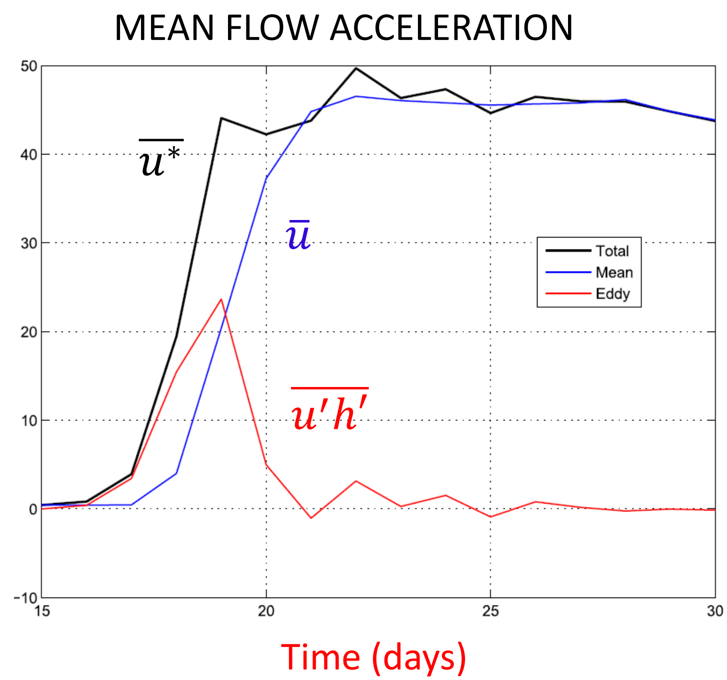


However, we can get superrotation now.

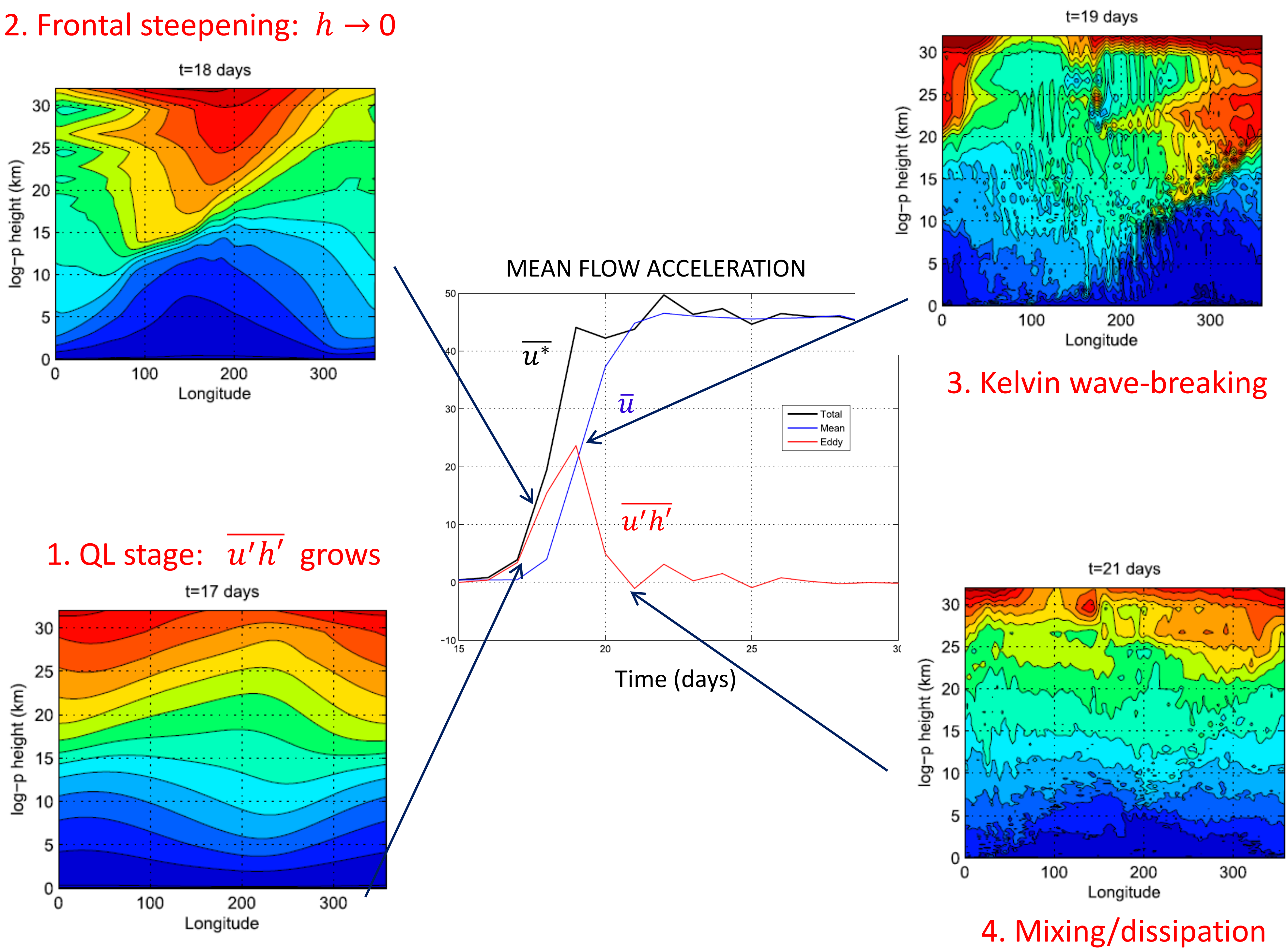




Isentropic diagnostics (near level max. acceleration, h is isentropic density now)

