A model experiment of the quasi-biennial oscillation

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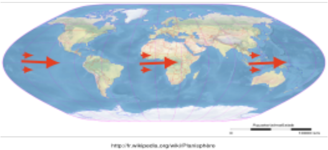
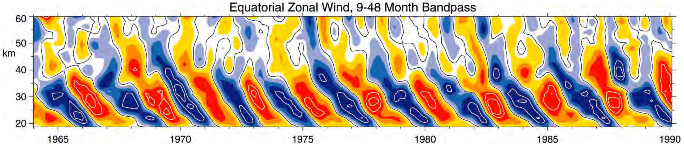
The atmospheric quasi-biennial oscillation (QBO)





Time-height section. Red : eastward wind. Blue : westward wind.

Baldwin et al. 2001

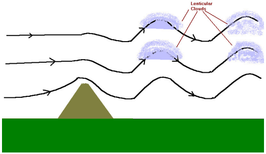
- Zonal (azimuthal) wind in the equatorial stratosphere (16-50 km).
- Amplitude $20\ \mathrm{m \cdot s^{-1}}$
- Period about 28 months: not linked to any astrophysical forcing.
- Internal gravity waves/mean flow interaction. $T_{\mathrm{wave}} < 2\mathrm{days}$
- Similar winds on Jupiter and Saturn.

Internal gravity waves : geophysics

- Internal gravity waves propagate in stably stratified fluids subjected to gravity
- Inertia of the fluid. Restoring mechanism: gravity

- Ubiquitous in geophysical and astrophysical flows
- Lenticular clouds

i.pinimg.com/



http://www.prh.noaa.gov/hnl/pages/events/lenticular/

- Mixing in the ocean

Internal gravity waves

- Minimal model: Euler equation, mass conservation, incompressibility:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p + \rho \mathbf{g}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\nabla \cdot (\mathbf{v}) = 0$$

Constant Brunt-Väisälä frequency:

$$N^2 = -\frac{\mathrm{d}\rho_0}{\mathrm{d}z} \frac{g}{\rho_0}$$

- Linearisation of the equations
  Plane waves solutions
  $\mathbf{v}_0 = \mathbf{v} \exp(i(\mathbf{k}.\mathbf{x} - \omega t))$

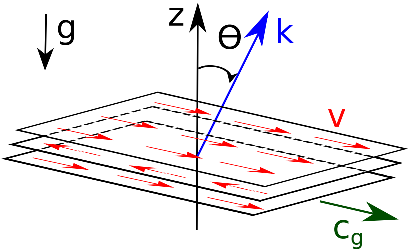
- Dispersion relation:

$$\omega^2 = N^2 \frac{k_x^2 + k_y^2}{k_x^2 + k_y^2 + k_z^2}$$

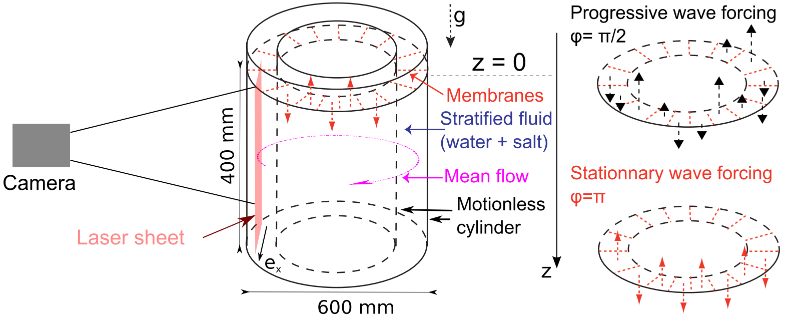
$$\omega = N|\sin(\theta)|$$

Anisotropic and dispersive waves

- From incompressibility: $\mathbf{v}.\mathbf{k} = \mathbf{0}$
  Group velocity direction: $\mathbf{c_g}.\mathbf{k} = \mathbf{0}$



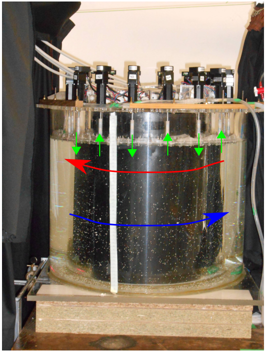
Experimental setup (1)

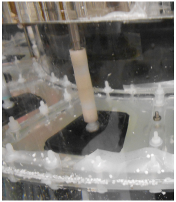


Plumb and McEwan (1978) , Otobe et al. (1998)

