A Flavour of Equatorial Dynamics

From Super-rotation to the MJO

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Lyon, 2019

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Three Parts

1. Super-rotation on giant planets with deep jets.
2. Super-rotation on terrestrial planets
   - Are all Venus-like planets like Venus?
3. Moist Processes and the MJO.
   - The MJO is a prograde *pattern*, so morally it is super-rotation…

Super-Rotation

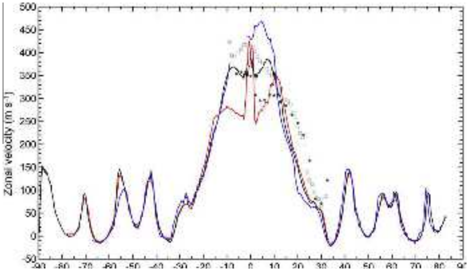
A planetary atmosphere super-rotates if it has prograde zonal flows with more angular momentum about the rotation axis than does stationary air at the equator, Ωa^2.

- Angular momentum: $m(\vartheta) = a\cos\vartheta(\Omega a\cos\vartheta + \overline{u})$
- Local superrotation: $s(\vartheta) = \dfrac{m}{\Omega a^2} - 1$
- Global superrotation: $S = \dfrac{\int \rho m \, \mathrm{d}V}{\int \rho \Omega^2 \cos^2\vartheta \, \mathrm{d}V} - 1,$ or perhaps: $S' = \int \rho s(\vartheta) \, \mathrm{d}V$
  Read (1986)
- We use the local definition, $s(\vartheta)$, (simpler and more restrictive).
- In practice, this means prograde flows at the equator, otherwise inertially unstable.

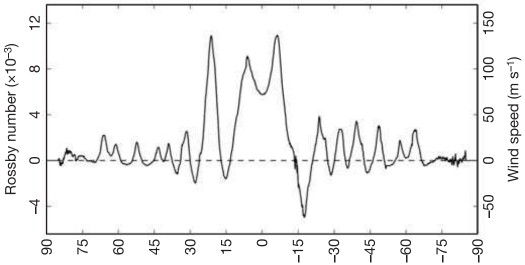
Examples and the Need for Eddies

Super-rotation on Saturn and Jupieter

Saturn



Jupiter



Hide's Result

Angular momentum conserved by axi-symmetric flow:

$$\underbrace{\frac{Dm}{Dt}}_{\text{conserves } m} = \underbrace{\nu\nabla^2 m}_{\text{diminishes } |m|}$$

Axi-symmetric flow cannot produce an extremum of m in the interior of the fluid. Therefore no super-rotation in axi-symmetric flow.

Solar System Bodies

Planetary Body	Super-rotates?
Mercury	N/A
Venus	**Yes!**
Earth	Sometimes (QBO)
Mars	Sometimes
Jupiter	**Yes!**
Saturn	**Yes!**
Titan	**Yes!**
Uranus	No
Neptune	No
Hot Jupiters	Yes, probably.

Two distinct groups of strong super-rotators:

1. Slowly rotating terrestrial planets:
   Venus + Titan.
   Large Rossby number, $Ro \geq 1$

2. Fast rotating giant planets:
   Jupiter and Saturn.
   Small Rossby number $Ro \ll 1$

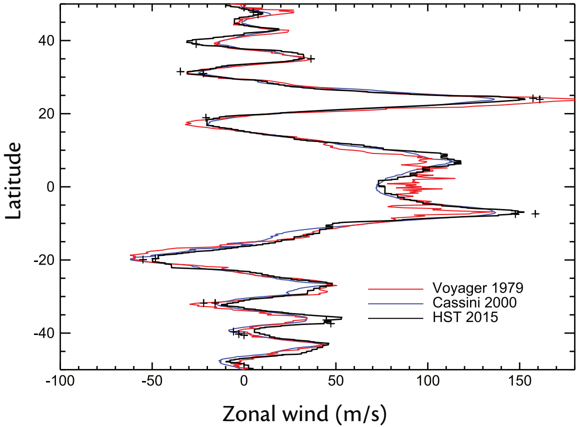
Jupiter from Space

(Enhanced color from 3 images, by K. M. Gill)

Characteristics:

- Weather, clouds
- Jets!
- Global organization.
- Very zonal flow.

Jupiter and its Jets



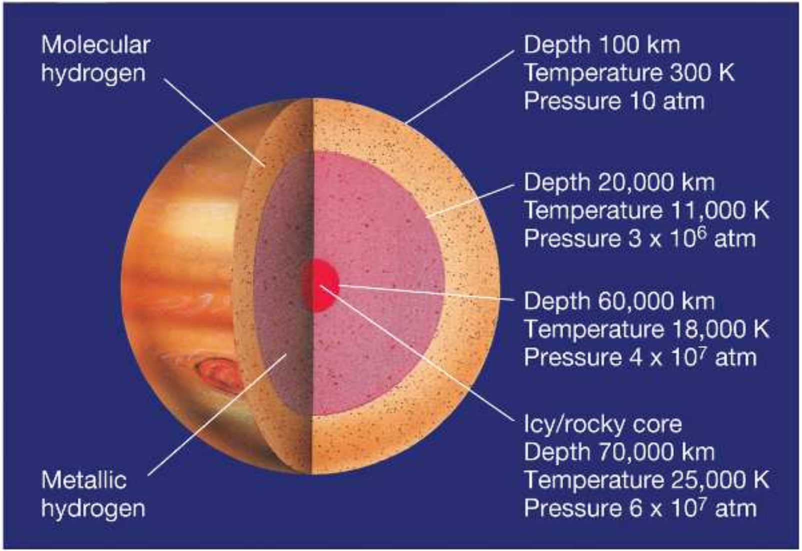
– Strong, sharp jets. Barotropically unstable $\beta - \partial^2 U/\partial y^2$ changes sign.

– Superrotates.

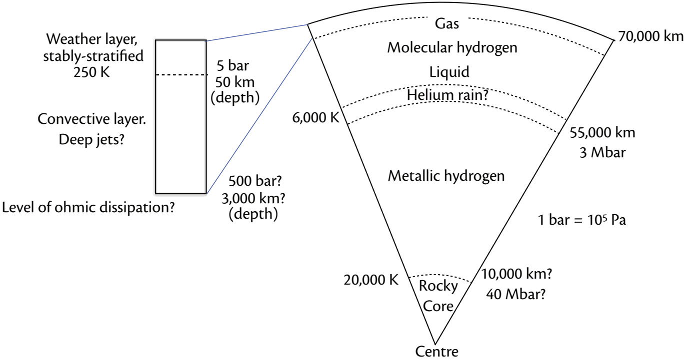
– Multiple super-rotating jets in the tropics.

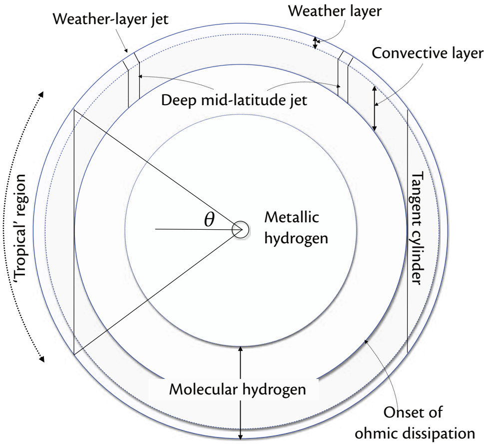
Structure of Jupiter

Popular science book.

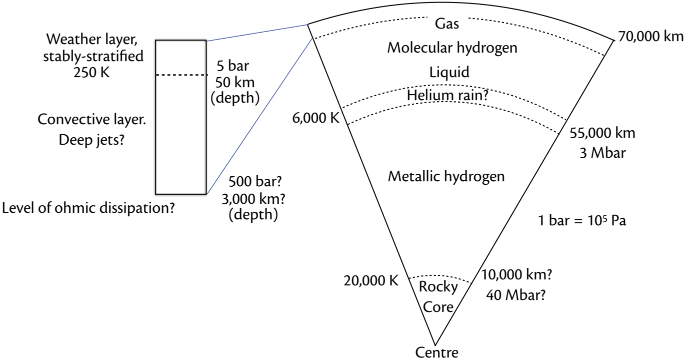


Structure of Jupiter



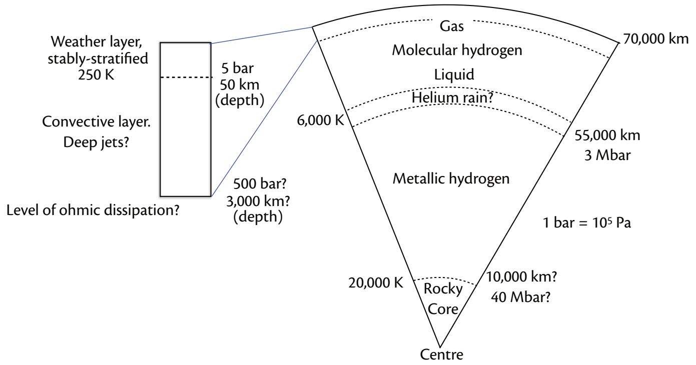


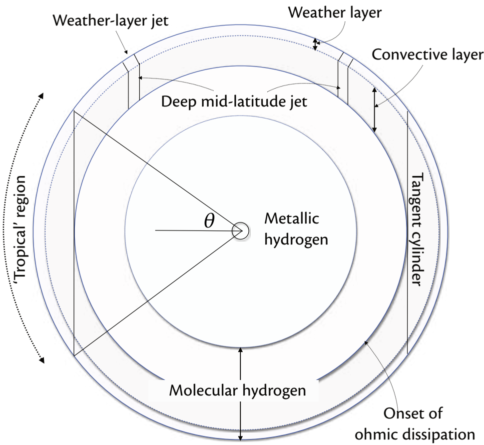
STRUCTURE OF JUPITER



www.cakecrumbs.me

Structure of Jupiter





Weather-layer Jets

Easy! — (if Shallow)