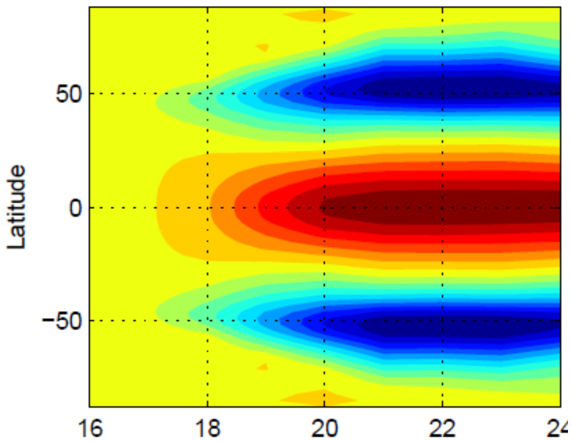


- Initially only $\overline{u'h'}$ grows, as in the shallow-water model
- After 1-2 days, $\bar{u}$ starts growing. The Eulerian-mean acceleration dominates the change in $\overline{u^*}$ at equilibration as $\overline{u'h'}$ dissipates



2. Frontal steepening: $h \rightarrow 0$
t=18 days
log−p height (km)
Longitude
1. QL stage: $\overline{u'h'}$ grows
t=17 days
log−p height (km)
Longitude
MEAN FLOW ACCELERATION
$\overline{u^*}$
$\bar{u}$
$\overline{u'h'}$
Total
Mean
Eddy
Time (days)
t=19 days
log−p height (km)
Longitude
3. Kelvin wave-breaking
t=21 days
log−p height (km)
Longitude
4. Mixing/dissipation

Vertical-mean wind change

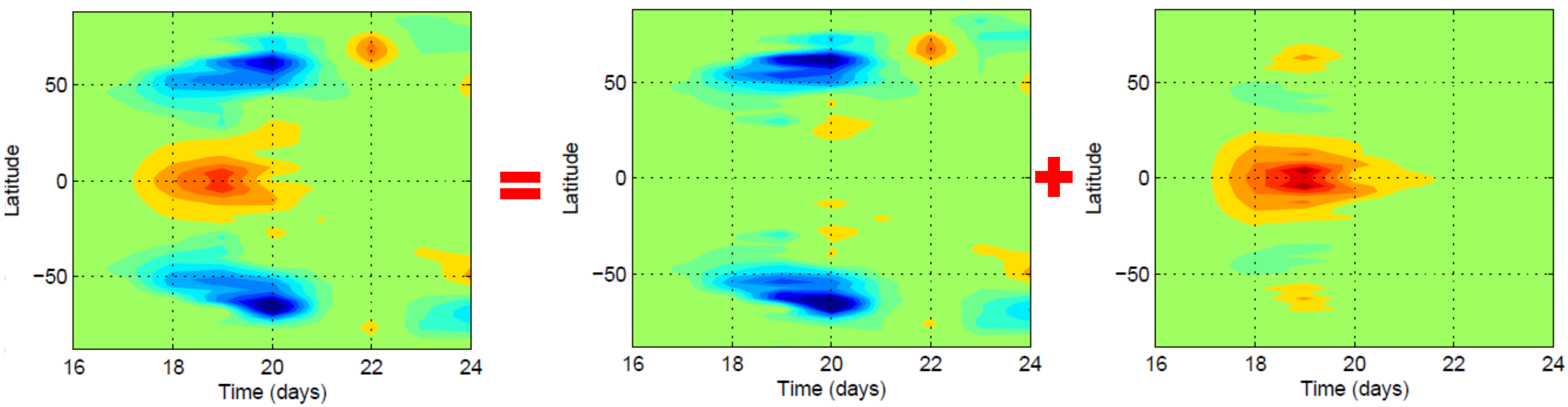


- Though formally adiabatic, ***thermal*** **dissipation is key to changing** $\bar{u}$
- Eulerian acceleration imparted by ***cross-isentropic advection*** during Kelvin wave breaking $\overline{u'h'} \rightarrow \bar{u}$

Vertical-mean eddy acceleration

Vertical-mean vorticity flux

Vertical-mean cross-isentropic advection



Would need to parameterize Kelvin wave breaking to get this in shallow water model

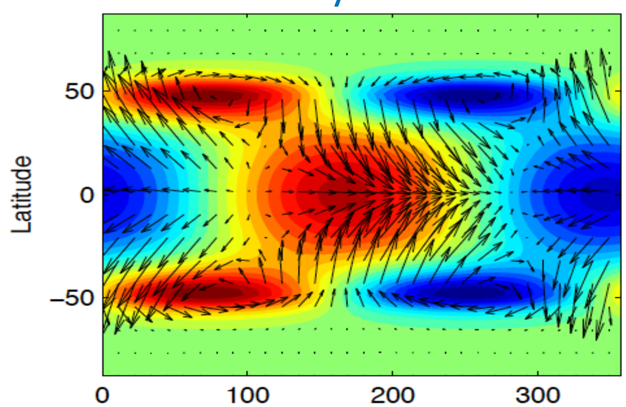
Superrotation in rapidly-rotating planets

Results strongly reminiscent of Showman & Polvani (2011)

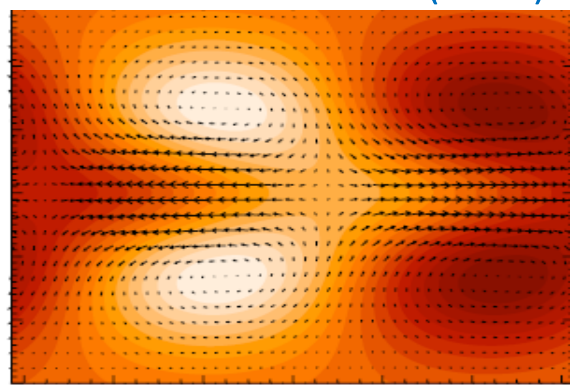
-> Forced Matsuno-Gill response to day/night heating contrast

-> Superrotation driven by Kelvin-Rossby interaction

Kelvin-Rossby Unstable Mode



Showman & Polvani (2011)



Main differences:

