Interplay of Planetary Waves and Stochastically forced Turbulence in the Tropics on a Shallow Earth

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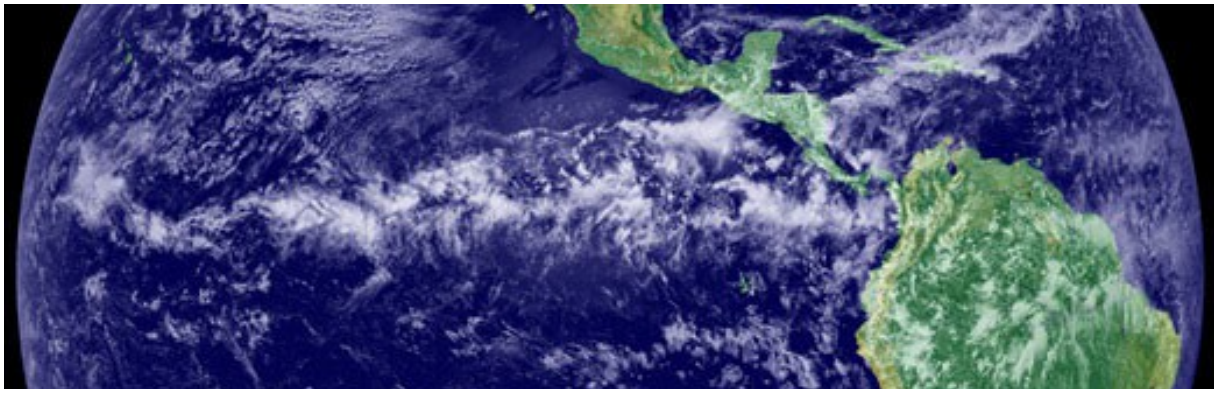

A band of clouds forms in the inner tropical convergence zone (Physics Today).

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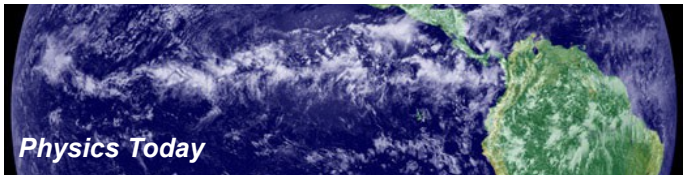
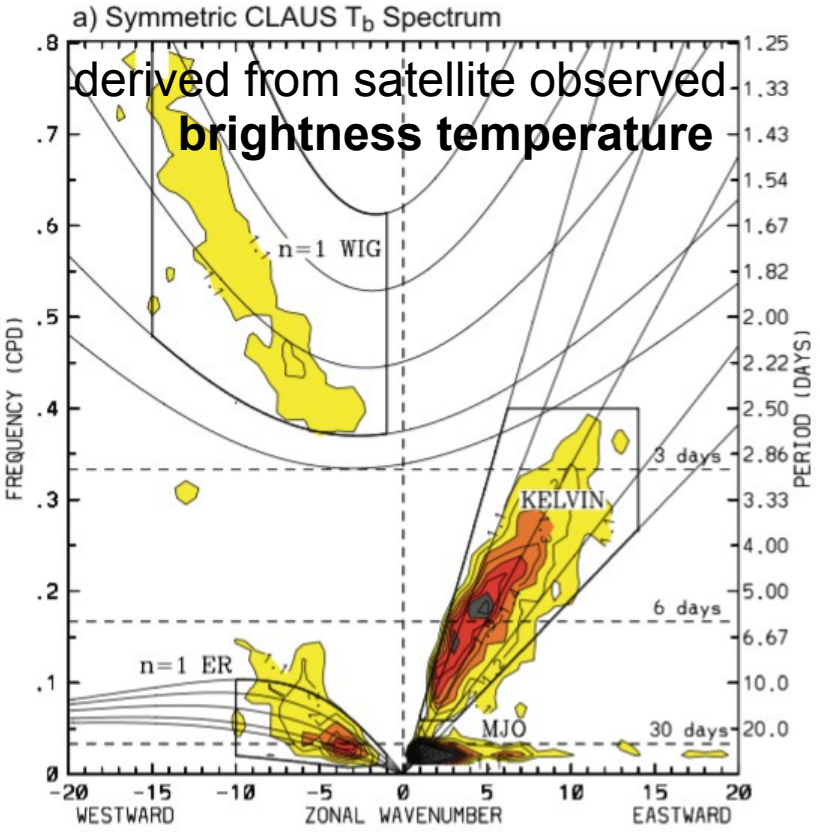
Physics at the equator: from the lab to the stars in Lyon, Oct. 2019

I) Approach to Moist Equatorial Waves

*… from the **perspective of small scale stochastic forcing***

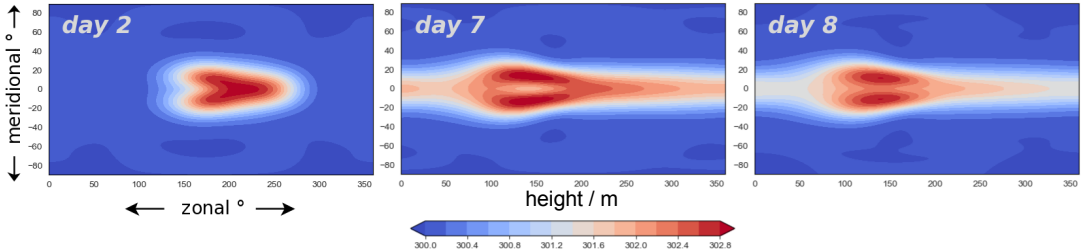




Wheeler & Kiladis (1999)

Validation Experiment

We excite a dry solution as calculated by *Gill (1980)*



We include **moisture** in the equations & ask:

- Can small **scale stochastic forcing** initiate **self-aggregation**?
- How does **a moist upscale energy cascade** emerge?
- What kind of stochastic forcing excites **planetary waves**?

I) Moist Shallow Water Equations

Vorticity $\zeta_t = -\nabla\cdot(\zeta \boldsymbol{v}) + f^{\zeta}$

Divergence $\delta_t = (\nabla\times\zeta \boldsymbol{v})_m - \Delta E + f^{\delta}$

Height $h_t = -\nabla\cdot(h \boldsymbol{v}) - L q^{+}/\tau_q + f^{h} + S^{h}$

Moisture $q_t = -\nabla\cdot(q \boldsymbol{v}) - q^{+}/\tau_q - q^{-}/\tau_e + f^{q}$

Dry Energy $E = gh + \frac{\|\boldsymbol{v}^2\|}{2}$

open system due to **asymmetry** and **nonlinearity**

$\tau_q = 0.1$ day $\qquad \tau_e = 10.0$ days

Stochastic noise

- *Random kicks*
- *Lévy-process*
- Conservative

Background Literature

Kuskin (2006)
Randomly forced nonlinear PDEs and statistical hydrodynamics in 2 space dimensions

The code is based on **dynamical core** developed by *Schaeffer et al. (2013, 2018).*
Used in experiments including **moisture** following *Gill (1982)* by *Suhas et al. (2015, 2017)*.