- Väisälä-Brunt angular frequency $N = \sqrt{-\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\frac{g}{\rho_0}} \sim 1.5 - 2.2 \,\mathrm{rad}\cdot\mathrm{s}^{-1}$
- Forcing period $T \sim 15 - 38$ s.
- Forcing amplitude: $M \leq 15$ mm.
- Visualisation : particles, laser sheet. Particle image velocimetry (PIV)

Experimental setup (2)



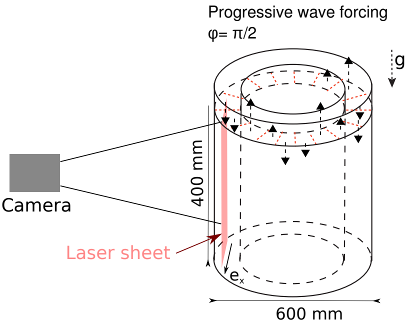


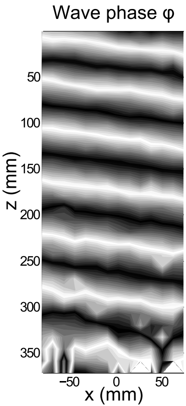
- Each membrane is controlled individually
- Continuous injection of fluid at the bottom and sucking of the fluid at the top (very small velocity $< 10\ \mu m \cdot s^{-1}$): linear stratification despite of the mixing by the membrane motion.

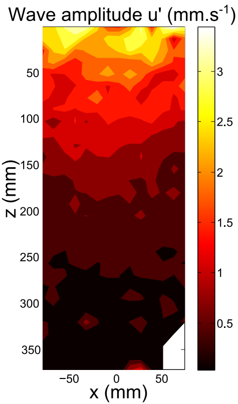
Wave

Progressive wave forcing: 1 wave

Fit in each point of the horizontal velocity: $u(t) = u' \sin(\omega t + \varphi) + \bar{u}$
with u': wave amplitude, φ: initial phase, $\bar{u}$: mean flow





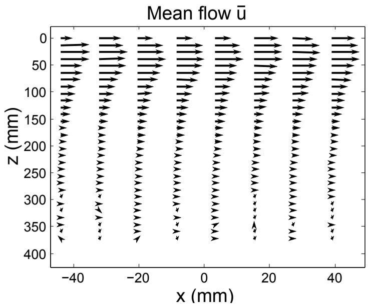


Forcing amplitude: $M = 8$ mm, $T = 28$ s, $N = 1.5$ rad $\cdot$ s^{-1}, steady state.

$\Longrightarrow$ The component oscillating at ω has a wave structure.

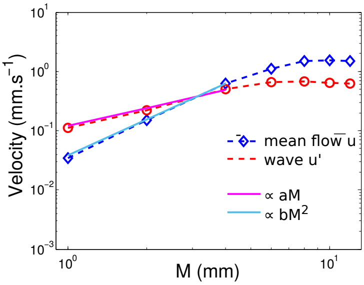
Mean flow

Fit in each point of the horizontal velocity: $u(t) = u' \sin(\omega t + \varphi) + \bar{u}$
with u': wave amplitude, φ: initial phase, $\bar{u}$: mean flow



Forcing amplitude: $M = 8$ mm, $T = 28$ s, $N = 1.5$ rad $\cdot$ s^{-1} steady state.

Analysis using Fourier Transform.



Forcing amplitude: $T = 38$ s, $N = 1.5$ rad $\cdot$ s^{-1}, $z = 64$ mm, steady state.

⟹Always a mean flow.
⟹The mean flow is homogeneous in the x direction (2D).
⟹Mean flow proportional to the square of the wave at low M.

Variation with forcing amplitude M

- Navier-Stokes equation (3D):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \eta\Delta\mathbf{v} + \rho\mathbf{g}$$

- Velocity: $\mathbf{u} = (u' + \bar{u})\mathbf{e_x} + (v')\mathbf{e_z}$
- Model from Navier-Stokes:

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial F}{\partial z} + \nu\frac{\partial^2 \bar{u}}{\partial z^2} - \gamma\bar{u}$$

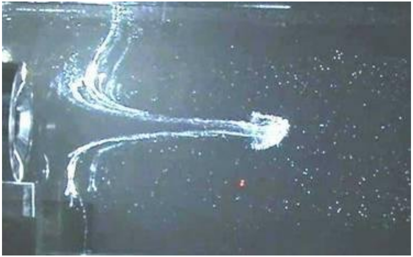
with ν: kinematic viscosity
γ: wall friction

$$F = \overline{u'v'}$$

with v' the vertical wave component

- Prediction of the model: in steady state, without feedback of the mean flow on the wave: $\bar{u} \propto (u')^2$.
  In agreement with experimental data.