-Forced/unforced

-**Small thermal Ro**

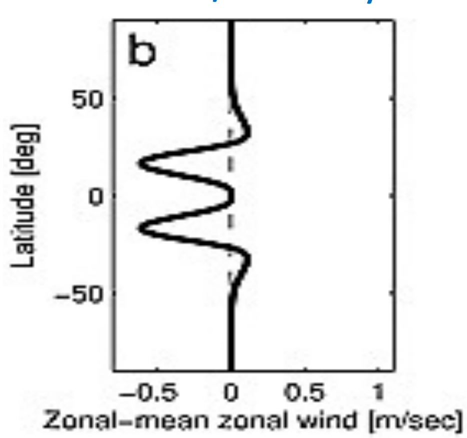
-**Narrow tropics**

Similar to us, their standard shallow-water model does not superrotate

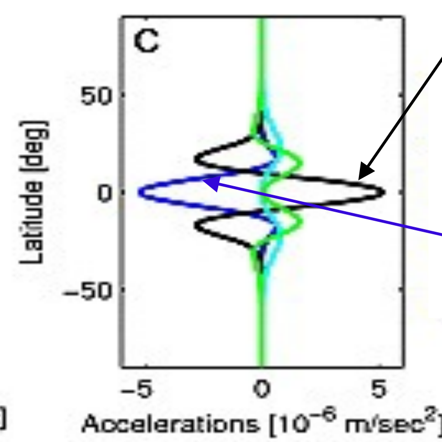
Vorticity flux only vanishes near the equator, but is **negative elsewhere** (weak vorticity source) and **cannot accelerate the flow**

SP11 achieved superrotation by adding vertical advection

Zonal wind/vorticity flux



Acceleration



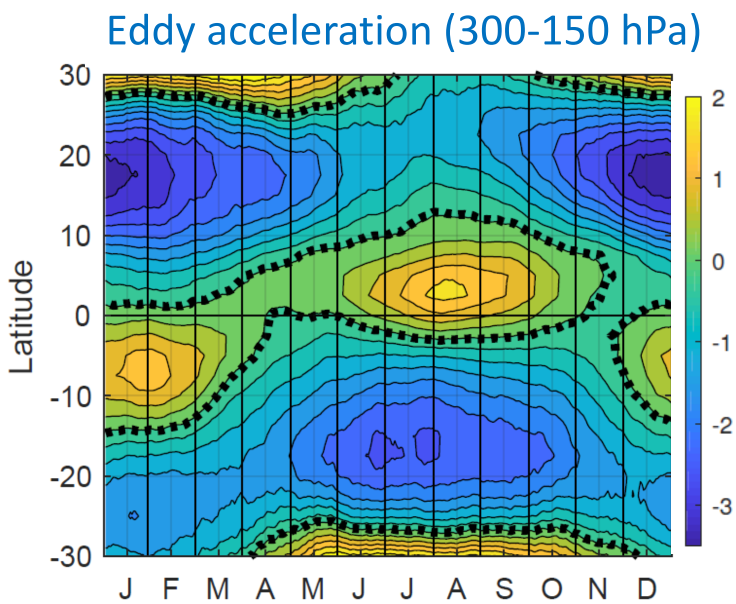
Eddy meridional momentum convergence

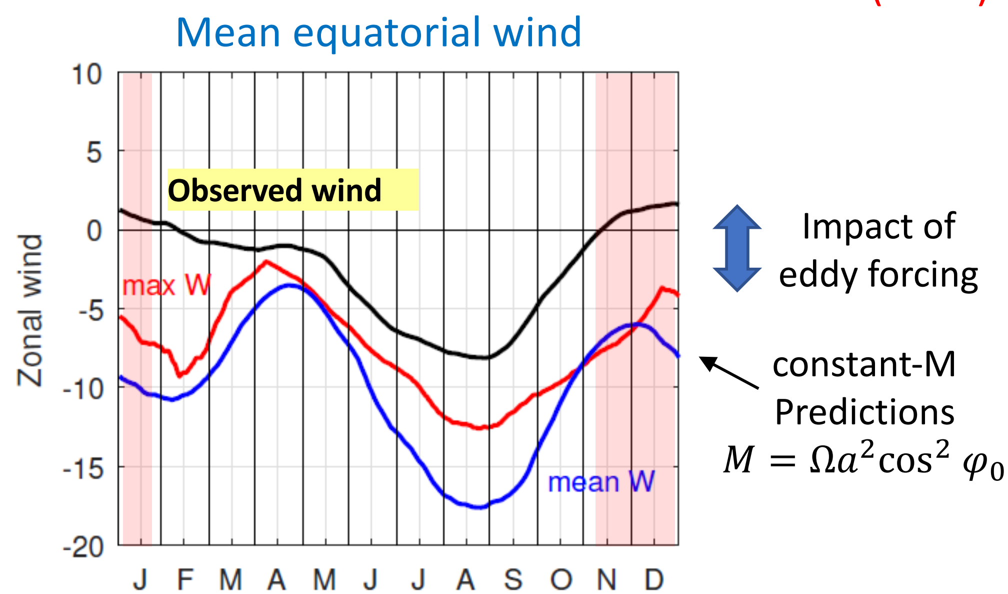
Eddy "vertical" momentum convergence $\overline{u'D'}$

Showman & Polvani (2010)