I) Moist Shallow Water Experiments

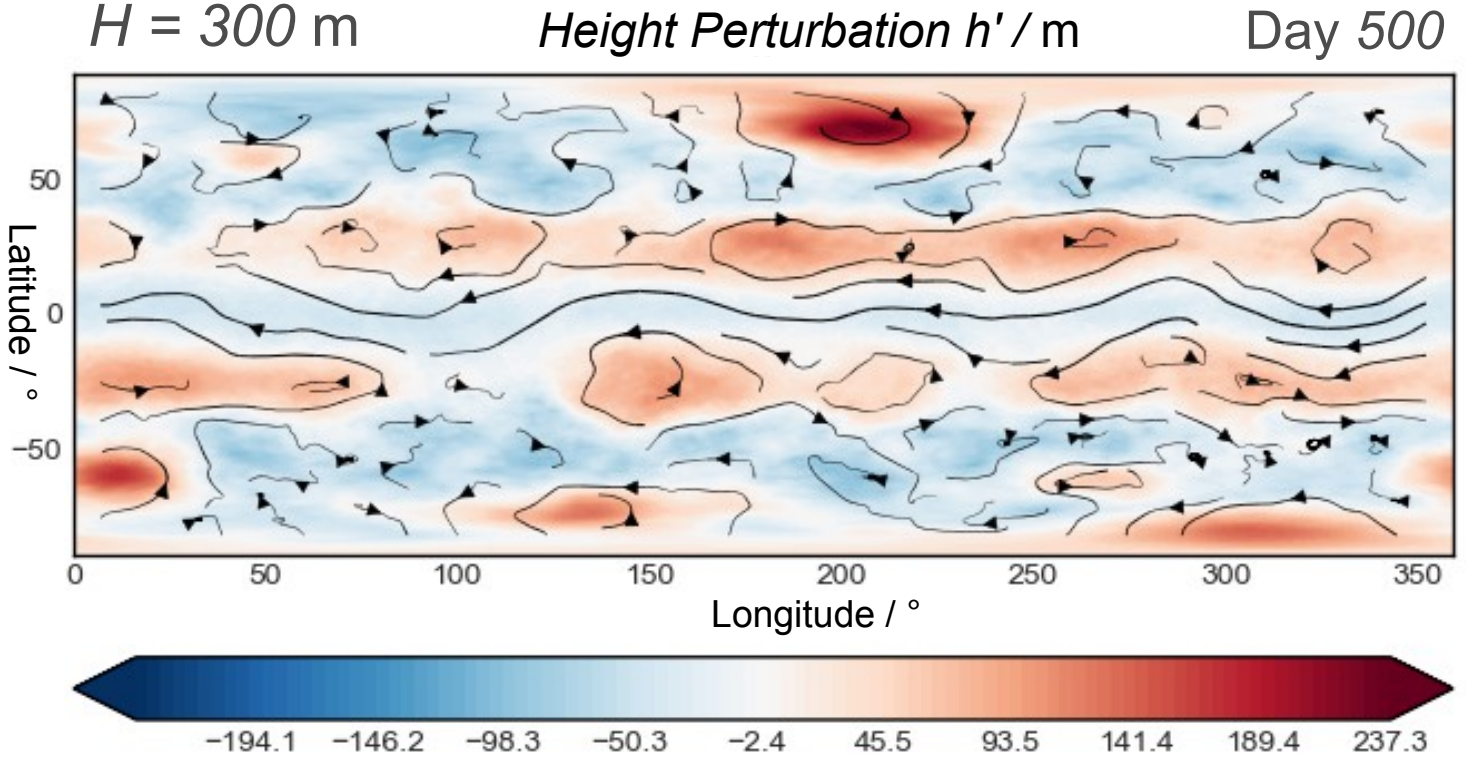
Overall, we ran **48 experiments** varying parameters of

- stochastic forcing **time-scale**
- stochastic forcing **spatial scale**
- **moisture** parameterization

Experiment	Dry/Moist	**Noise** on	
1	**Dry**	Vorticity	*Single-layer shallow water*
1.b	Passively **Moist**	Vorticity	
2	**Dry**	Divergence	
2.b	Passively **Moist**	Divergence	
3	**Moist**	Moisture	
4	**Dry**	Vorticity	*3-level pressure tendency*

I) Shallow Water Flow on the Sphere

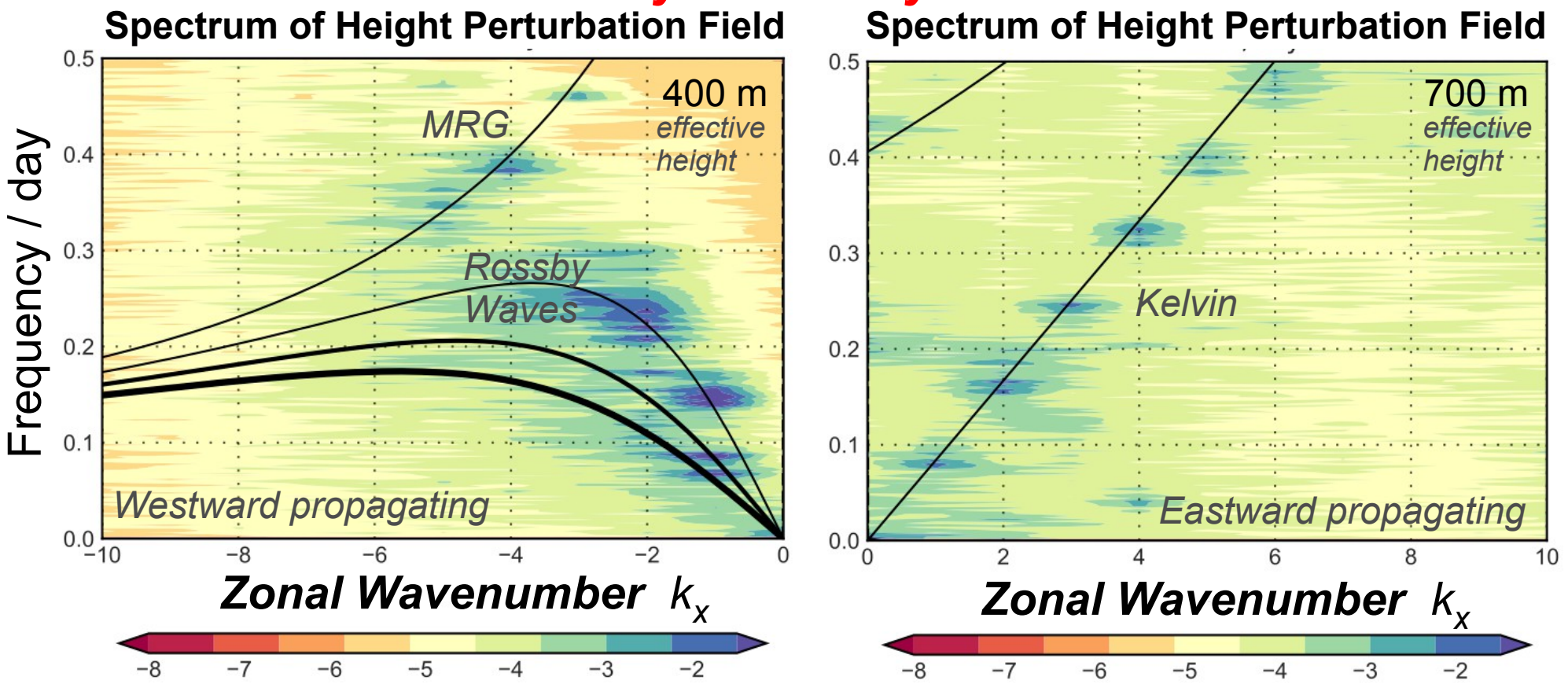
Experiment 1) ***Dry – vorticity forced***



Easterlies *in the* ***tropics*** *and* ***westerly jets*** *with* ***(anti-)cyclones*** *in the* ***extra-tropics.***

II) Space Time Spectra of Equatorial Waves

Dry – vorticity forced

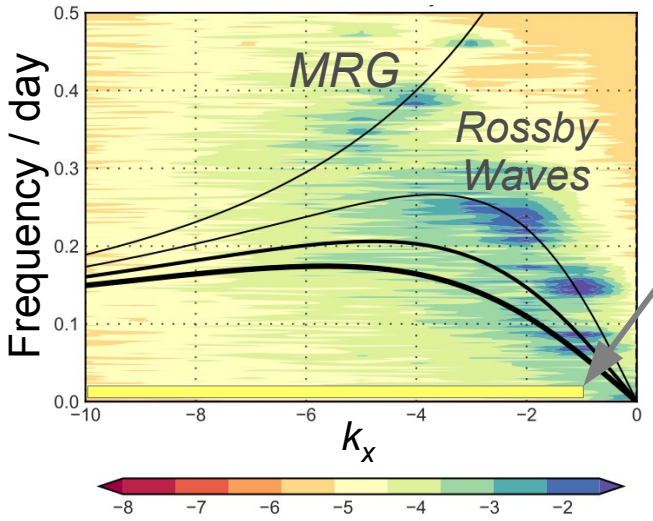


- **Very discrete waves -** reproduced from *Suhas* et al. (2015).
- **Following linear dispersion** relations without removing background noise (needed to do in observations by *Wheeler and Kiladis*, 1999)
- **No low frequency modes** as observed in nature