Two-dimensional beta-plane turbulence! (Rhines, Williams, Vallis & Maltrud)

$$\frac{DQ}{Dt} = F - D, \qquad Q = \beta y + \zeta$$

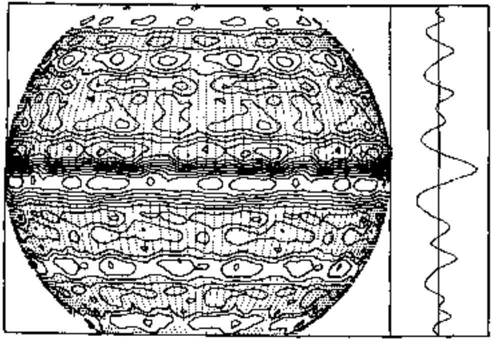
Here, β is dues to differential rotation:

$\beta = \partial f / \partial y = 2\Omega \cos\vartheta / a.$

$$\frac{D\zeta}{Dt} + \beta v = F - D,$$

$$\text{Jet Scale} = \sqrt{\frac{U}{\beta}} \sim \sqrt{\frac{100}{(2\Omega \cos\vartheta / a)}}$$

$$\sim \sqrt{\frac{100}{5 \times 10^{-12}}} \sim 5000\,\text{km}$$

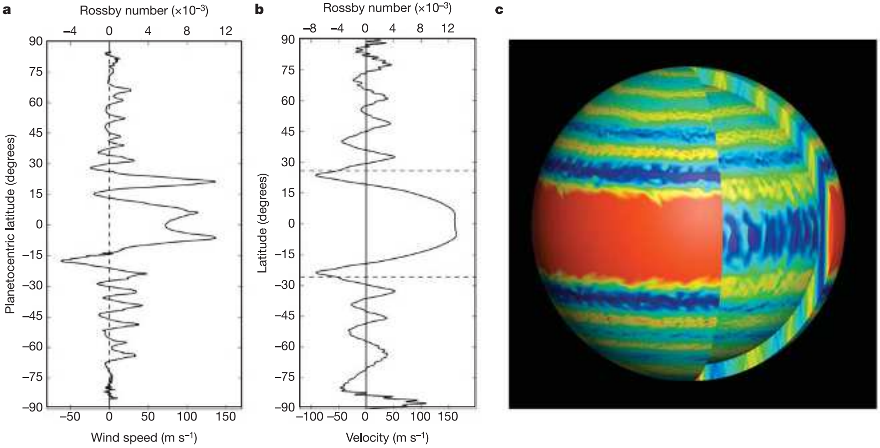


Williams (41 BN) (before now! i.e., 1978)

Deeper Jets

Taylor-columns (Busse, Schubert etc)

Simulation by
Heimpel
Zonal velocity
Equatorial jet
too broad.



Observed | Modelled | Modelled

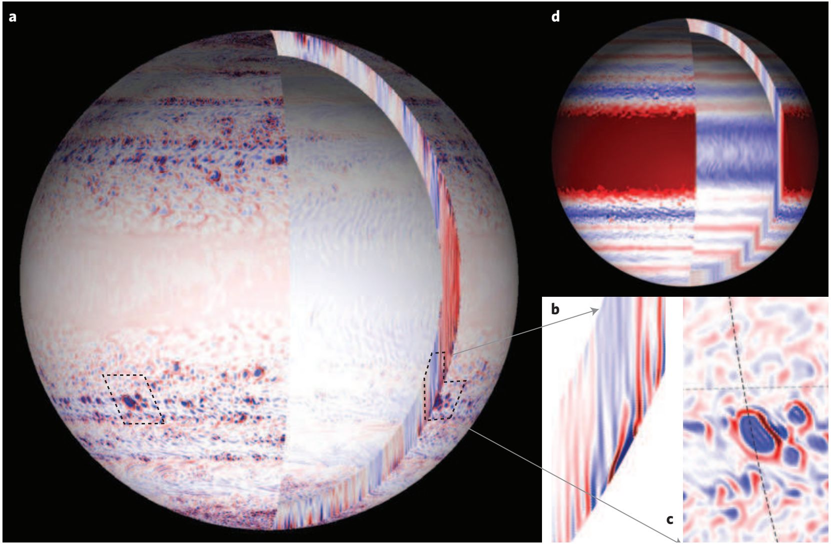
Possible lack of eddies in weather layer?

(Heimpel)

Deeper Jets

Taylor-columns?

Vorticity and velocity:
With stratified upper layer
Heimpel, Arnou et al:



Jovian Superrotation

The topographic beta effect for deep jets

Potential vorticity of a column:

$$\frac{DQ}{Dt} = 0, \qquad Q = \left(\frac{\zeta + 2\Omega}{h}\right).$$

$$Q = \left(\frac{\zeta + 2\Omega}{h}\right) \approx \left(\frac{\zeta + 2\Omega}{H}\right),$$

or, approximately,

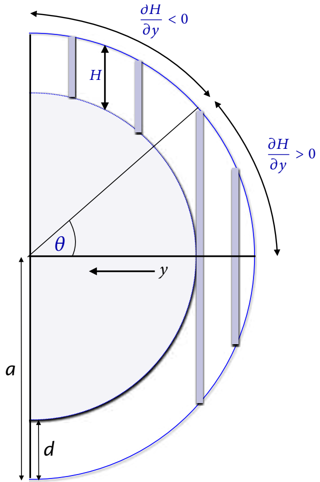
$$\frac{D\zeta}{Dt} + \beta^* v = 0 \qquad \text{where} \qquad \beta^* = -\frac{2\Omega}{H}\frac{\partial H}{\partial y}.$$

Just like flow in the weather layer!

$\beta^* > 0$ region insider the tangent cylinder (the extra-tropics)

$\beta^* < 0$ and outside the tangent cylinder, in the tropics.

$$\cos\theta = \frac{(a - d)}{a}$$



Jovian Superrotation

The topographic beta effect for deep jets

Homogenize the potential vorticity to give:

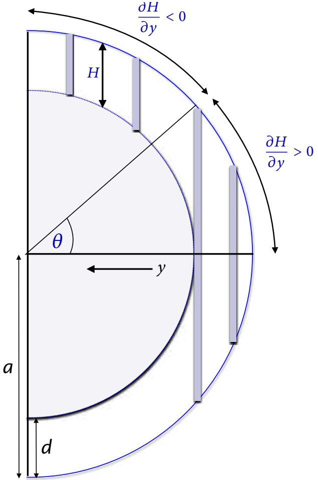
$$\frac{\partial \overline{u}}{\partial y} = A - |\beta^*|y, \qquad \text{or} \qquad \overline{u} = Ay - \frac{1}{2}|\beta^*|y^2 + B,$$

If $\partial\overline{u}/\partial y = 0$ at $y = 0$ then $A = 0$. $\overline{u}$ is a maximum at $y = 0$.

$\overline{u}$ is max at equator.

Jet width $\sim \sqrt{\overline{u}/\beta^*}$

NB: Weather-layer jets might not coincide with deep jets!



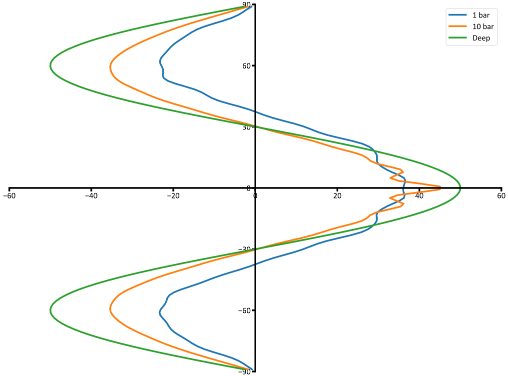
If $d = 3,500$ km and $a = 70,000$ km then $\theta = 18°$ (Jupiter).