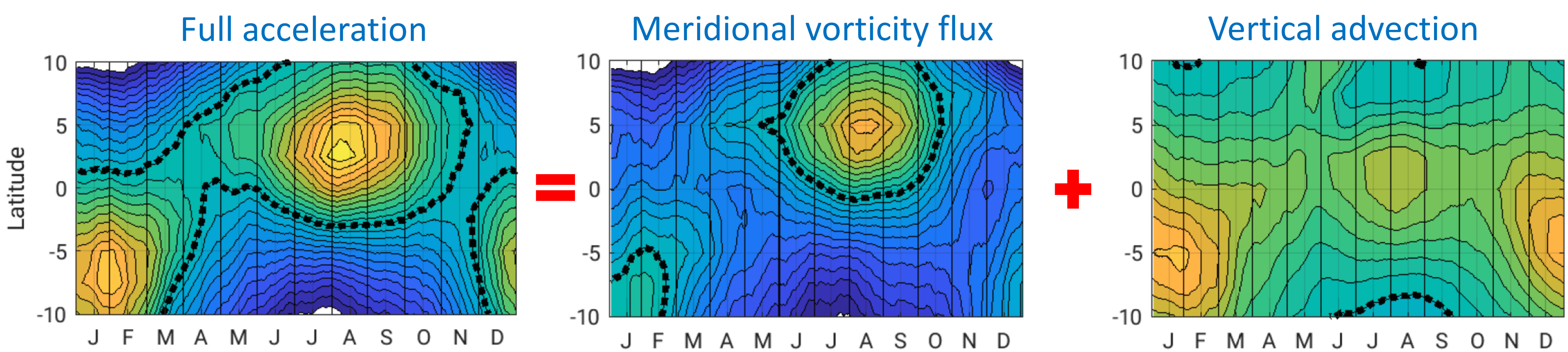
Dynamics of weak superrotation on Earth

Zurita-Gotor (2019)





- Year-round westerly acceleration at the Equator, specially during solstices
- Only weak superrotation (i.e., relative to ITCZ) due to import of low *M* by Hadley cell



Dynamics of weak superrotation on Earth

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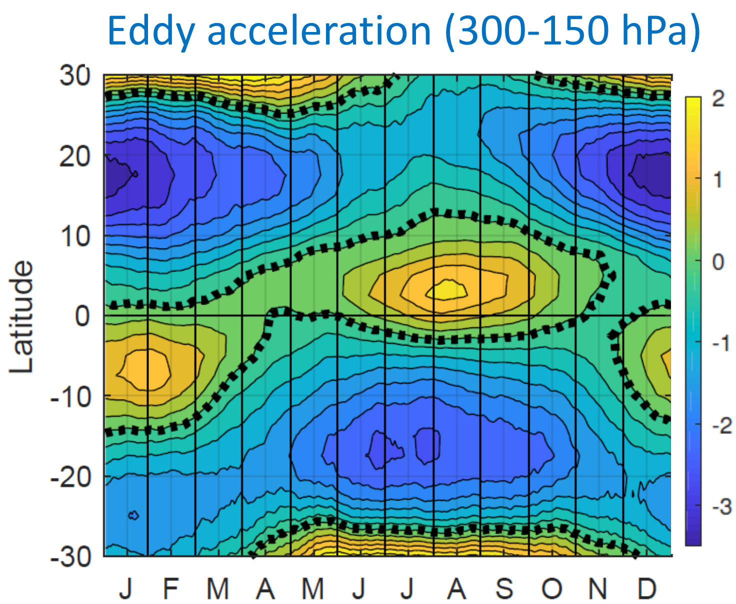