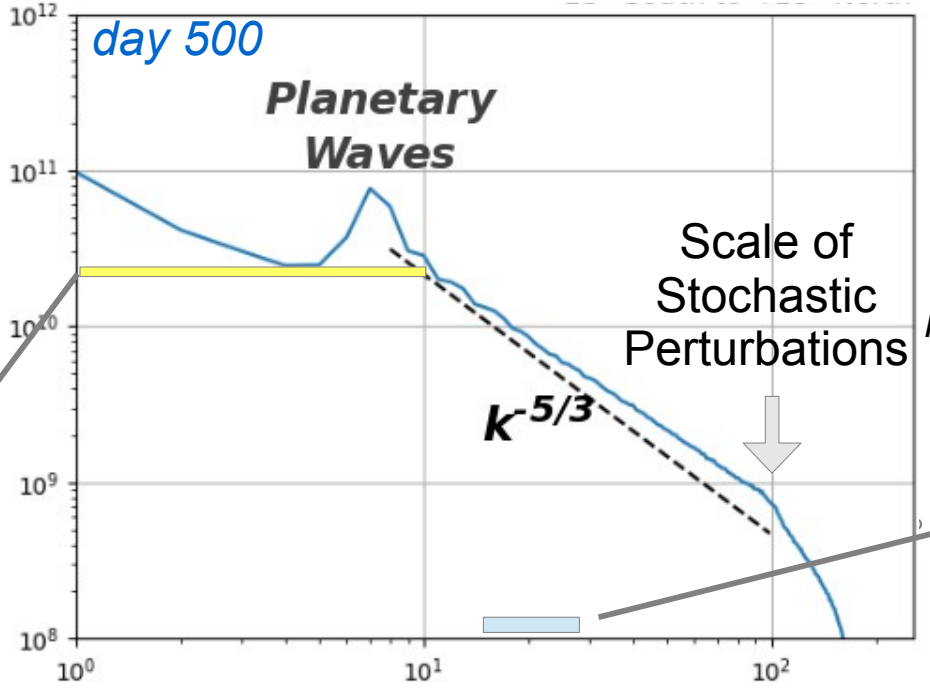
II) Spectra of Waves & Turbulence in Tropics

Experiment 1) ***Dry – vorticity forced***

Space-Time Spectrum of Height Perturbation



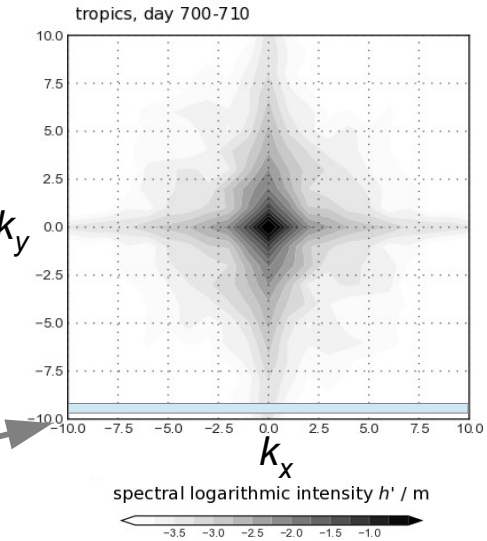
1D Zonal Spectrum of Kinetic Energy



Zonal Wavenumber k_x

Horizontal 2D Spectrum of Height Perturbation

in *20° x 20°* area

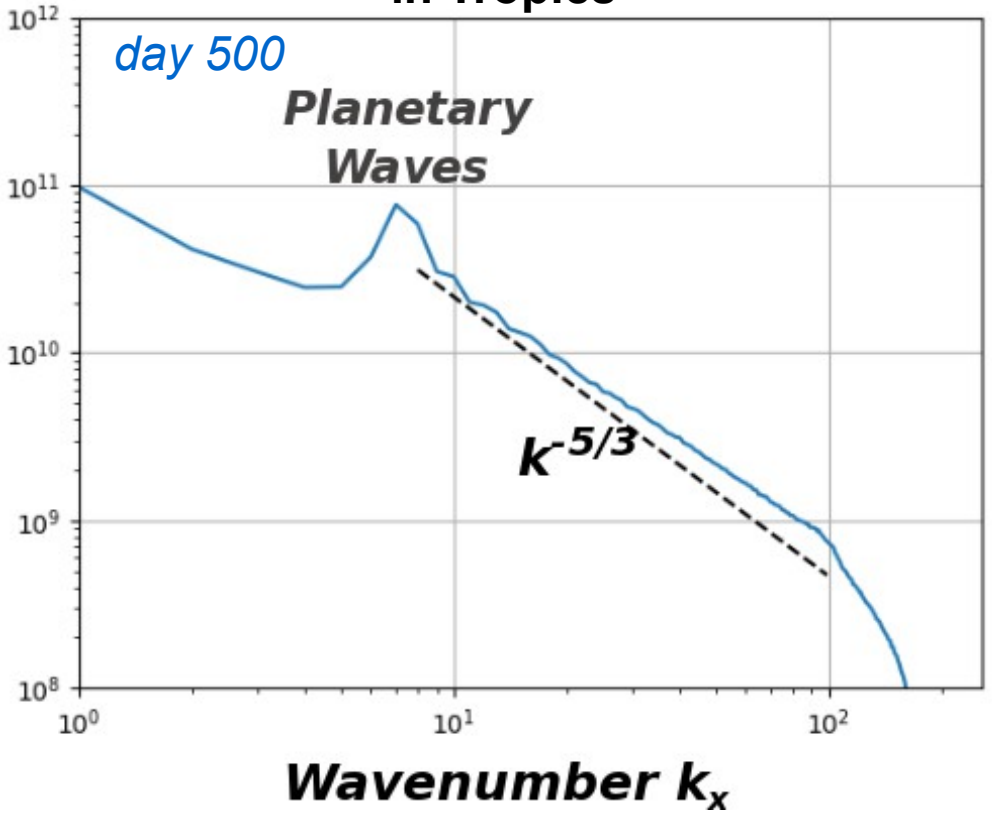


Isotropic flow occurs below the scale of planetary waves.

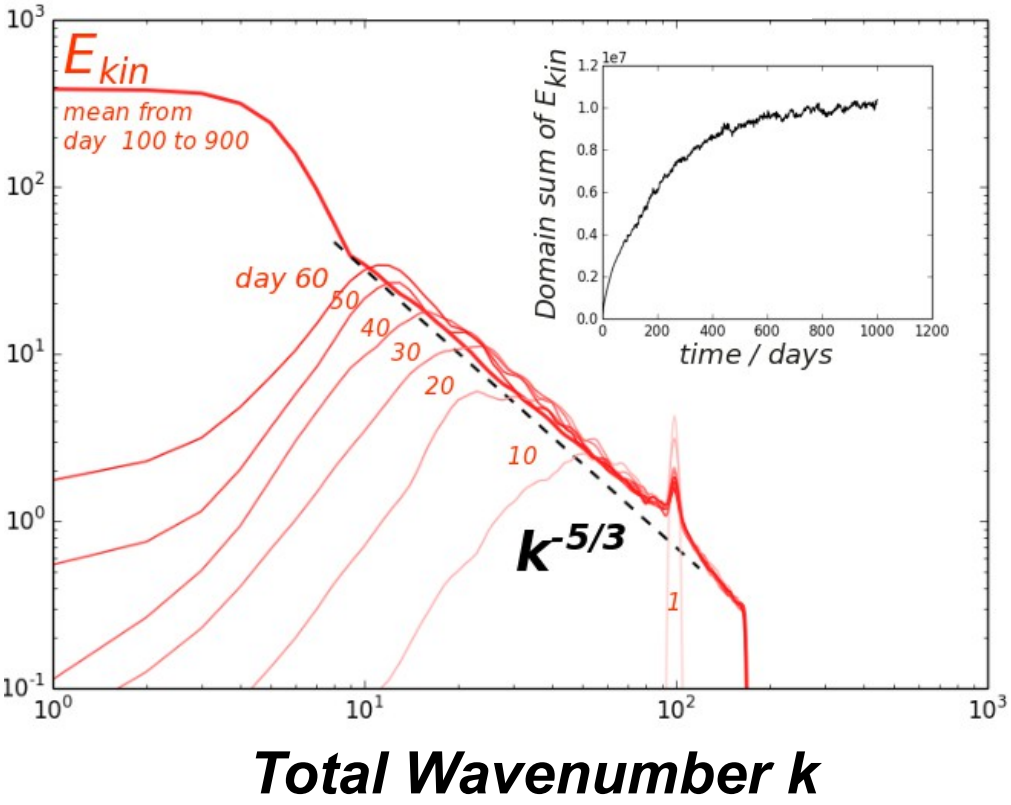
II) Spectral Kinetic Energy Evolving in Time