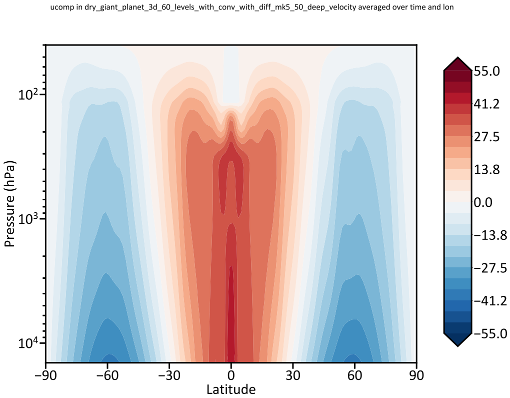
If $d = 9,000$ km and $a = 58,000$ km then $\theta = 32°$ (Saturn).

cf. Yano et al (2005), Vallis (2019)

Prescribe a Deep Flow

On a Weather-layer Jupiter GCM

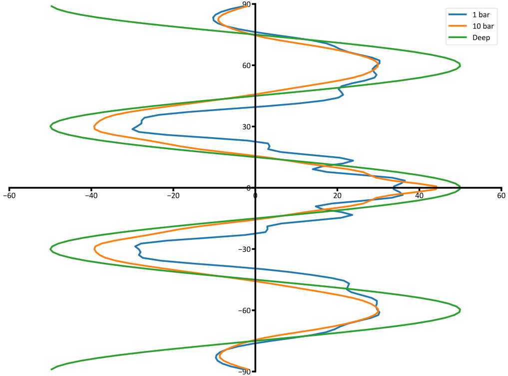


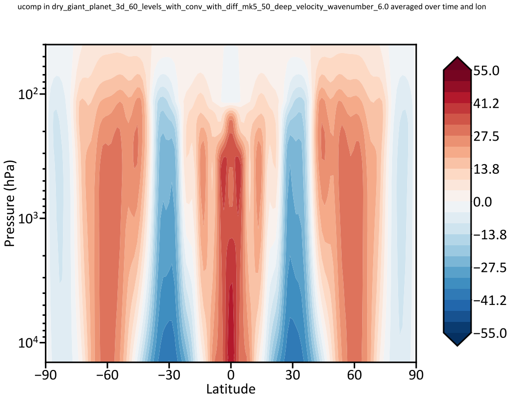


Thomson and Vallis, (in prep).

Prescribe a Deep Flow

On a Weather-layer Jupiter GCM





At upper levels the jets break down into more jets.

Preliminary, but demonstrates some decoupling between deep and shallow jets.

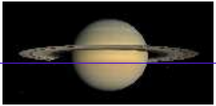
Thomson and Vallis, (in prep).

Jovian possibilities

Consistent with observations

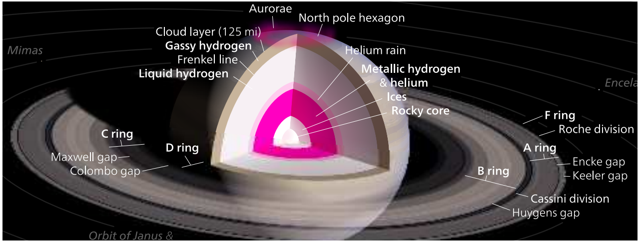
1. Jets descend to about 3000 km at all latitudes (Kaspi et al interpretation of Juno).
   Deep jets coincident with shallow jets.
2. Weather-layer jets are *decoupled* from deep jets at all latitudes.
   Weather-layer jets are shallow, beta-plane turbulence.
3. Mid-latitude jets are shallow, equatorial jets are deep.
4. Evidence that the equatorial and mid-latitude jets have a different character? – *Saturn!*

Saturn



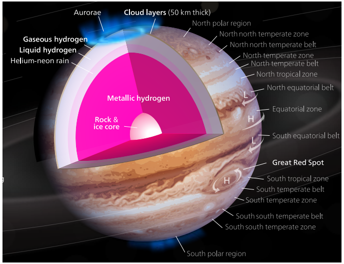
Natural light, Cassini, equinox.

Saturn:



- $a \approx 58,000$ km
- Sidereal day $\approx$ 10.5 hours
- Mass = 95 Earths = 5.7×10^{26} kg.
- Hydrogen becomes metallic at $r \leq 0.5a$? (30,000 km from surface).

Jupiter:

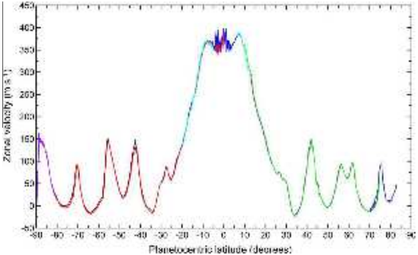


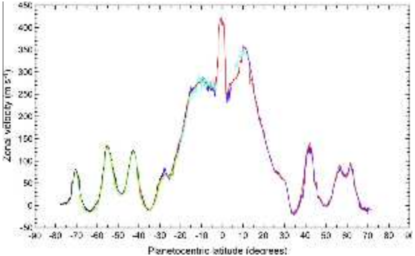
- $a \approx 70,000$ km
- Sidereal day $\approx$ 10 hours
- Mass = 318 Earths = 1.9×10^{27} kg.
- Hydrogen becomes metallic at $r \approx 0.8a$ (15,000 km from surface).

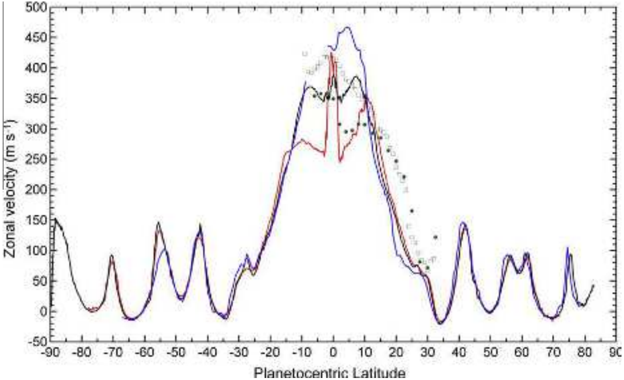
Saturn

Transition to metallic hydrogen is deeper, because the planet is smaller.

Equatorial winds are stronger and wider than Jupiter! (But maybe not wide enough? Being quantitative is difficult.)





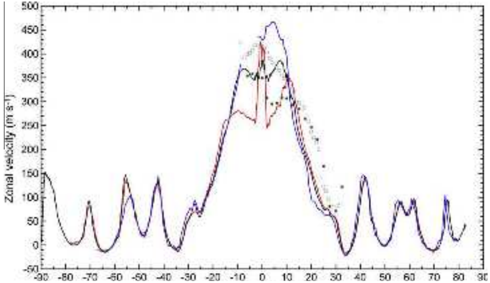


Various estimates from Garcia et al (2011), Cassini mission.

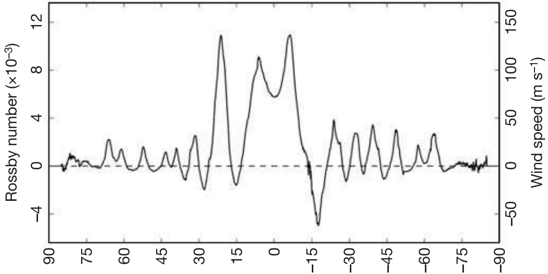
Saturn and Jupiter

Observed Zonal Winds

Saturn



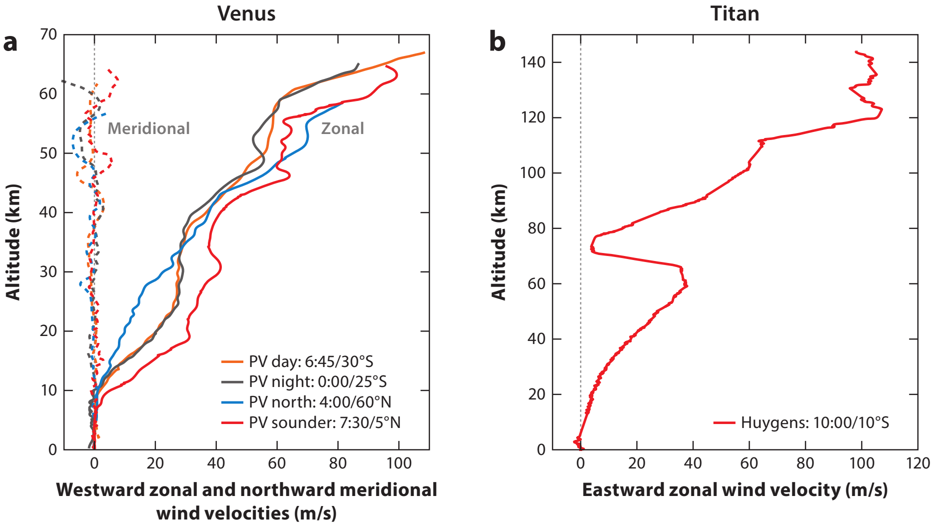
Jupiter



- Wider and stronger equatorial winds on Saturn.
- Suggests equatorial winds are connected to the deep.
- Mid-latitude zonal winds are stronger too…

Slowly Rotating Terrestrial Planets

Venus and TItan

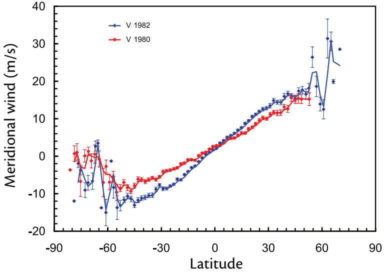


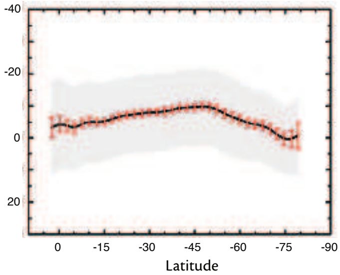
Vertical profiles of winds. Read & Lebonnois (2018), data from Counselman (1980) & Bird (2005).