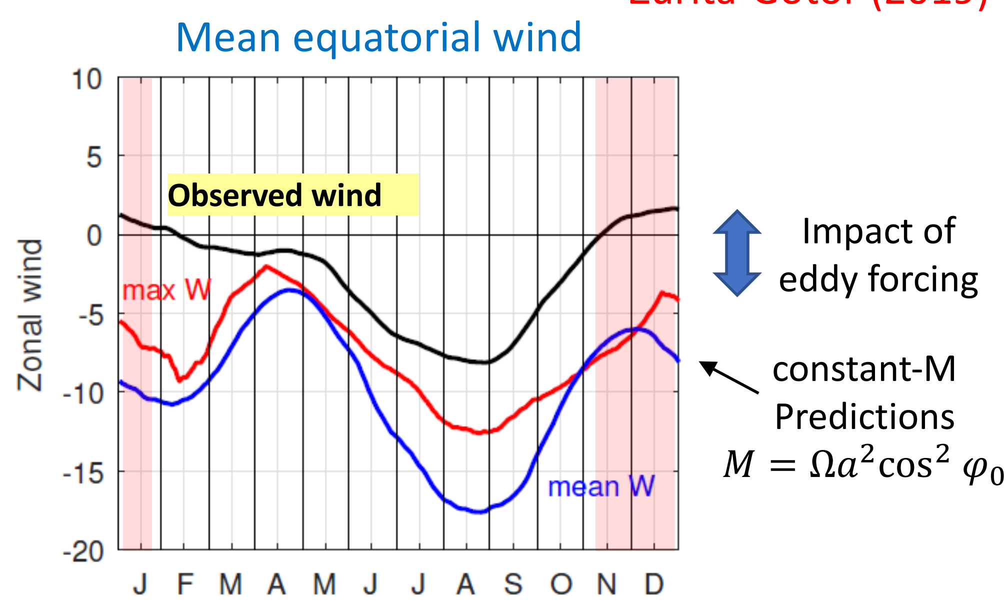

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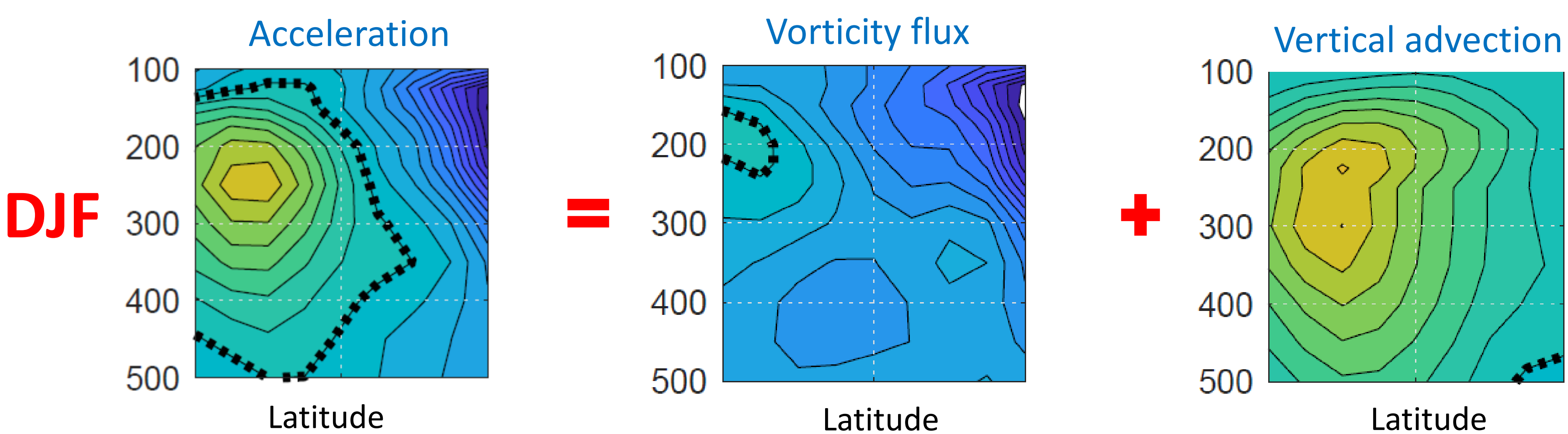
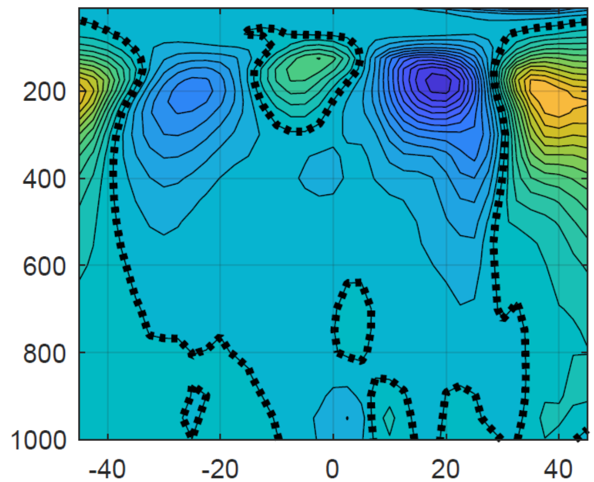


DJF

$$-\frac{\partial \overline{u'v'}}{\partial y} = \overline{v'\xi'} - \overline{u'D'}$$

Merid. convergence $-\partial_y \overline{u'v'}$



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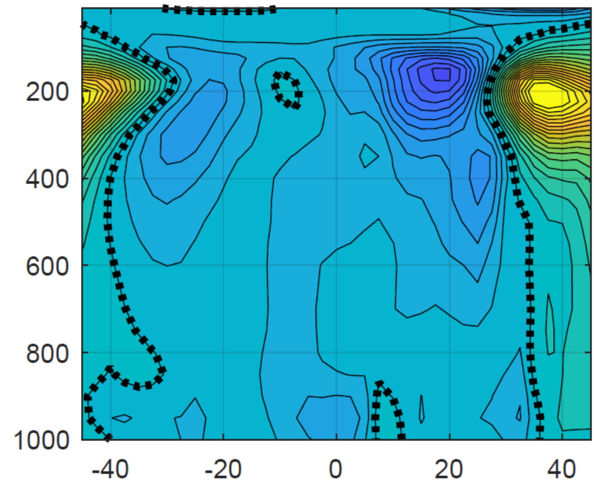
Vorticity flux $\overline{v'\xi'}$



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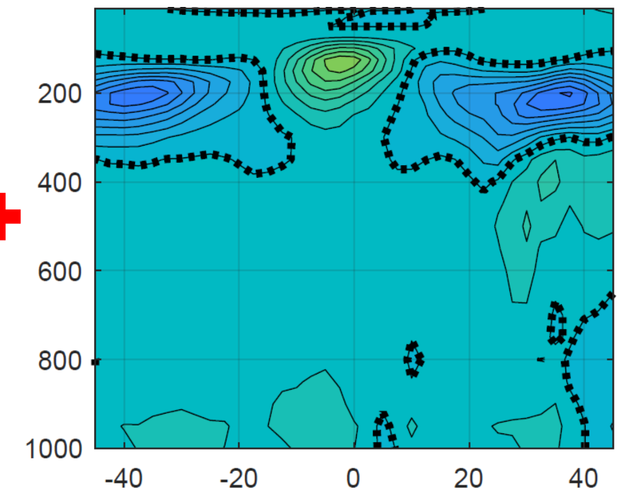
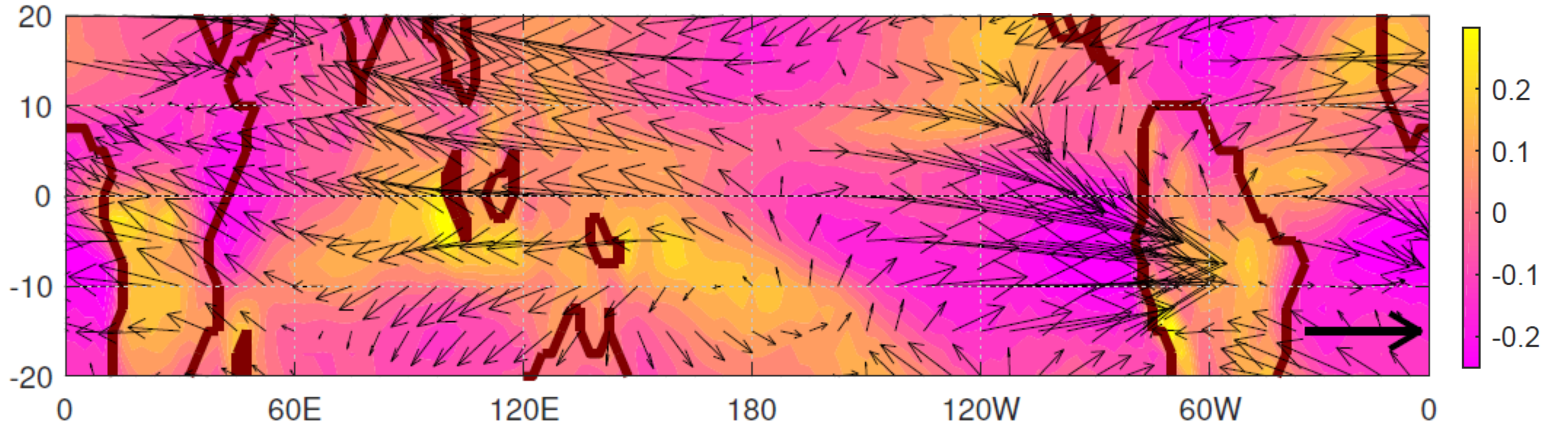
Divergence forcing $-\overline{u'D'}$