Experiment 1) ***Dry – vorticity forced***

1D Zonal Spectrum of Kinetic Energy in Tropics



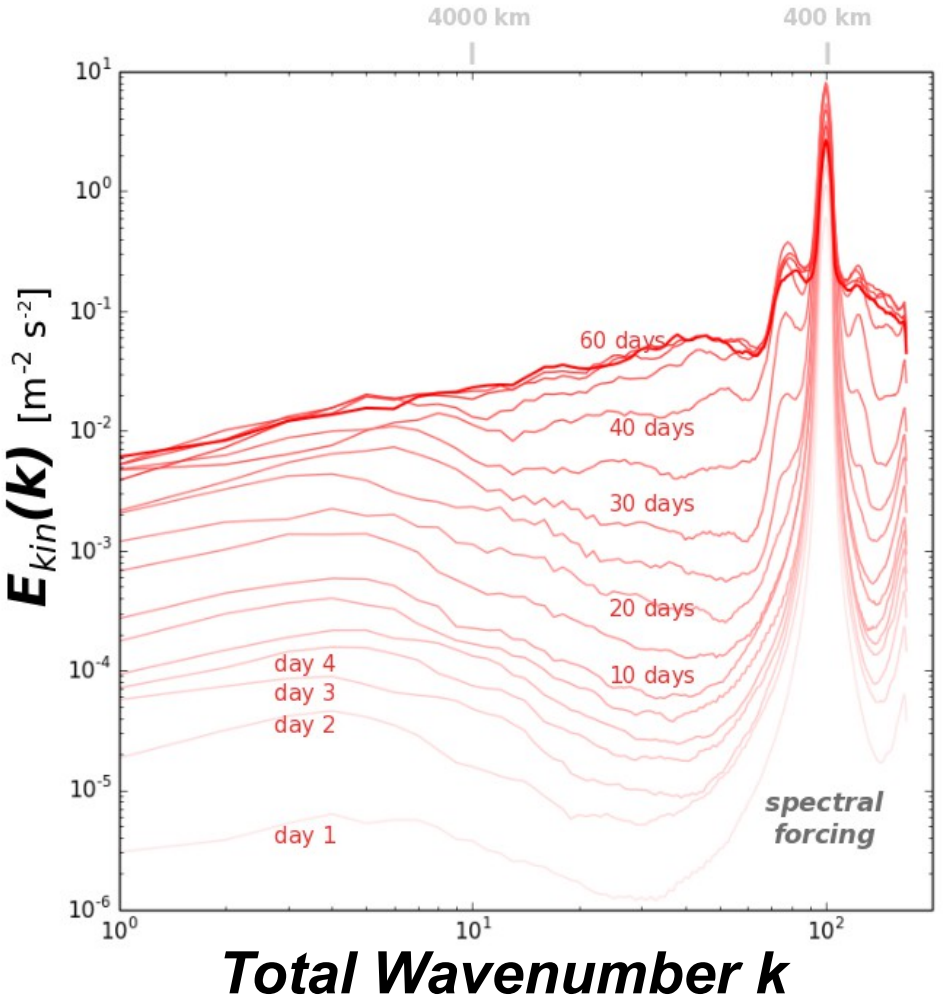
Global Spectrum of Kinetic Energy



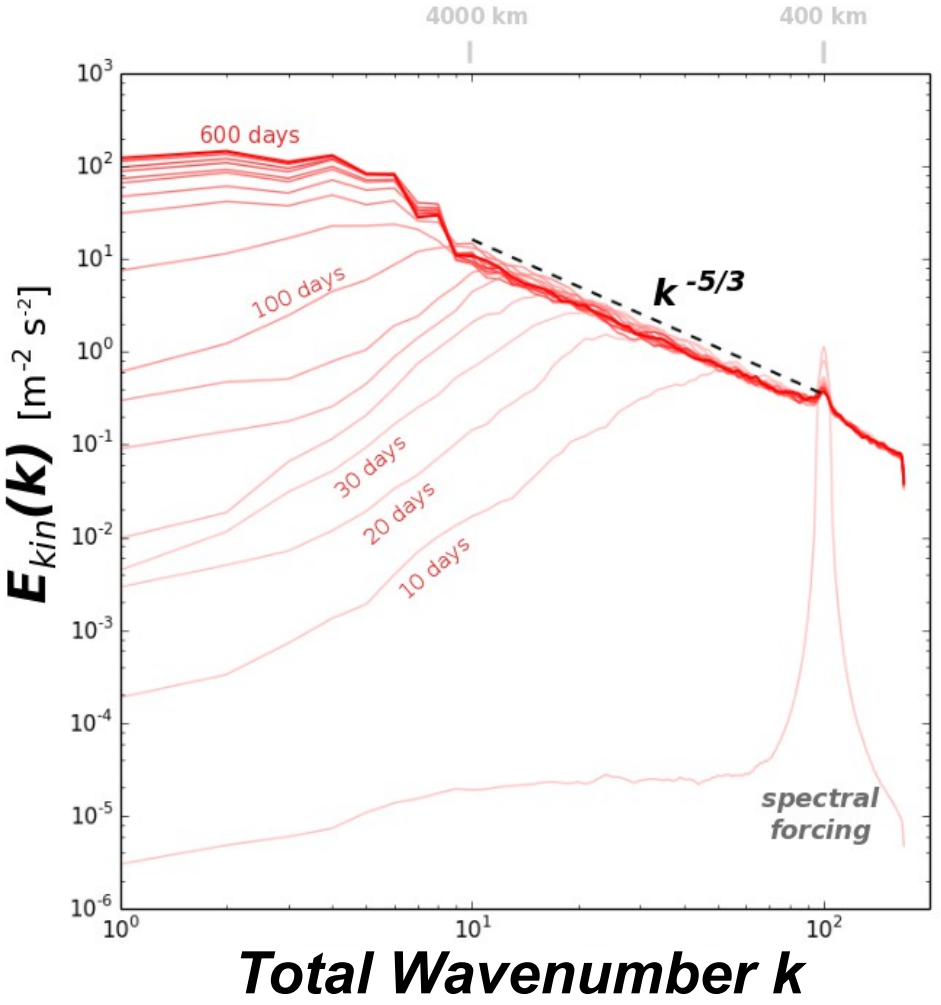
Upscale cascade occurs in evolution of vorticity forced dry flow.