Venus, cloud-tracked wind

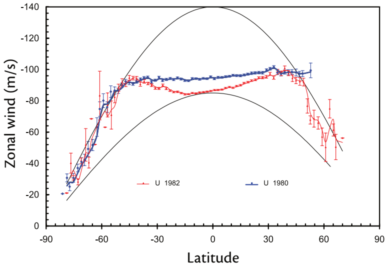
Dayside only

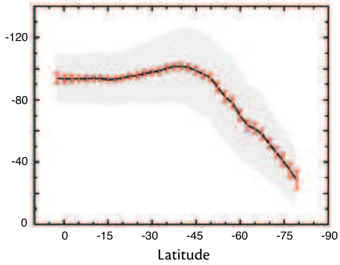
Venus Express orbiter.





Meridional wind





Zonal wind

Limaye (2007),

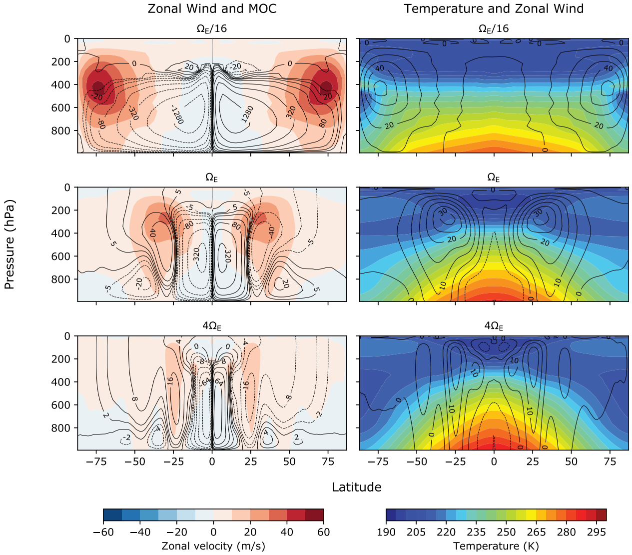
Khatuntsev et al (2013)

Zonal Winds and Hadley Cell for Low Rotation

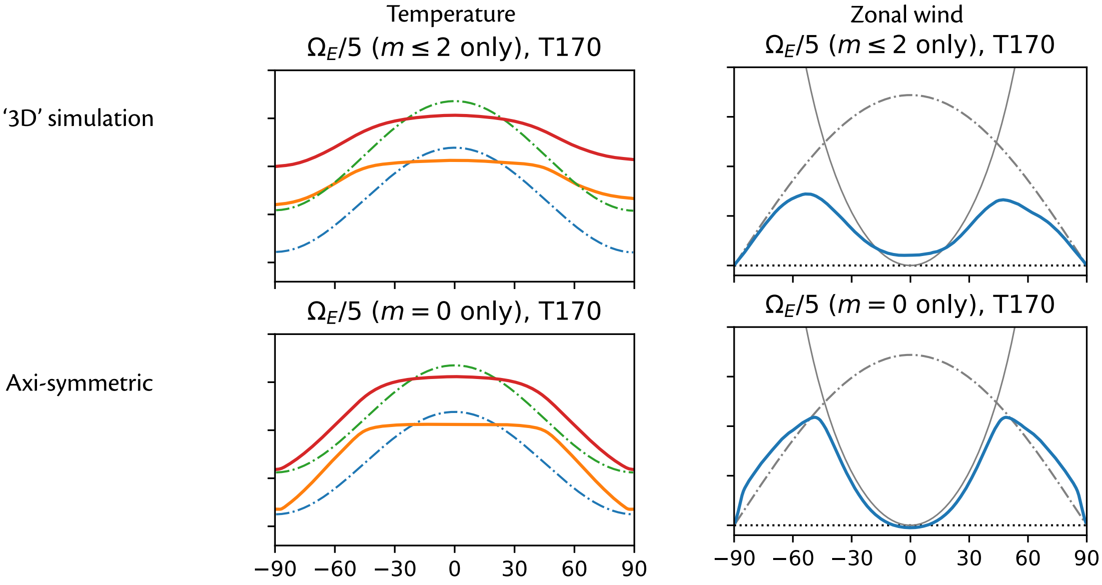
Earth-like planet, vary rotation.

- Hadley Cell extends polewards.
- Temperature gradient flattens.
- Zonal wind super-rotates as rotation falls.
- Venus GCMs vary considerably and are sensitive to parameter choices.

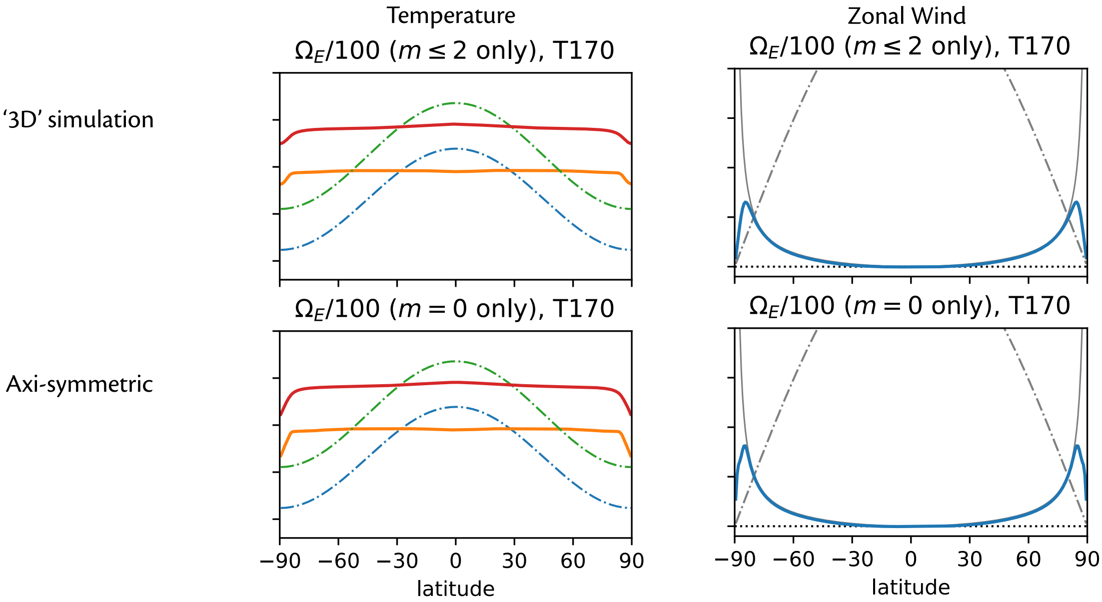
(Simulations by J. Eager with Isca.)



Temperature and Velocity, $\Omega = \Omega_E/5$

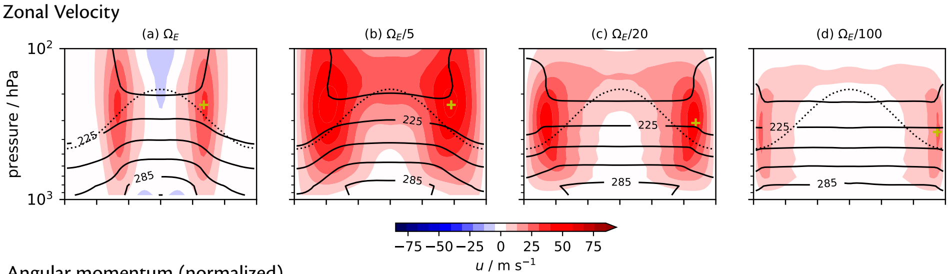


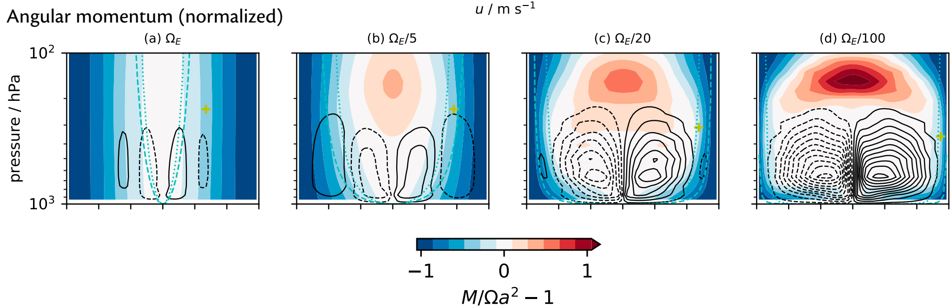
Temperature and Velocity, $\Omega = \Omega_E/100$