Generation of an Eulerian mean flow by a wave



Acoustic streaming, W. Dridi et al.

Examples of mean flows induced by waves:

- Motion of liquid metals in electromagnetic pumps
- Eckhart acoustic streaming

- Decomposition of the velocity:

$$\mathbf{v} = \underbrace{\overline{\mathbf{V}}}_{\text{mean flow}} + \underbrace{\mathbf{v}'}_{\text{wave}} \qquad (1)$$

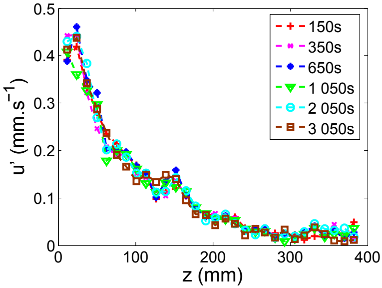
- Momentum equation (Navier-Stokes):

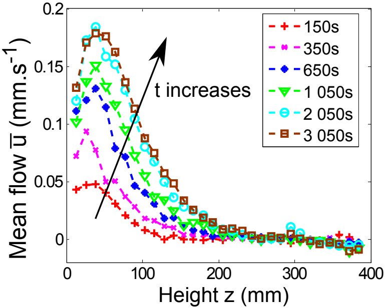
$$\rho \frac{\mathrm{d}\bar{V}_i}{\mathrm{d}t} = -\partial_i p + \eta \Delta V_i - \frac{\partial(\rho \overline{v_i' v_j'})}{\partial x_j} \qquad (2)$$

- To generate a mean flow using waves in a steady regime, the waves must vary spatially (due to dissipation for example).

Linear regime

Linear regime, $M = 2$ mm



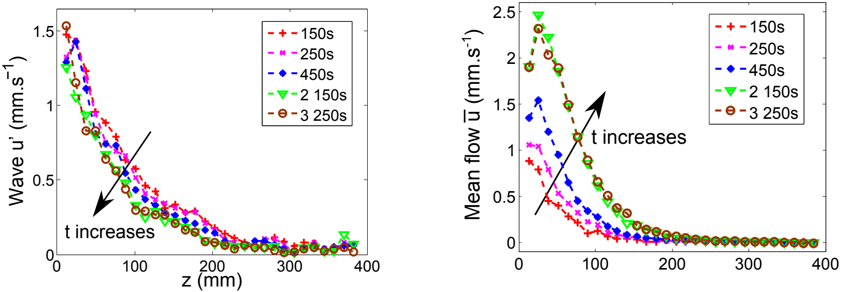


Forcing amplitude: $T = 38$ s, $M = 2$ mm, $N = 1.5$ rad $\cdot$ s^{-1}.

Constant wave amplitude u' : no feedback .
Mean flow increases and reaches a steady state in a timescale much longer than T.

Non-linear regime

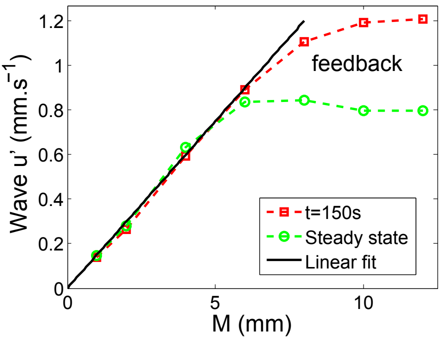
Non-linear regime, $M = 8$ mm



Forcing amplitude: $T = 38$ s, $M = 8$ mm, $N = 1.5$ rad $\cdot$ s^{-1}.

The wave amplitude u' is not constant : feedback of the mean flow on the wave.
Mean flow increases and reaches a steady state.

Feedback



Forcing amplitude: $T = 38$ s, $N = 1.5$ rad $\cdot$ s^{-1}, $z = 64$ mm.