II) Spectral Kinetic Energy Evolving in Time

Exp. 2) ***Dry - divergence forced***



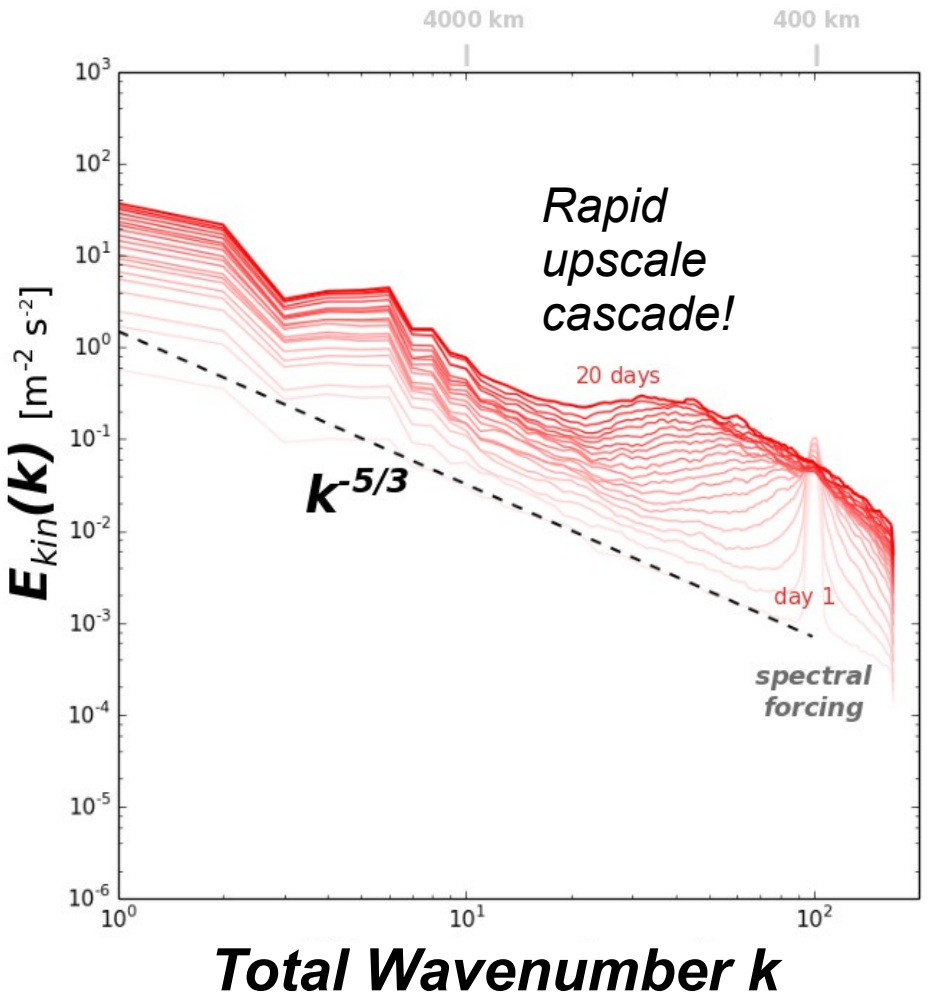
Exp. 1) ***Dry - vorticity forced***



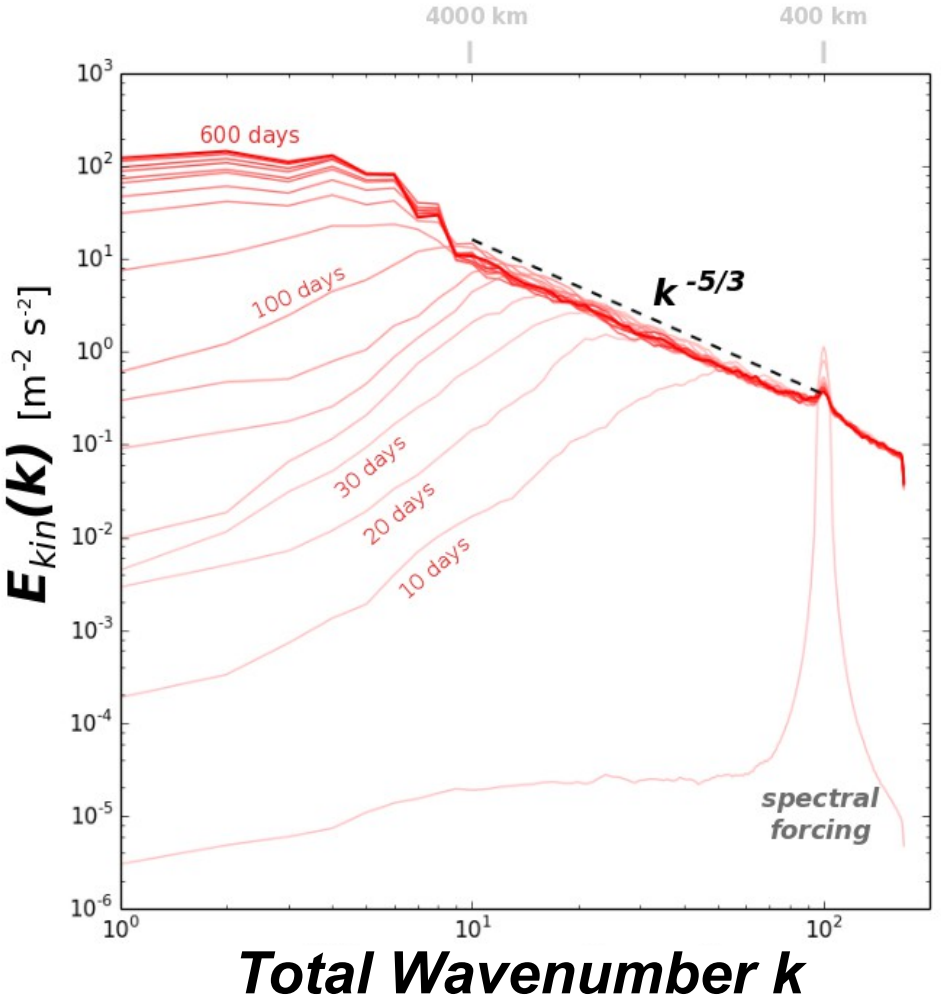
No upscale cascade beyond forcing magnitude in divergence forced dry flow.

II) Spectral Kinetic Energy Evolving in Time

Exp. 3) ***Moist - moisture forced***



Exp. 1) ***Dry - vorticity forced***



Rapid upscale cascade occurs in moisture forced moist flow.

II) Energy Evolution

Kinetic Energy

kinetic energy **momentum flux**

$$\partial_t E_{kin} = -\boldsymbol{v}\cdot\nabla\phi \; - \; u^2\partial_x u \; - \; v^2\partial_y v \; - \; uv\left(\partial_x v + \partial_y u\right) \; [\mathrm{m^2\ s^{-2}\ /\ s}]$$

advection of **potential energy** | zonal **shear** | meridional **shear** | **strain rate**

Geopotential

time delayed

$$\phi = gh = g\int dt\left(\partial_t h\right) = g\int dt\left(-\nabla\cdot(h\boldsymbol{v}) - L[q-q_s]^+ / \tau_q\right)$$

Moisture

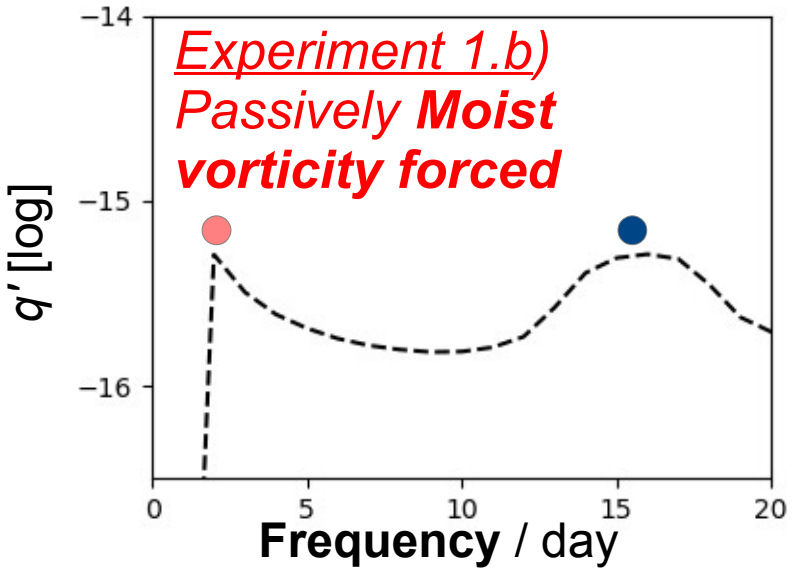
time delayed

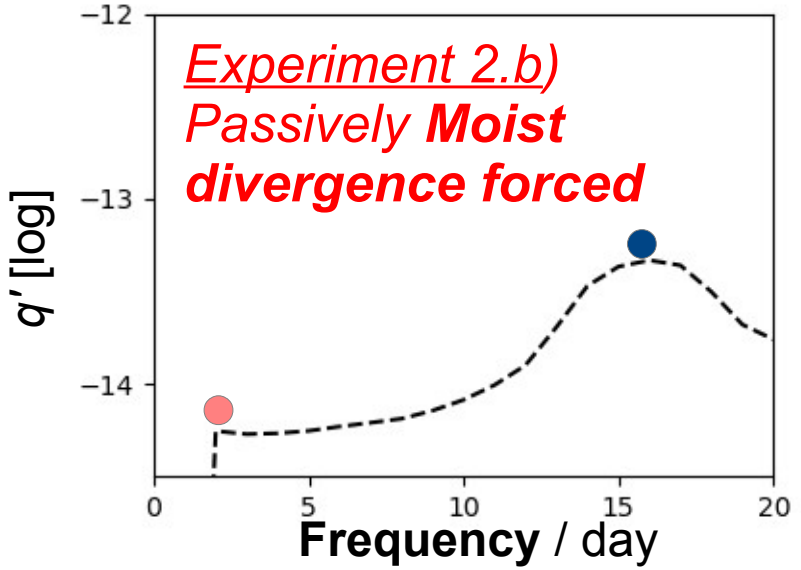
$$q = \int dt\left(\partial_t q\right) = \int dt\left(-\nabla\cdot(q\boldsymbol{v}) - q^+ / \tau_q - q^- / \tau_e\right)$$

Production term $\sim L q^+ / \tau_q$ *in kinetic energy occurs in advection of geopotential.*

This effect is especially strong in regions of ***low wind speed****.*

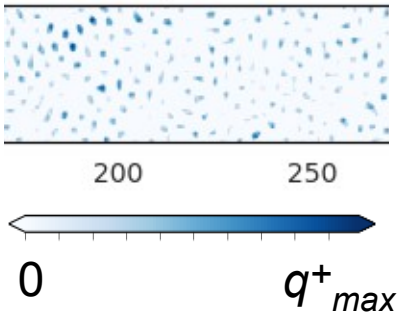
II) High frequency FFT for moisture *(day 1)*