Super-Rotation

Spontaneously appears at low rotation

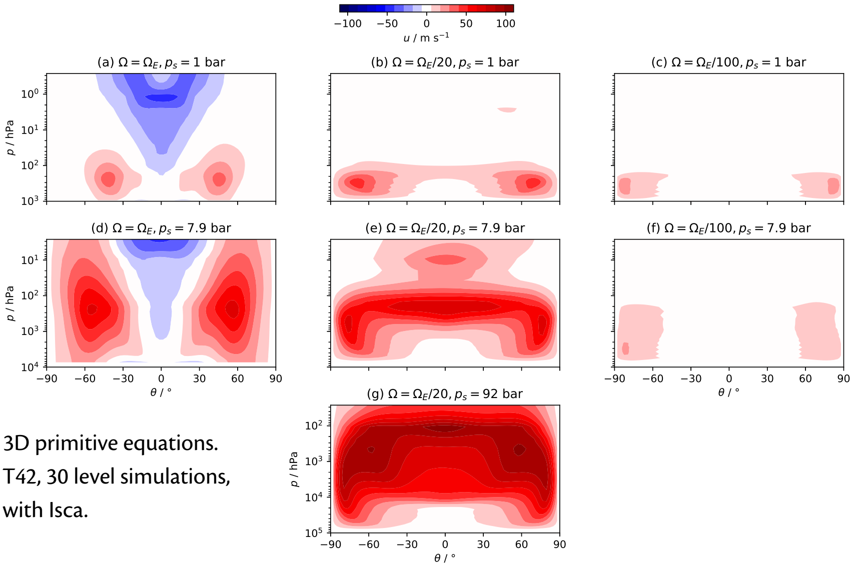




$M = (u + \Omega a \cos\vartheta) a \cos\vartheta$

Although super-rotating, zonal wind itself dies at very low rotation.

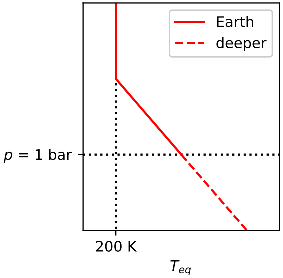
Zonal Mean Zonal Wind, Simulations



3D primitive equations. T42, 30 level simulations, with Isca.

(i) As Ω decreases, Hadley cell widens; zonal winds strengthen, then, at very low rotation rates, weakens.

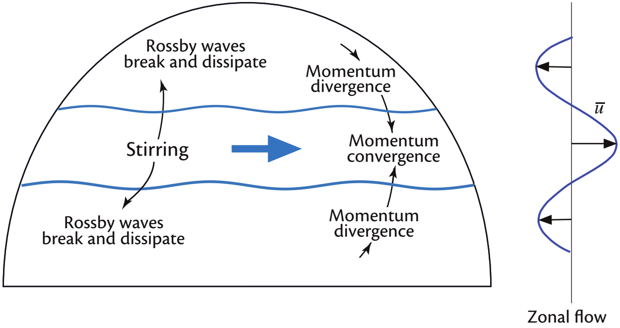
(ii) As p_s increases: super-rotating layer strengthens and deepens (between Hadley cell and tropopause)



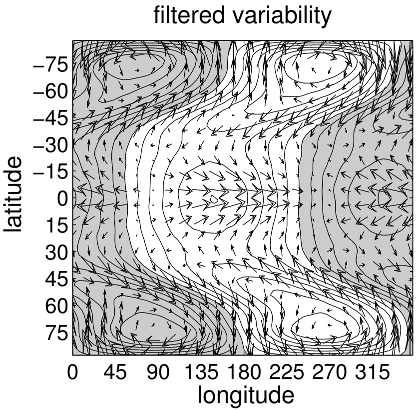
Super-Rotation

Mechanisms

Rossby Waves



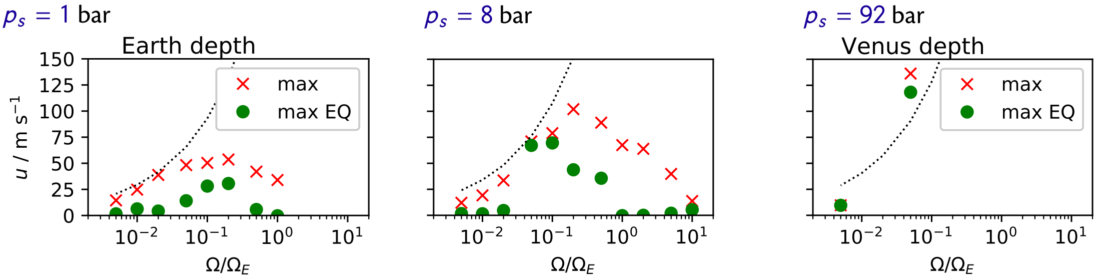
Rossby-Kelvin Wave interaction



At low rotation Kelvin waves and Rossby waves can interact to give an instability that converges momentum to the equator. (Potter, Mitchell & Vallis 2014).

Zonal Wind vs Ω

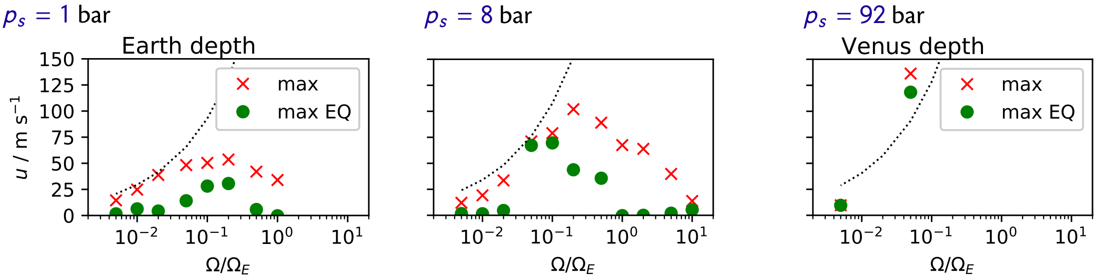
Although super-rotating, winds collapse as $\Omega \rightarrow 0$



Although super-rotating, solutions collapse at very low rotation rates, even at Venus depth.

ZONAL WIND VS Ω

Although super-rotating, winds collapse as $\Omega \to 0$

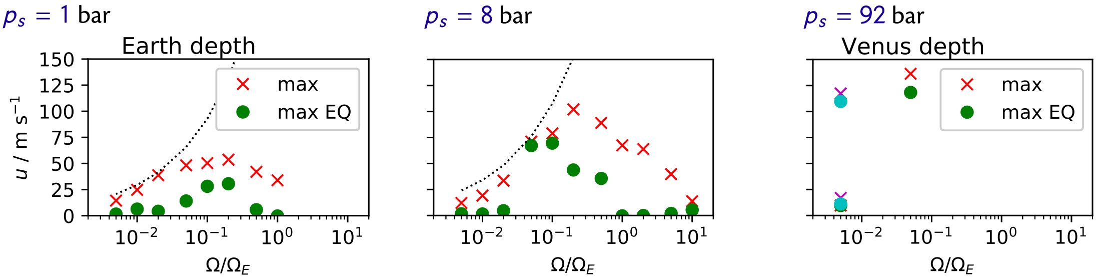


Although super-rotating, solutions collapse at very low rotation rates, even at Venus depth.

But wait!

Zonal Wind vs Ω

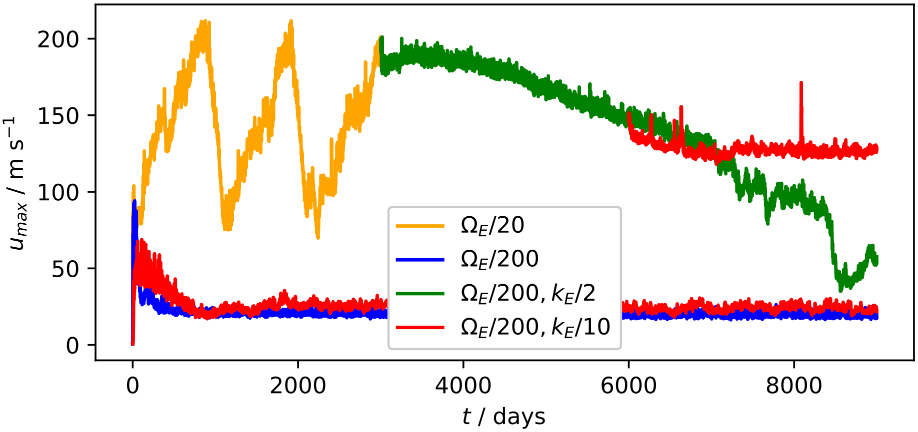
There is a second, highly super-rotating solution



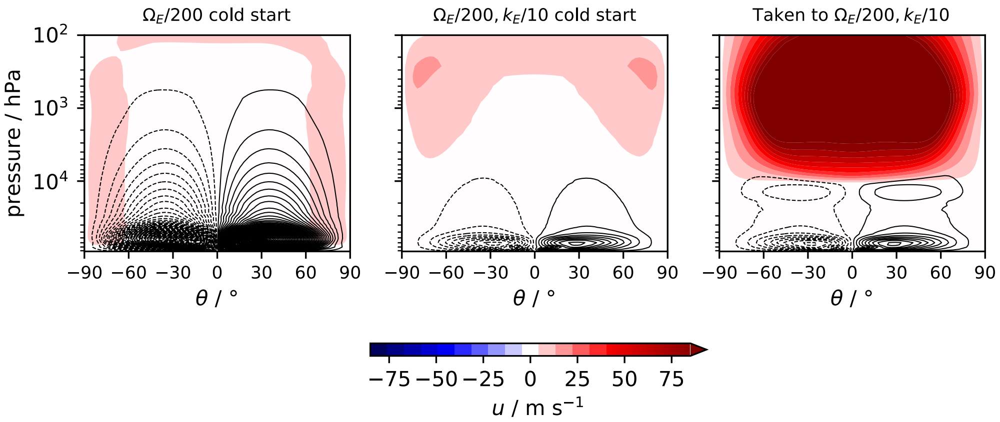
An additional solution exists at very low rotation rates at Venus depth.