- Model from Navier-Stokes, mass conservation, in Boussinesq and WKB approximations (from Plumb and McEwan 1978):

$$F(\xi) = F(0) \times \exp\left(-\int_0^{\xi}\left[\frac{1}{(1-\bar{U})^4}\right]d\xi\right)$$

$$F = \overline{u'v'}$$

$$c = \frac{\omega}{k_x} \quad \text{horizontal phase velocity}$$

$$d = \frac{k_x c^4}{N^3 \nu} \quad \text{vertical dissipation length}$$

$$\xi = \frac{z}{d}$$

$$\bar{U} = \frac{\bar{u}}{c}$$

$\Longrightarrow$Negative feedback of mean flow on wave

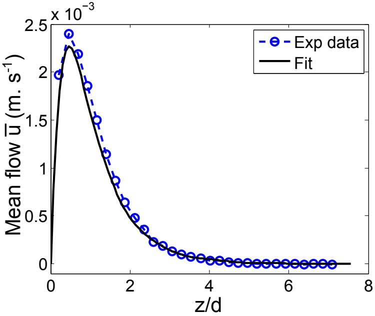
$\Longrightarrow$Feedback increases the momentum transfer from wave to mean flow at $z = 0$

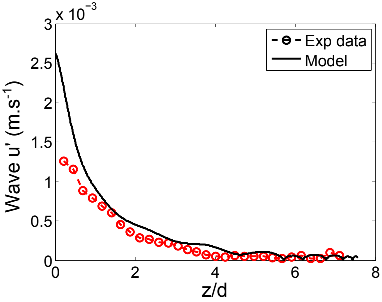
Comparison with the model

$$\frac{\partial \bar{U}}{\partial t} = -d\frac{\partial F}{\partial \xi} + \nu d^2 \frac{\partial^2 \bar{U}}{\partial \xi^2} - \gamma \bar{U}$$

$$F(\xi) = F(0) \times \exp\left(-\int_0^{\xi}\left[\frac{1}{(1-\bar{U})^4}\right]\mathrm{d}\xi\right)$$

Comparison with the model. One single fitting parameter $F(0)$.





Forcing amplitude: $T = 38$ s, $M = 8$ mm, $N = 1.5$ rad $\cdot$ s^{-1}, steady state.

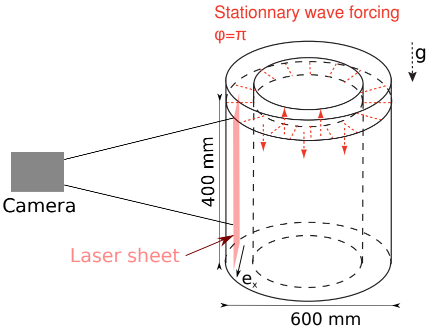
Good agreement between the model and experimental data

Progressive wave forcing: conclusion

- Dissipation of the wave due to viscosity
- Generation of a mean flow
- Negative feedback of the mean flow on the wave
- 1D model sufficient

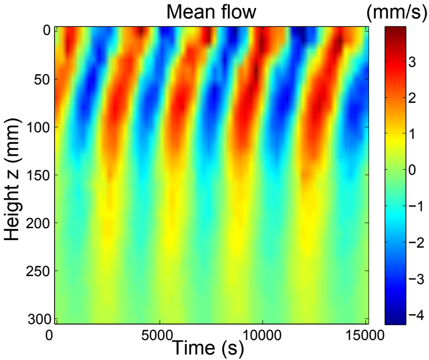
Stationnary wave forcing

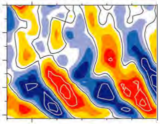
Stationnary wave forcing: 2 waves



Mean flow

Stationnary wave forcing: 2 waves





Oscillation in the atmosphere

Forcing amplitude: $M = 13.5$ mm, $T = 17$ s, $N = 1.5$ rad $\cdot$ s^{-1}, $T_{QBO} = 3\,200$ s

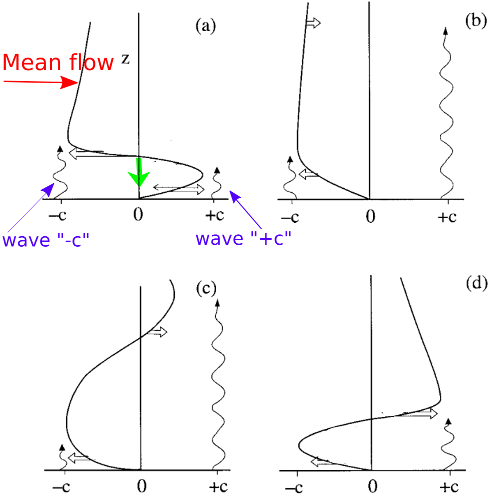
$\Longrightarrow$ No mean flow below threshold forcing amplitude.

$\Longrightarrow$ Observation of an oscillation similar to the one of the atmosphere

Basic mechanisms of the QBO

- 2 internal gravity waves with opposite phase velocity in the azimuthal direction
- Dissipation of the waves.