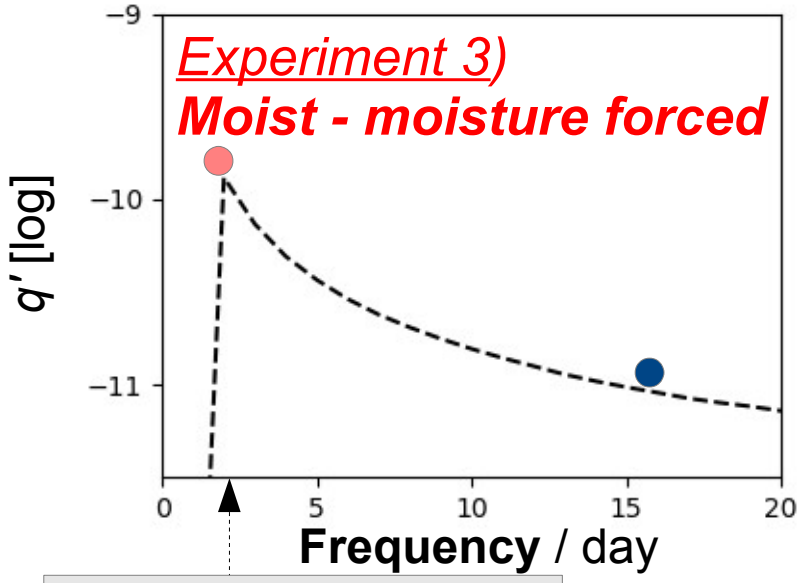


Experiment 3

day 7, q^+ snapshot



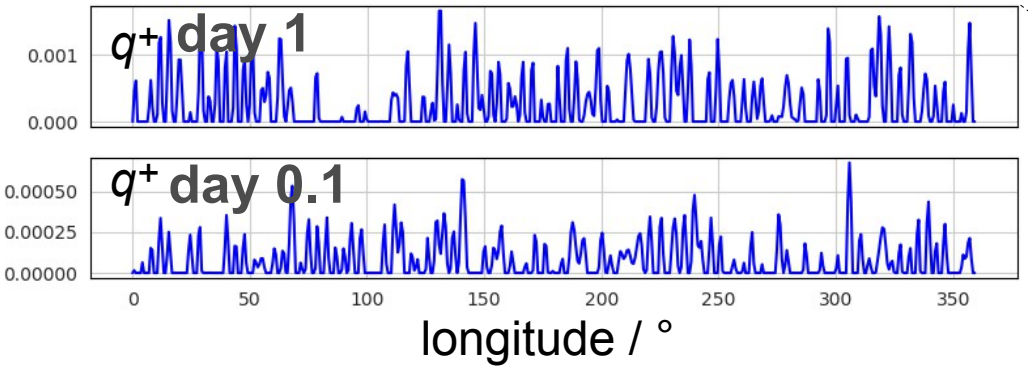
*The **moist forced** flow carries a strong **low frequency** signal on day 1.*

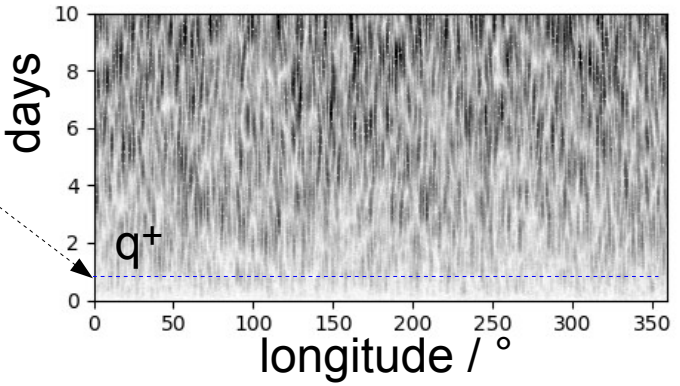


II) Evolution of moisture in zonal direction

Zonal series of q^+ at Equator, y=0

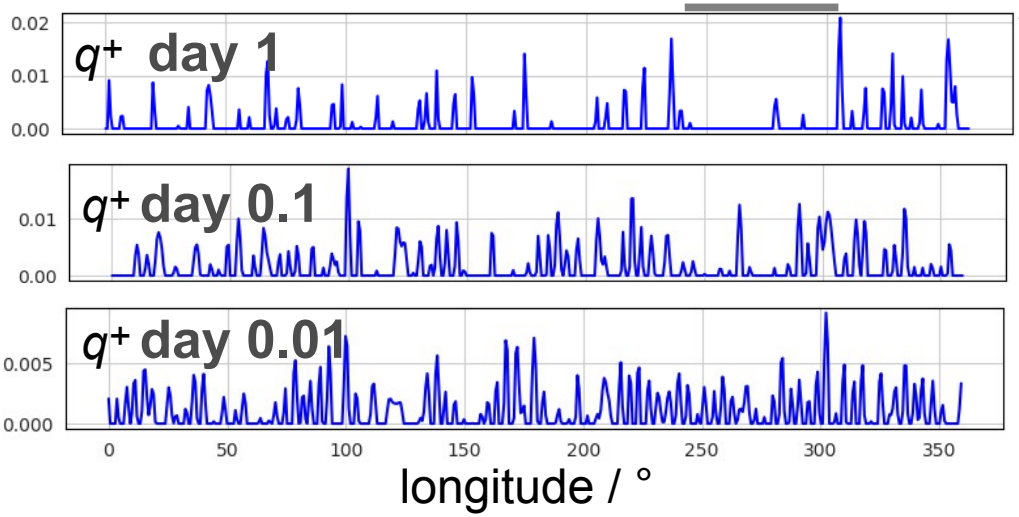
Experiment 2.b) ***Moist - divergence forced*** $\tau_q, \tau_e = \infty$ days

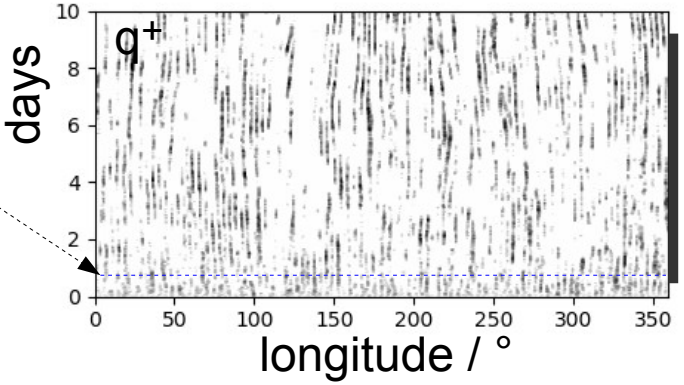




Experiment 3) ***Moist - moisture forced*** $\tau_q = 0.1$ day $\tau_e = 10.0$ days