Exactly same parameters, different initial conditions.

Different states appear with different initial conditions, provided the damping is small (radiative relaxation timescales are long).



Multiple states in Venus-like atmospheres

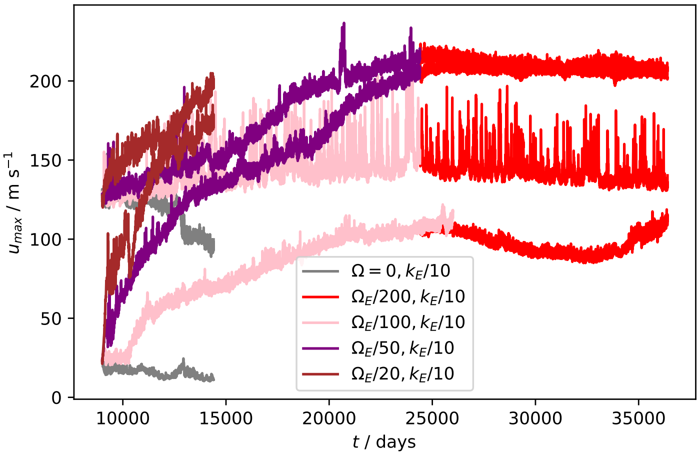


Rightmost two plots have same parameters.

Superrotation above main Hadley Cell with Venusian value of zonal wind. (cf Caballero et al; Lebonnois, 2016).

Multiple states in Venus-like atmospheres

Three possible solutions? More?



- Different initial conditions and pathways.
- Maybe many solutions?
- Superrotation is long-lived even if Ω is set to zero (grey).
  - A possible jet-like state at zero rotation? (grey lines)

Venus-like Planets are not all like Venus

- Superrotation is a generic feature at low rotation.

  Interaction of Kelvin and Rossby waves. (Mitchell, Vallis, Potter, Zurita-Gotor & Held, etc.)

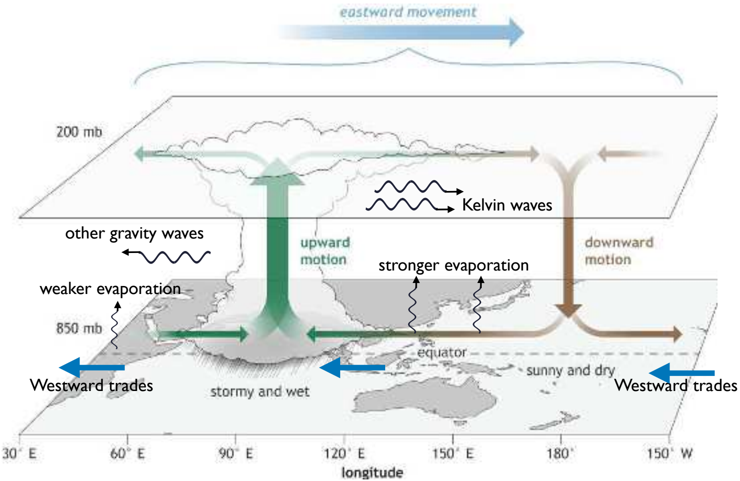
- One branch of solutions show zonal wind falling at low rotation rates, collapsing almost completely at Venusian rotation rates.

  Another branch shows high super-rotation at very low rotation rates (possibly even at zero rotation!). Occurs above Hadley Cell and In deep atmospheres only in our simulations.

- Exoplanets with slow rotation may not all behave like Venus!

The Madden–Julian Oscillation

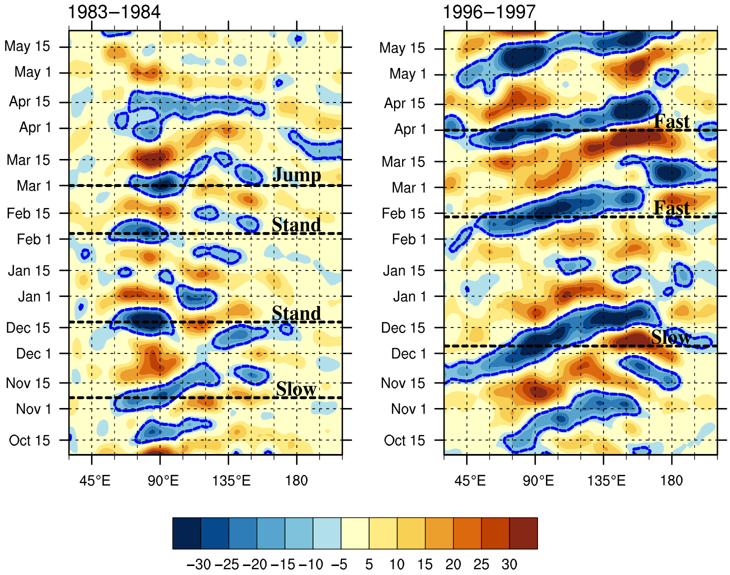
Intraseasonal oscillations and MJO: ‘*has been and still remains a holy grail … basic physical processes are still unknown*’ — Fuchs and Raymond (2017) being rather bleak.



Adapted from figures in NOAA and Met Office web sites

MJO Evolution

Outgoing Longwave Radiation



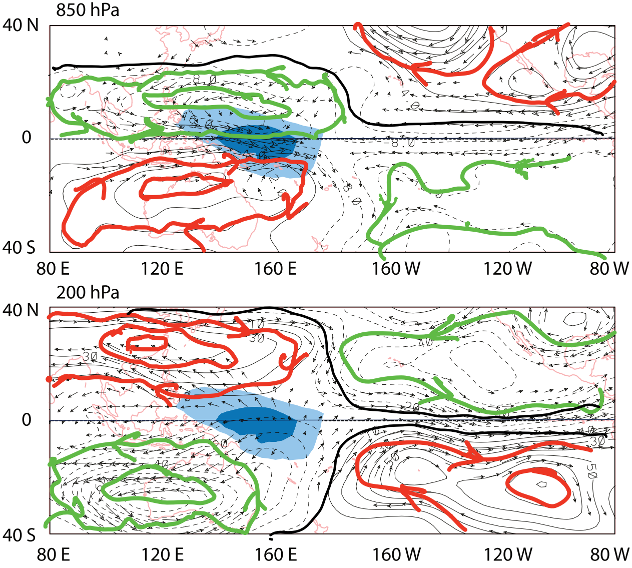
Hovmüller Plots, OLR averaged over 10° S–10° N

Blue is less OLR, more high cloud.

Wang et al 2019.

Observed Structure

Composite MJO structure



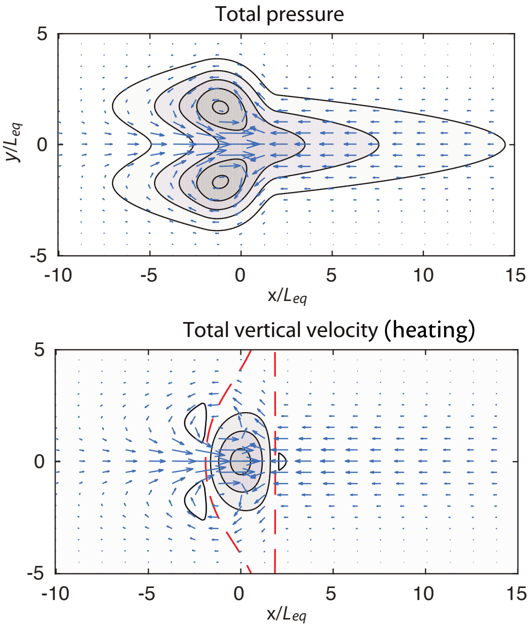
Kiladis

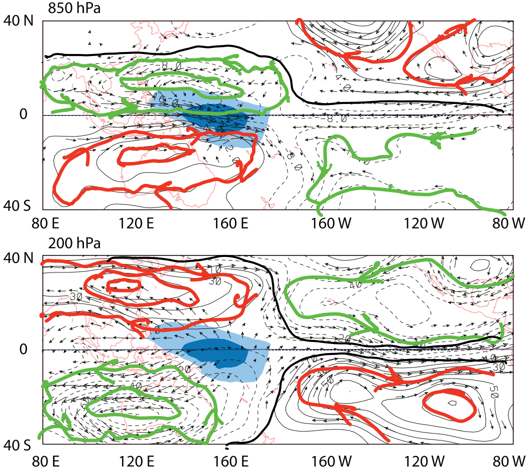
Clouds (OLR) – blue.