DJF Upper-level eddy divergence and wind vectors



$\overline{u'D'} < 0$

yellow: divergence
magenta: convergence

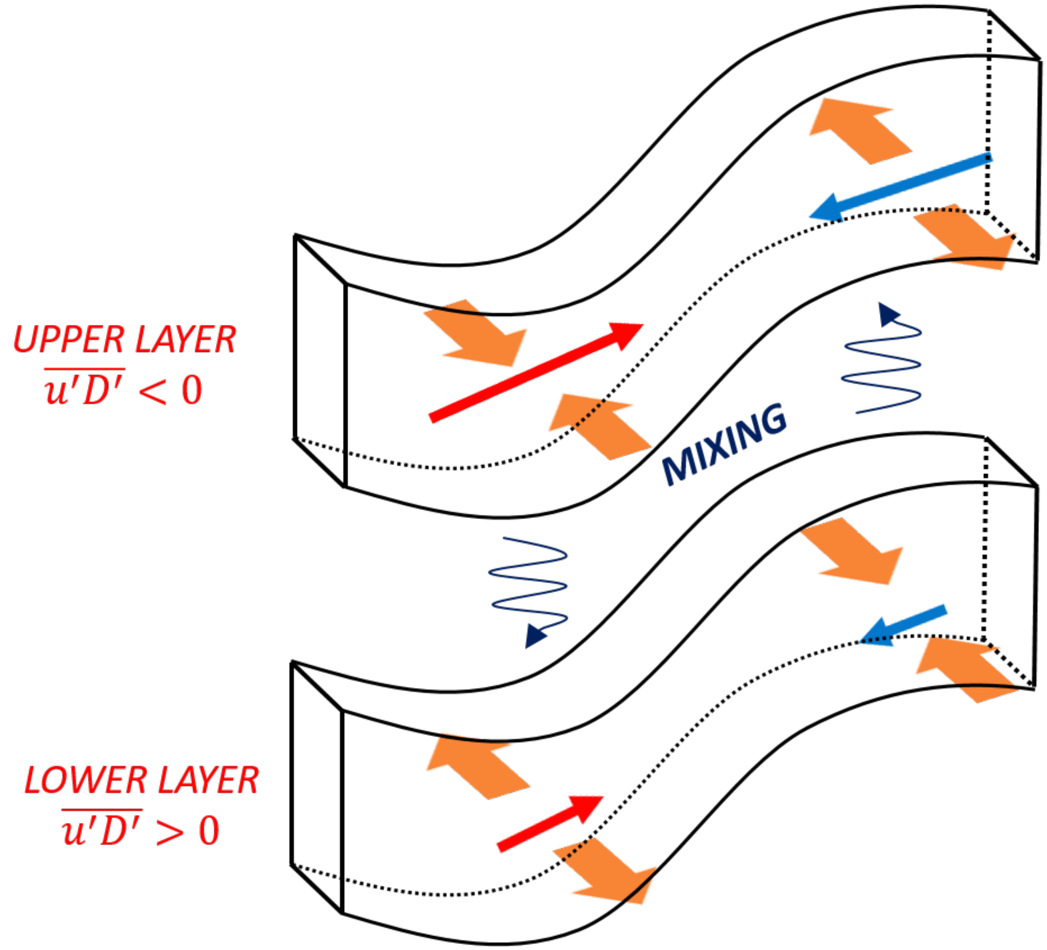
Meridional convergence

$$-\frac{\partial \overline{u'v'}}{\partial y} = \overline{v'\xi'} - \overline{u'D'}$$

Full eddy acceleration

$$\vec{\nabla} \cdot \vec{F} = \overline{v'\xi'} - \overline{\omega' \frac{\partial u'}{\partial p}}$$

Vertical advection/mixing key for irreversibility

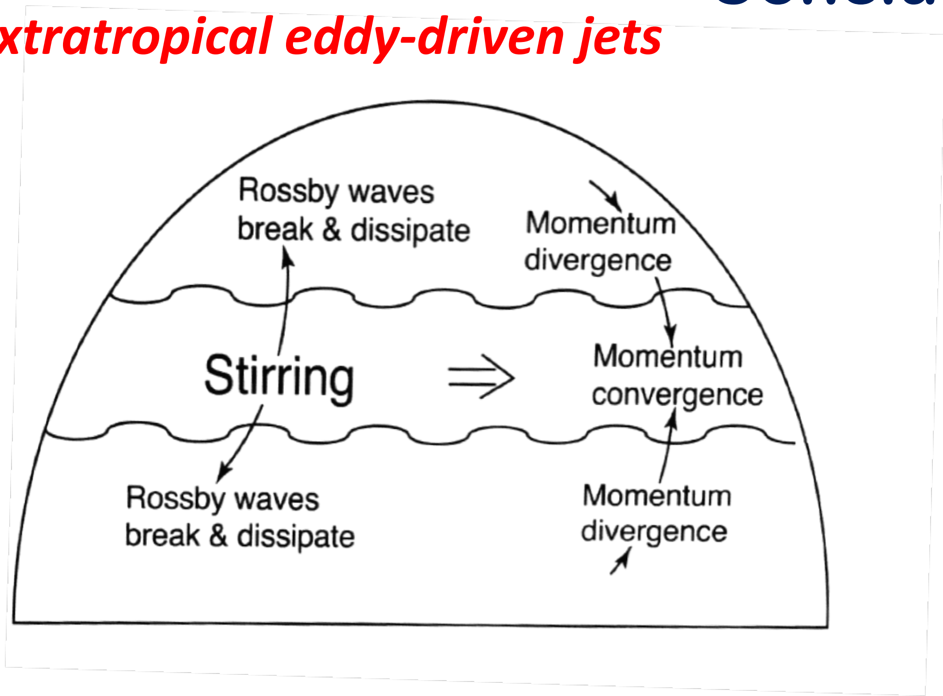


With no vertical shear, the upper-layer acceleration $\overline{u'D'} < 0$ would be compensated by lower-layer deceleration $\overline{u'D'} > 0$