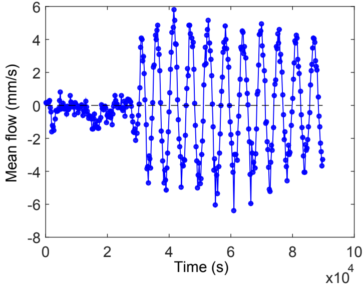
- The transfer of momentum from the wave to the mean flow is more efficient if c and the mean flow V are of the same sign (positive feedback).
- Momentum transfered from a wave to the mean flow can not be transfered above.
- Dissipation of the large mean flow gradient.

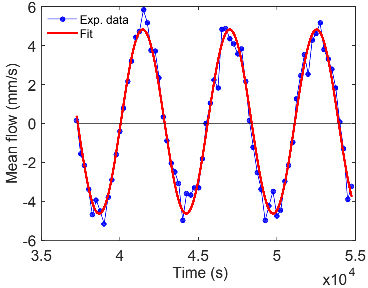


Adapted from Baldwin et al 2001

Mean flow at a given height

Height: $z = 88$ mm



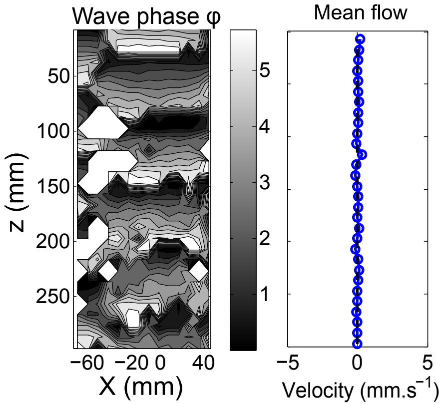


Forcing amplitude: $M = 13.5$ mm, $T = 15$ s, $N = 1.5$ rad $\cdot$ s^{-1}, NaCl

Cosine fit : $V = A\cos(\omega t + \Phi) + V_n$

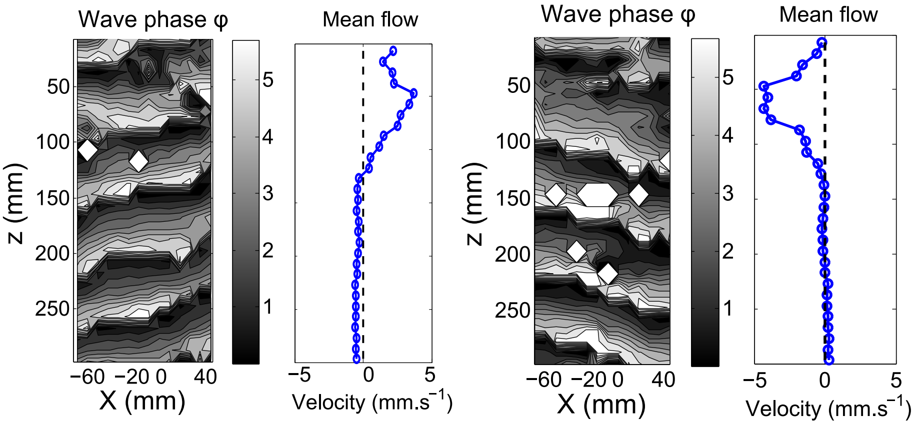
The mean flow may appear after a very long time.
Sine oscillation of the mean flow.

Wave without mean flow



Initial state, no mean flow.