LInes – streamfunction

1. Dipole structure slightly west of clouds (blue).
2. First baroclinic mode in vertical.
3. Pattern moves slowly eastward at a few meters/sec.
4. Instantaneous picture (snapshot) is a mess — lots of isolated convection etc.

Matsuno–Gill Pattern



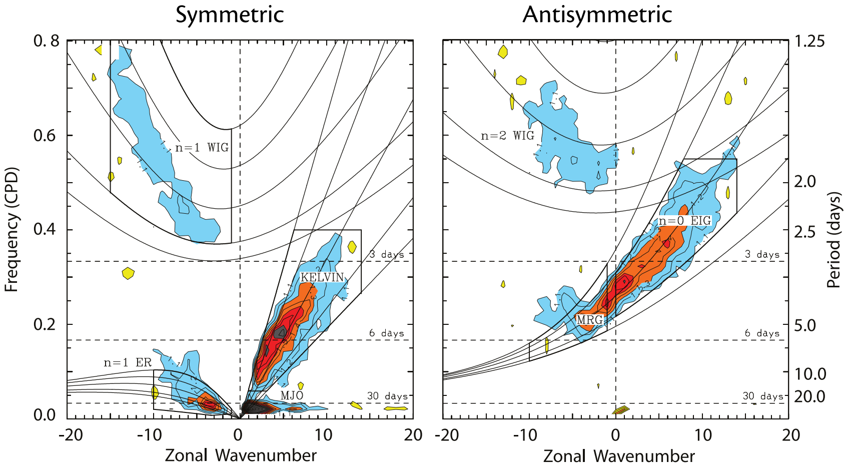


Suggests that the MJO is similar to a propagating Matsuno–Gill pattern.

Convergence is almost coincident with the heating.

Observed Spectra

Wheeler–Kiladis diagram



Kiladis et al (2009)

cloud brightness

(i) Gravity, Kelvin and Rossby waves.

(ii) Additional signal at low frequency and large scale.

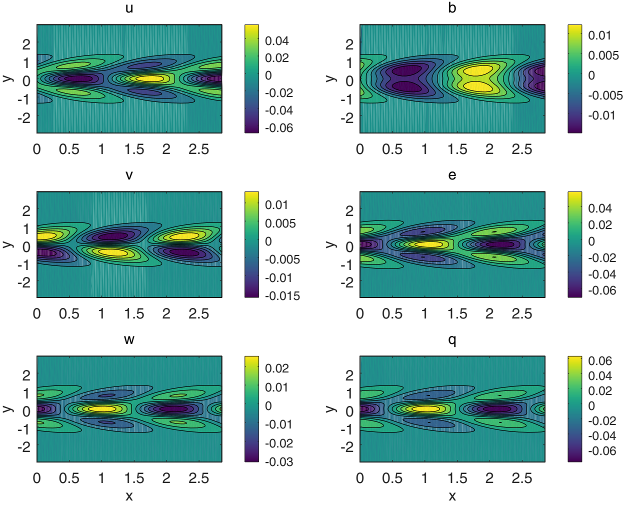
Previous Theories and Mechanisms

'Moisture Modes'

- Restrict vertical structure to a single baroclinic mode.
- Make an assumption about horizontal scales and/or structure – e.g., small number of horizontal modes, or build in a Matsuno–Gill structure.
- Make some parameterization of convection.
- Seek to solve semi-analytically and/ore reduce to ODEs. Look for travelling disturbances/large-scale instabilities/waves etc

Fuchs & Raymond, Majda, Stechman and collaborators, Soebl

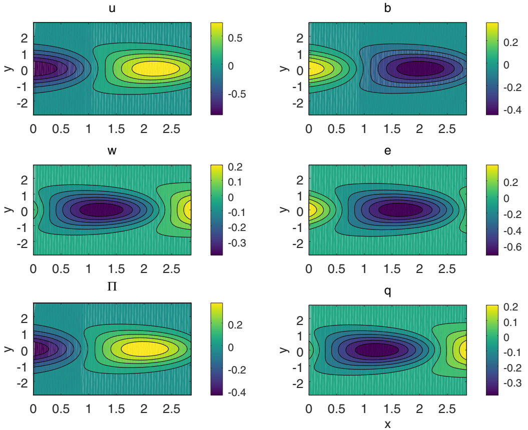
Modes like this:



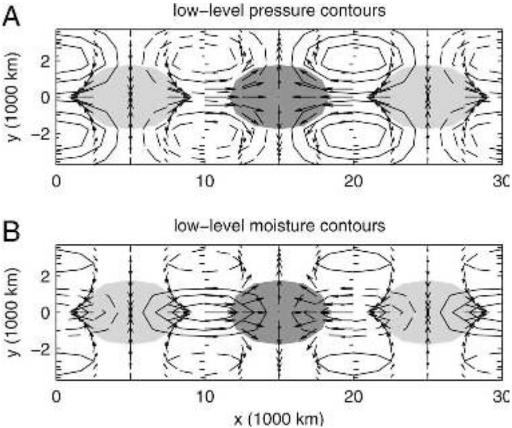
From Fuchs and Raymond 2017.

Moisture Modes

Examples



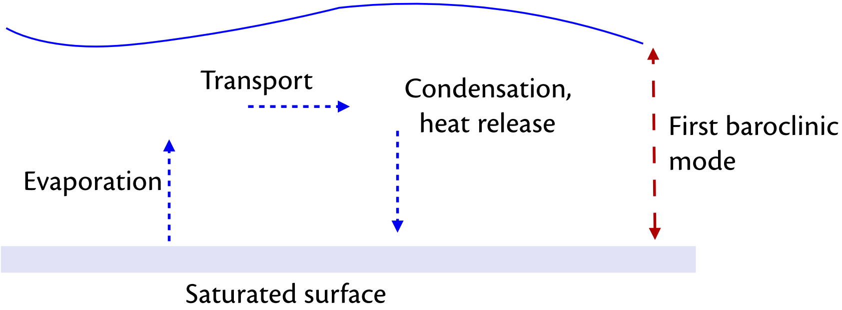
Planetary wave 1 from Fuchs and Raymond 2017.



Wavenumber 2 MJO mode from the Majda–Stechman skeleton model

METHODOLOGY

Moist Shallow-Water System



- Simple, explicit model.
  - Constrain vertical structure (gives shallow water equations).
  - Add a moisture variable, with evaporation, advection, condensation.
  - Do not constrain the horizontal structure.

    Do not parameterize convection, except in a very basic way.
- Solve, analyze and hopefully understand the resulting system.

Moist Shallow Water Equations

Represent the first baroclinic mode of the primitive equations

$$\text{momentum:} \qquad \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{u} = -g\nabla h - r\boldsymbol{u}, \tag{1}$$

$$\text{thermodynamic:} \qquad \frac{\partial h}{\partial t} + \nabla \cdot (h\boldsymbol{u}) = -L_v C - \lambda_r (h - h^*) \tag{2}$$

Moist tracer:

$$\frac{\partial q}{\partial t} + \nabla \cdot (q\boldsymbol{u}) = E - C, \tag{3}$$

where E = evaporation, C = condensation, L_v = latent heat of vaporization.

$$E = C_D |\boldsymbol{u}| (q_{\mathrm{surf}} - q),$$

$$C = \mathcal{H}(q - q_{\mathrm{sat}}) \frac{(q - q_{\mathrm{sat}})}{\tau}, \qquad q_{\mathrm{sat}} = q_0 \exp(-\alpha h)$$

Fast condensation (τ is small) on saturation ($\mathcal{H}$ = Heaviside function) determined by Clausius–Clapeyron.