With vertical shear, there is vertical advection and mixing

Conclusions

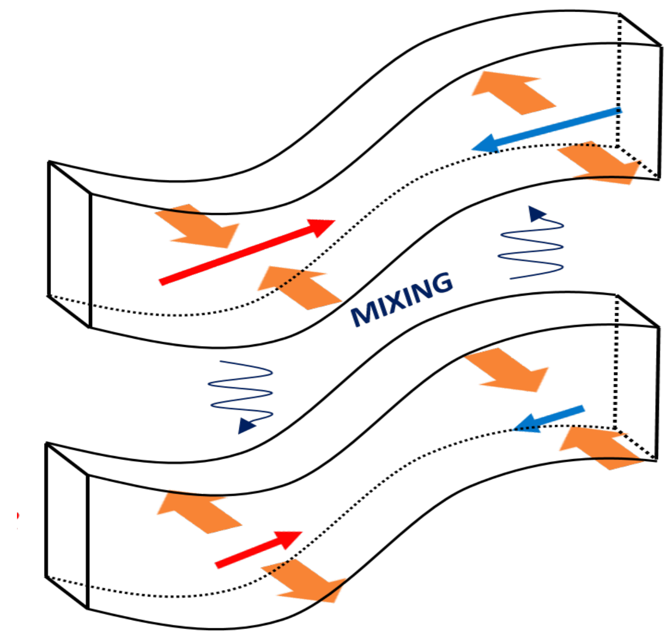
Extratropical eddy-driven jets



$$-\frac{1}{a\,cos^2\varphi}\frac{\partial(\overline{u'v'}cos^2\varphi)}{\partial\varphi} \approx \overline{v'\xi'}$$

- Governed by ***rotational*** dynamics, acceleration by ***upgradient*** vorticity fluxes (need a vorticity source)
- Mechanism: Rossby wave propagation
- Dissipation: breaking vorticity waves

Equatorial eddy-driven jets



$$-\frac{1}{a\,cos^2\varphi}\frac{\partial(\overline{u'v'}cos^2\varphi)}{\partial\varphi} = \overline{v'\xi'} - \overline{u'D'}$$

- Weak vorticity source → ***downgradient*** vorticity fluxes decelerate the flow.
- ***Divergent*** mechanism: eddy vertical overturning
- ***Diabatic*** dissipation, momentum mixing due to cross-isentropic advection

$$\bar{u}_t + \bar{v}^*[(a\cos\phi)^{-1}(\bar{u}\cos\phi)_\phi - f] + \bar{Q}^*\bar{u}_\theta - \bar{X}^* = -\bar{\sigma}^{-1}(\overline{\sigma' u'})_t + (\bar{\sigma} a \cos\phi)^{-1}\tilde{\nabla}\cdot\tilde{\mathbf{F}} \qquad (3.9.7a)$$

Isentropic EP relation

Andrews et al (1987)

Mass-weighted momentum

$$\frac{\partial}{\partial t}\left(\bar{h}\overline{u^*}\right) - \bar{h}\overline{v^*}\left(f + \bar{\xi}\right) + \bar{h}\overline{Q^*}\frac{\partial \bar{u}}{\partial \theta} = \frac{1}{a\cos\phi}\vec{\nabla}\cdot\vec{F}$$

Eulerian-mean momentum

$$\frac{\partial}{\partial t}\left(\bar{h}\bar{u}\right) - \bar{h}\overline{v^*}\left(f + \bar{\xi}\right) + \bar{h}\overline{Q^*}\frac{\partial \bar{u}}{\partial \theta} \approx \bar{h}^2\overline{v'q'} - \bar{h}\overline{Q'\frac{\partial u'}{\partial \theta}}$$

Eddy momentum

$$\frac{\partial}{\partial t}\left(\overline{u'h'}\right) \approx -\bar{h}^2\overline{v'q'} + \bar{h}\overline{Q'\frac{\partial u'}{\partial \theta}} + \frac{1}{a\cos\phi}\vec{\nabla}\cdot\vec{F}$$