Wave and mean flow



Permanent state, time t_1

$\Longrightarrow$ Feedback of the mean flow on the waves

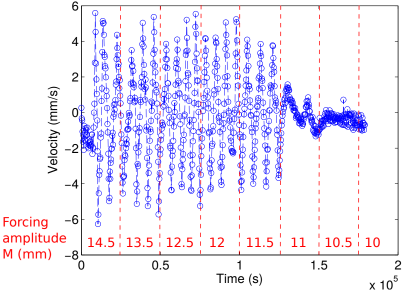
Permanent state, time $t_1 + 2\,500$ s

Influence of the stratification

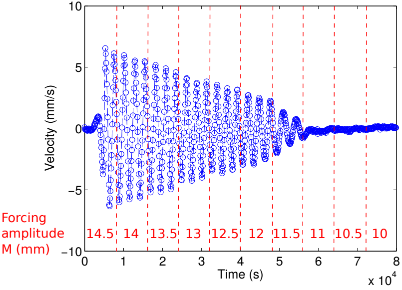
$T = 15$ s, $z = 30$ mm

Mean flow

$N = 1.5\ \mathrm{rad \cdot s^{-1}}$, NaCl



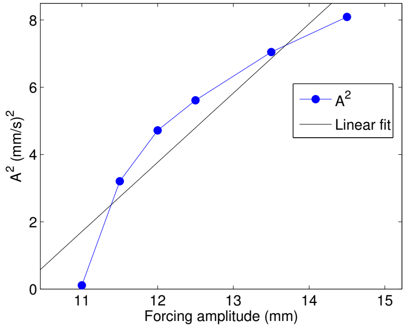
$N = 2.15\ \mathrm{rad \cdot s^{-1}}$, $MgCl_2$



Bifurcation

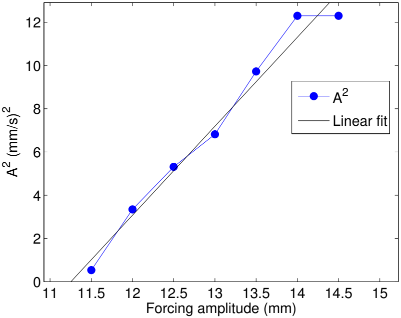
$T = 15$ s, A: vertical average of mean flow amplitude

$N = 1.5$ rad $\cdot$ s^{-1}, NaCl



Subcritical bifurcation

$N = 2.15$ rad $\cdot$ s^{-1}, $MgCl_2$



Supercritical bifurcation

$\Longrightarrow$Nature of the bifurcation depends on the parameters

Yoden and Holton (1988)

Dimensionless model

$$\frac{\partial U}{\partial T} = -\frac{\partial D}{\partial \xi} + \Lambda_1 \frac{\partial^2 U}{\partial \xi^2} - \Lambda_2 U \quad (3)$$

Forcing with two counter-propagating waves:

$$D(\xi) = \exp\left(-\int_0^\xi \frac{1}{(1-U)^4} \mathrm{d}\xi\right) - \exp\left(-\int_0^\xi \frac{1}{(1+U)^4} \mathrm{d}\xi\right) \quad (4)$$

with the scales:

- velocity: $c = \omega / k_x$
- length: dissipation length $d = (k_x c^4)/(N^3 \nu)$
- time: $cd/(F(0))$

and the parameters:

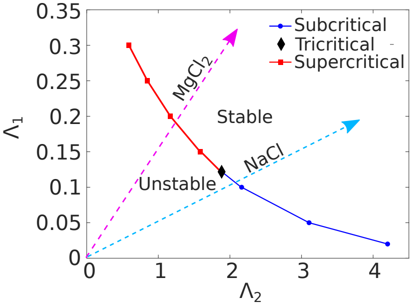
- $\Lambda_1 = \nu c/(F(0) d)$
- $\Lambda_2 = \gamma cd/(F(0))$

This model can be solved semi-analytically close to the threshold.

Threshold

$$\frac{\partial U}{\partial T} = -\frac{\partial D}{\partial \xi} + \Lambda_1 \frac{\partial^2 U}{\partial \xi^2} - \Lambda_2 U$$

- $\Lambda_1 = \nu c/(F(0)d)$
- $\Lambda_2 = \gamma c d/(F(0))$



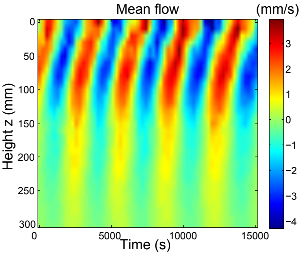
Yoden and Holton 1988

⟹Hopf bifurcation

⟹Nature of the bifurcation depends on the form of the dissipation

Conclusion

- A damped progressive internal wave induces a mean flow
- Feedback of the mean flow on the wave at high forcing amplitude
- Threshold for mean flow in symetric forcing (2 counter-propagating waves)
- Observation of mean flow reversal.
- Nature of the bifurcation is supercritical or subcritical, depending on dissipation



Perspectives:

- Link with atmospheric QBO: bifurcation in general circulation models (GCM) and in atmosphere.

B. Semin, N. Garroum, F. Pétrélis, S. Fauve, "Nonlinear saturation of the large scale flow in a laboratory model of the quasibiennial oscillation", Phys. Rev. Lett. 121, 134502 (2018)
B. Semin, G. Facchini, F. Pétrélis, S. Fauve, "Generation of a mean flow by an internal wave", Phys. Fluids, 28, 096601 (2016).