≈ 7.000 km



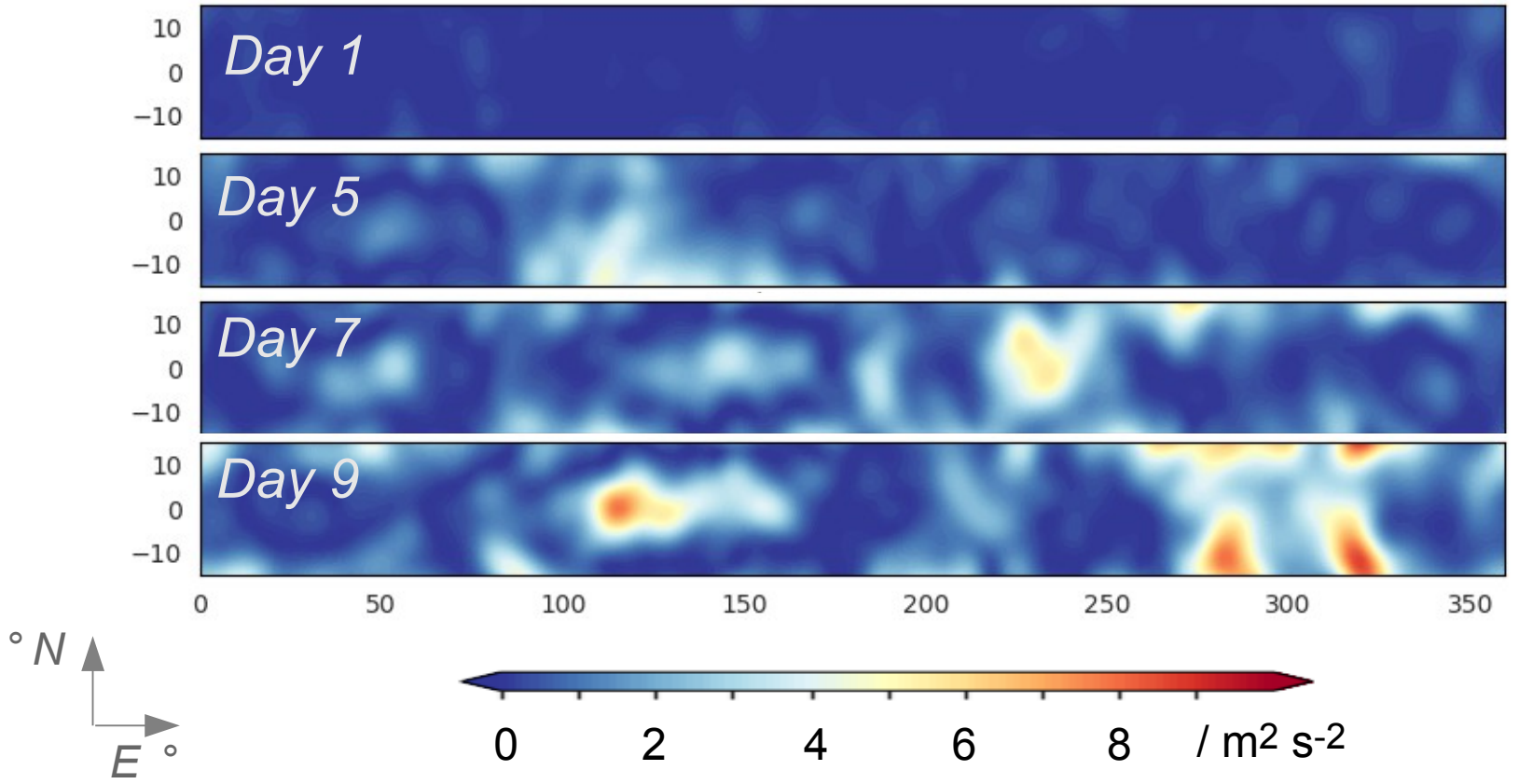


Dry regions *precondition & structure the moist flow at larger scales.*

III) Evolution of kinetic energy cascade

Experiment 3) ***Moist - moisture forced***

Moisture Forcing E_{kin} snapshots (**low pass filter** $k<28$)

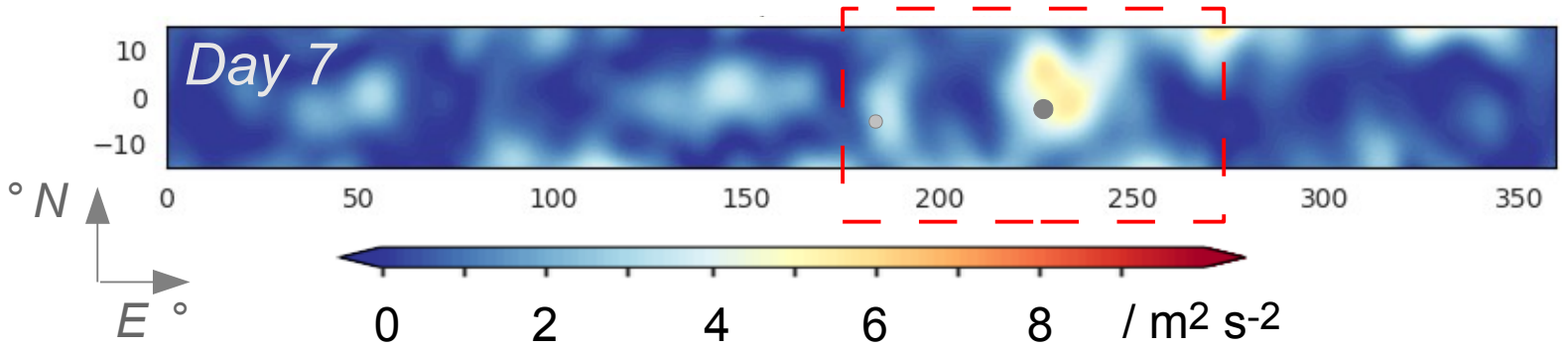


Kinetic energy grows ***upscale*** *rapidly.*

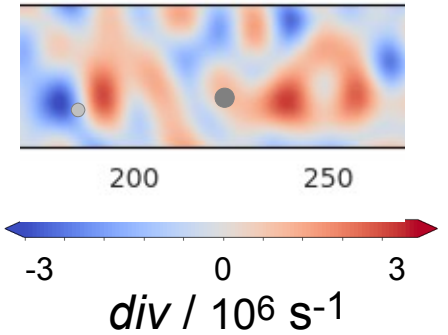
III) Coherent Structures in Upscale Cascade

Experiment 3) ***Moist - moisture forced***

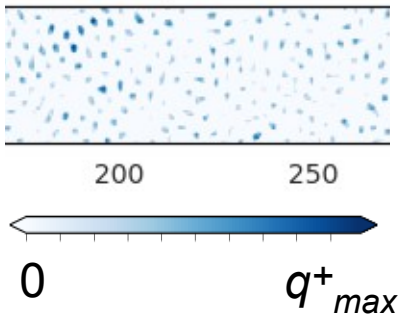
Moisture Forcing E_{kin} snapshots (**low pass filter** *k*<28)



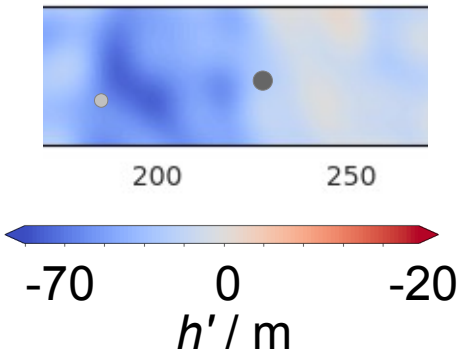
div snapshot (**low pass filter** *k*<28)



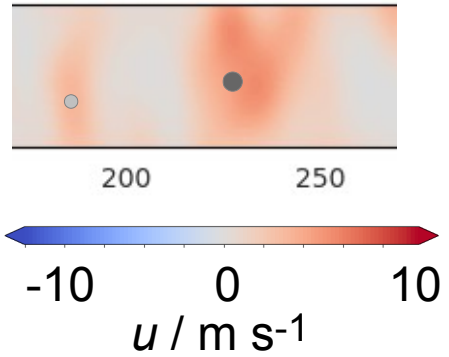
q^+ snapshot (**all scales**)



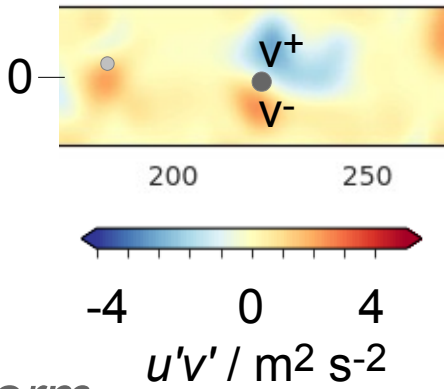
h' snapshot (**low pass filter** *k*<28)



u snapshot (**low pass filter** *k*<28)