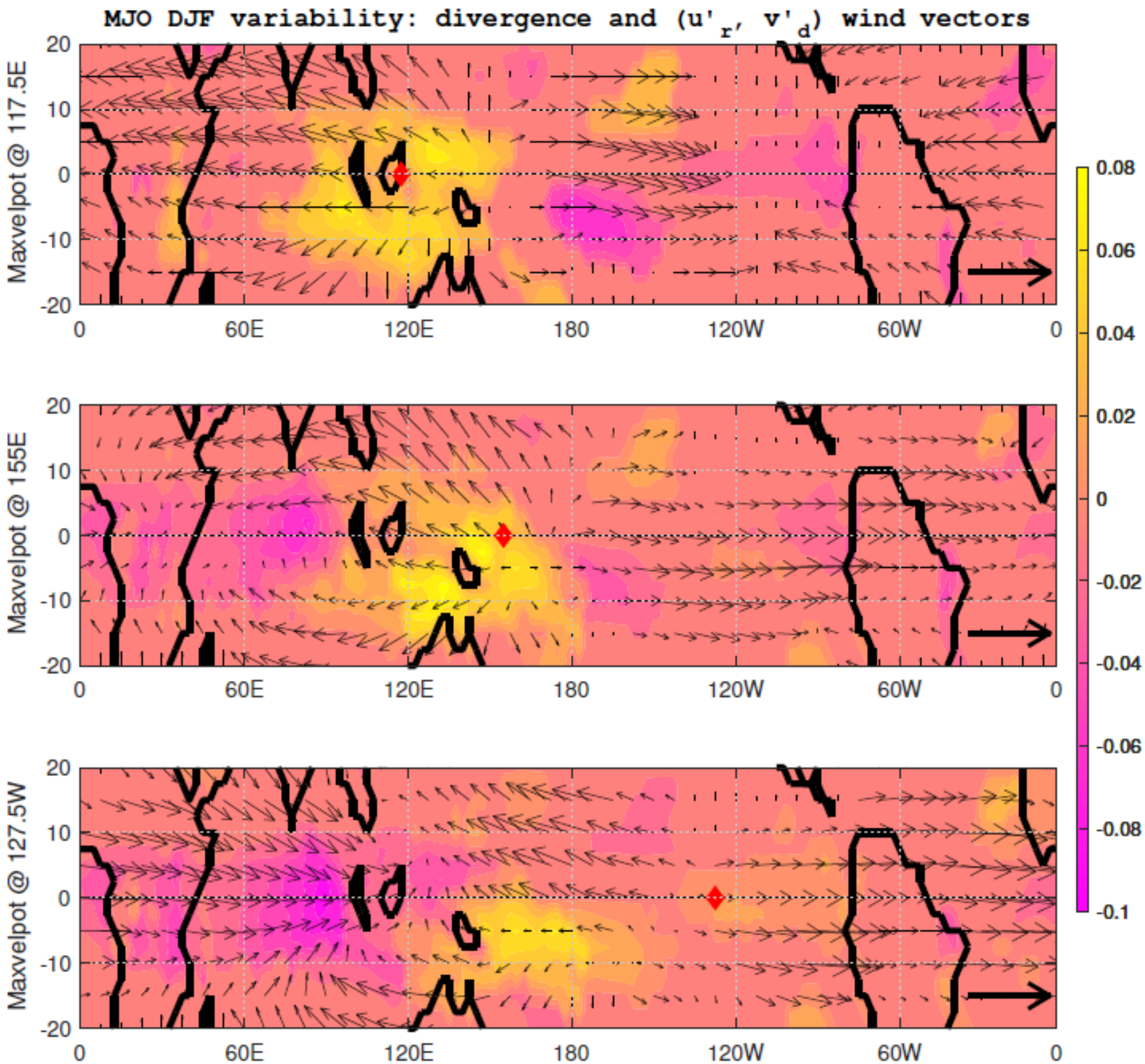
- Eliassen-Palm convergence forces mass-weighted momentum, not just $\bar{u}$
- *When* $\bar{h}^2\overline{v'q'} - \bar{h}\overline{Q'\frac{\partial u'}{\partial \theta}} = 0$, $\vec{\nabla}\cdot\vec{F}$ *only accelerates the eddy term* $\overline{u'h'}$*!*
- Changing the Eulerian-mean momentum requires mixing/dissipation:

$$\bar{h}^2\overline{v'q'} - \bar{h}\overline{Q'\frac{\partial u'}{\partial \theta}} \neq 0$$

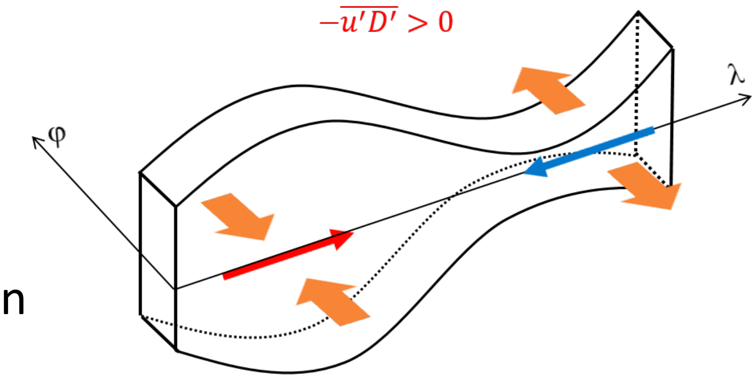
$\overline{v'q'}$ = Rossby wave breaking

$\overline{Q'\frac{\partial u'}{\partial \theta}}$ = cross-isentropic advection

MJO

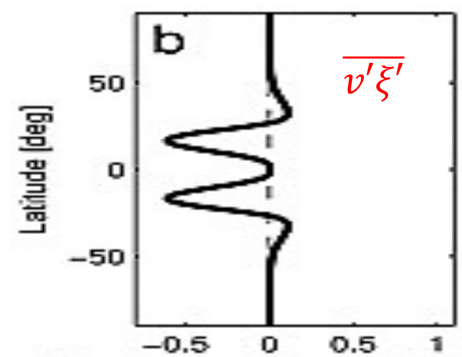

MJO DJF variability: divergence and (u'$_r$, v'$_d$) wind vectors
Maxvelpot @ 117.5E
Maxvelpot @ 155E
Maxvelpot @ 127.5W
20
10
0
-10
-20
0
60E
120E
180
120W
60W
0
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1

- Kelvin-Rossby instability (superrotation of small/slowly rotating planets)



  - Negligible vorticity flux: $\overline{v'\xi'} \approx 0$
  - Eddy momentum convergence due to $-\overline{u'D'}$, only change mass-weighted momentum $\overline{u'h'}$
  - Acceleration due to cross-isentropic advection in connection with Kelvin wave breaking

- Showman & Polvani (superrotation of tidally-locked hot Jupiters)



  - Vorticity flux small only at the Equator but <u>negative elsewhere</u> $\overline{v'\xi'} < 0$ (vorticity sink, not source!)
  - Meridional momentum convergence by $-\overline{u'D'}$ cannot change Eulerian-mean momentum
  - Acceleration requires vertical (cross-isentropic) advection

- Weak superrotation on Earth during the solstices

  - With weak vorticity source, also expect negative vorticity fluxes $\overline{v'\xi'} < 0$