NB: $\Theta = h - L_v q$ is adiabatically conserved:

$$\frac{\partial \Theta}{\partial t} + \nabla \cdot (\Theta \boldsymbol{u}) = 0 \qquad (\Theta \sim \text{'equivalent potential temperature'}) \tag{4}$$

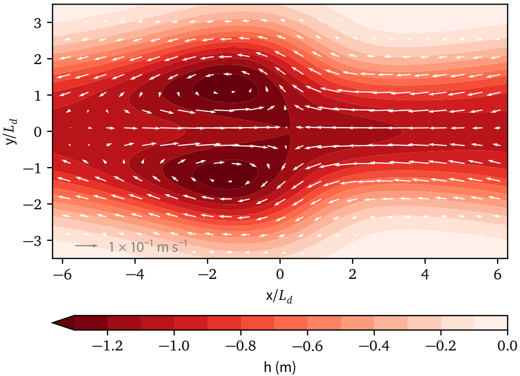
Solution with Moisture as a Passive Tracer

On a beta plane

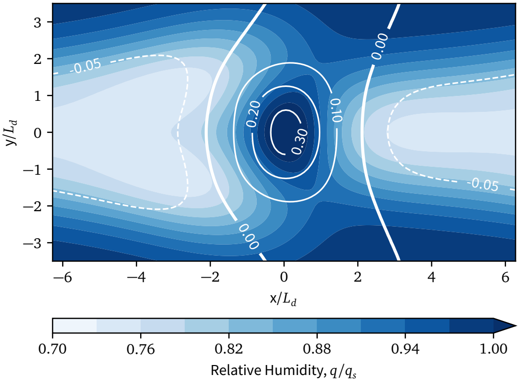
Imposed heat source, step forward time-dependent equations.

Moisture: White contour: $q/q_0 - 1$.

Height Field



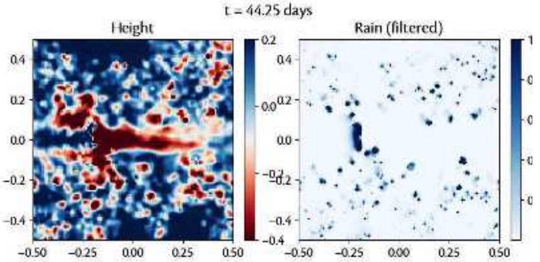
Blue – Relative Humidity

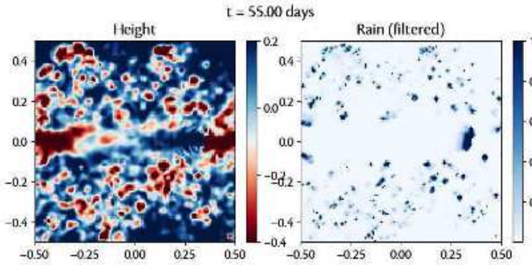


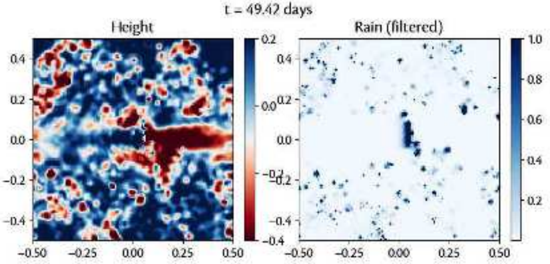
Relative humidity maximum slightly east of maximum heating.

Low relative humidity areas in east on equator, and in Rossby lobes.

Snapshots of an Unsteady Flow







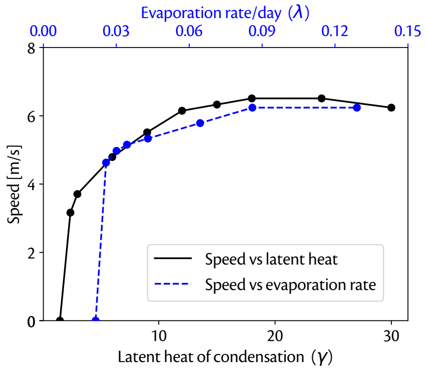
- Steady eastward progression of precipitation ***at about 5 m/s.***
- Speed dependent on evaporation and condensation parameters (less on Kelvin speed).
- WISHE helps, but eastward propagation can occur without it.
- Width of propagating region dependent on deformation radius, $\sqrt{\beta / c}$

Self Sustaining

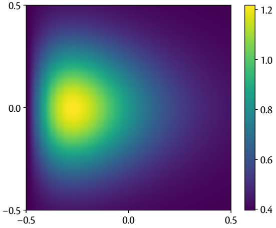
On the β-plane

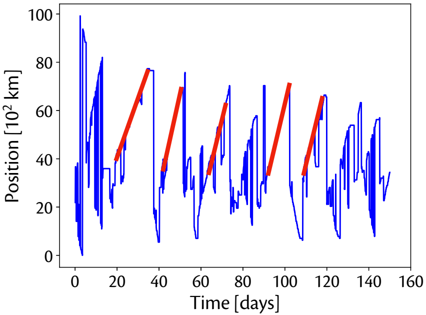
Speed and Recurrence

Speed vs Evaporation and vs Latent Heat



SST (Moisture)



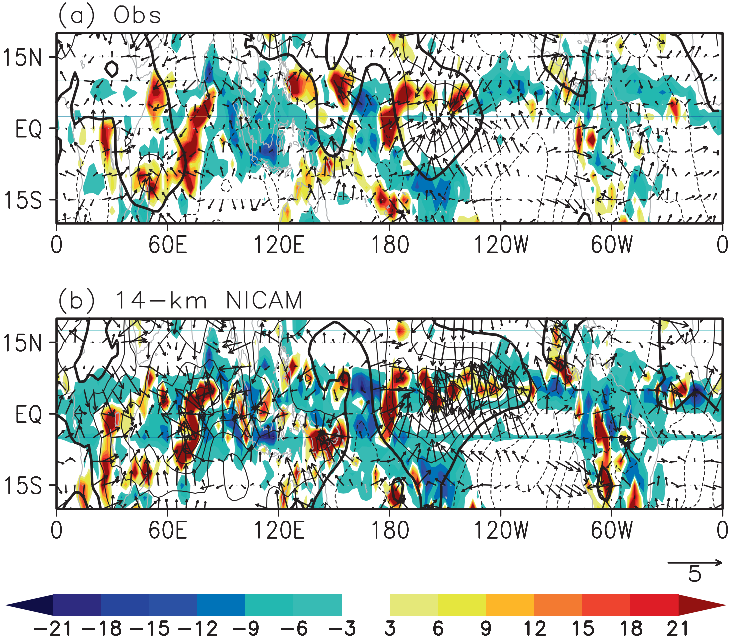


MJO position vs time.

Observations and High Res 3D Model (NICAM)

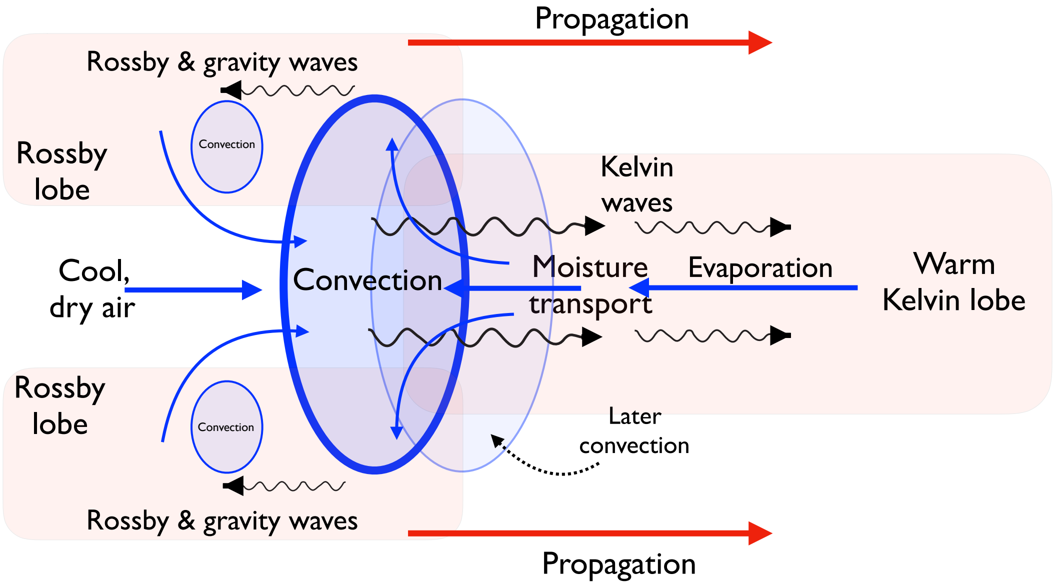
Almost-initial state (1 day).

Precipitation and velocity (925 hPa)



Liu et al 2009

Propagation Mechanism



Eastward Propagating Equatorial Disturbances

An MJO Mechanism

Why at the Equator?

(i) Equatorial aggregation via beta-plane ('Matsuno–Gill') dynamics. A disturbance at the equator leads to convergence, aggregation and self-maintained convection. (Not 'self'-aggregation.)

(ii) Off-equatorial disturbances generates rotational motion, less convergence.

Why Eastward?

(i) Convection at the equator produces convergence, and Kelvin waves that propagate east. Trigger more convection, more waves, and so on. (cf., waves in an excitable medium.)

(ii) WISHE (wind-induced evaporation). Can kill EPEDs by reversing the sign of mean wind.

Why don't all GCMs get MJOs?

(i) Involves self-sustaining convection and/or stochasticity.

- — Need to generate gravity waves and convergence of moist air.
- — A parametrization will reduce the gravity wave generation and the excitability.
- — cf. GCMs failure to simulate QBO, but some low-ish resolution CRMs can get an MJO.

(ii) Mechanism is rather sensitive to parameters (e.g. moisture availability).