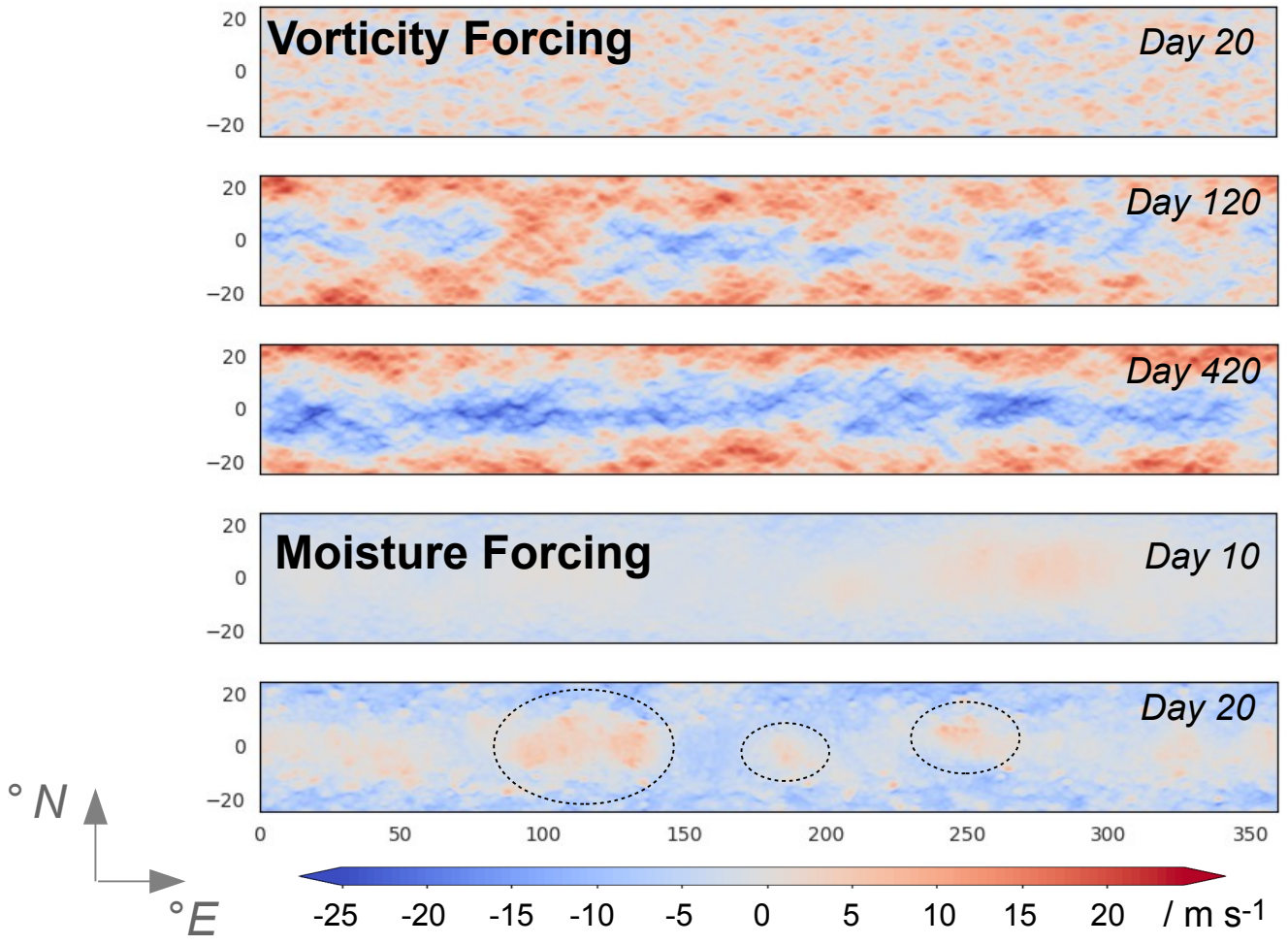
u'v' snapshot (**low pass filter** *k*<28)



Dynamically active ***kinetic energy structures*** *form, when* ***q+*** *is forced* ***randomly at much smaller scales.***

III) Coherent Structures in Upscale Cascade

Zonal Velocity *u* snapshots *(all scales)*



Experiment 1)

Dry - vorticity forced

Experiment 3)

Moist - moisture forced

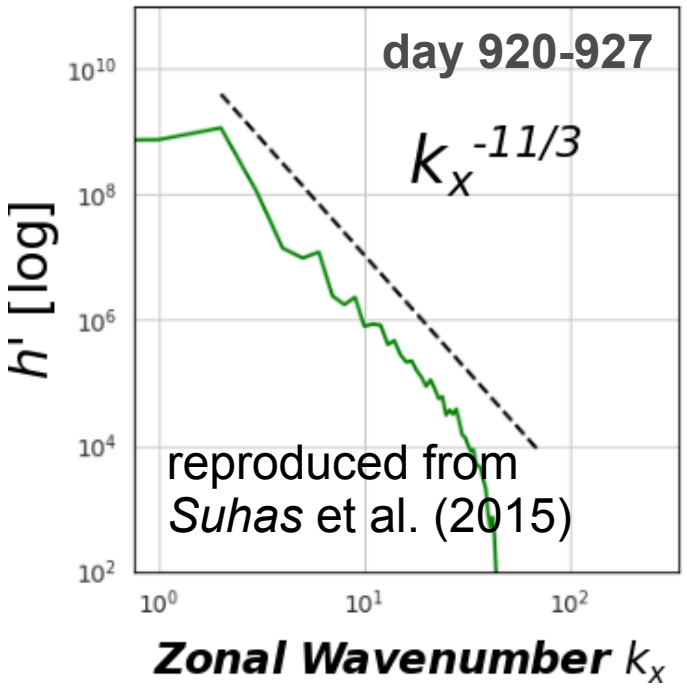
Eastward moving structures compare in scale and amplitude with simulations by *Yano et al. (1994)*

II) Zonal spectra of potential energy

Experiment 1)

Dry - vorticity forced

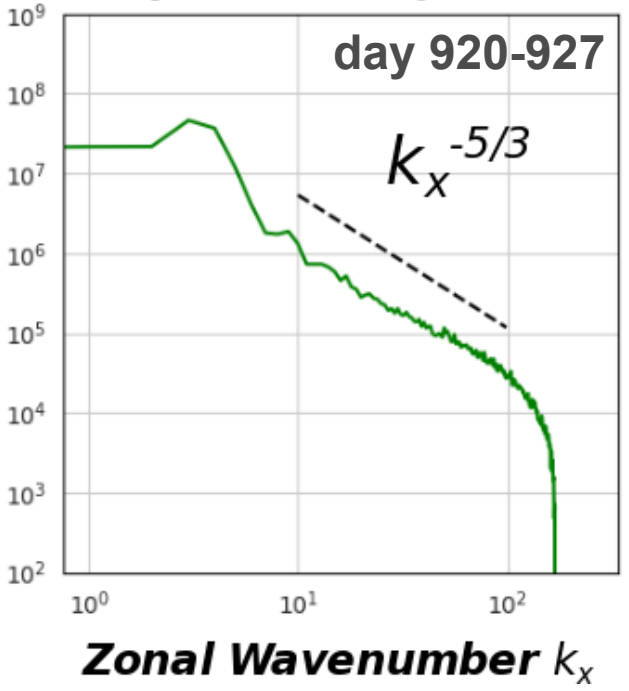
***poles**, vorticity forced*



Experiment 1)

Dry - vorticity forced

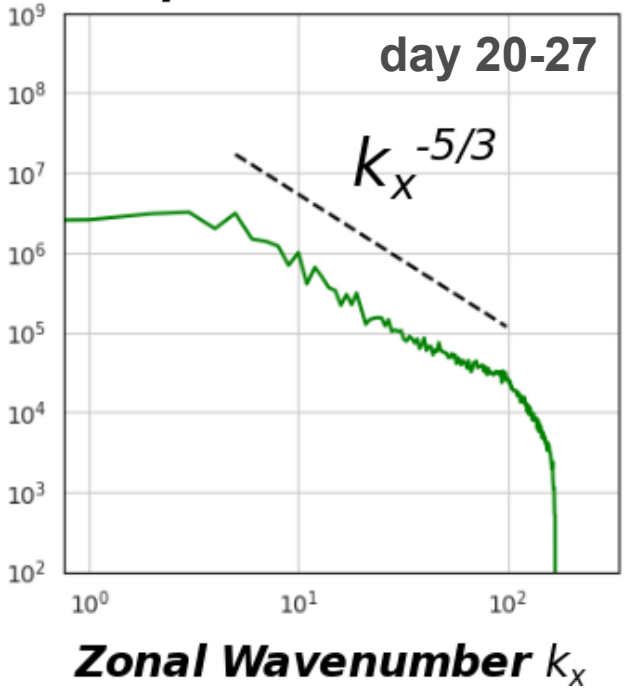
***tropics**, vorticity forced*



Experiment 3)

Moist - moisture forced

***tropics**, moist forced*

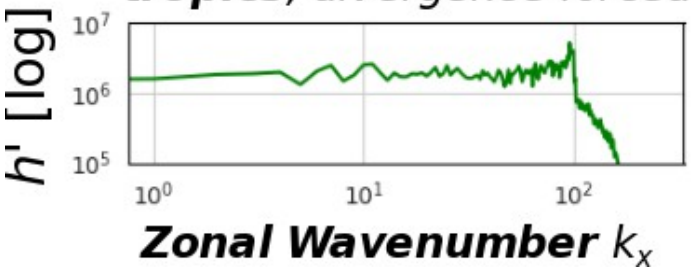


***Rapid upscale** cascade of **potential energy** in **moist forced** flow.*

Experiment 2)

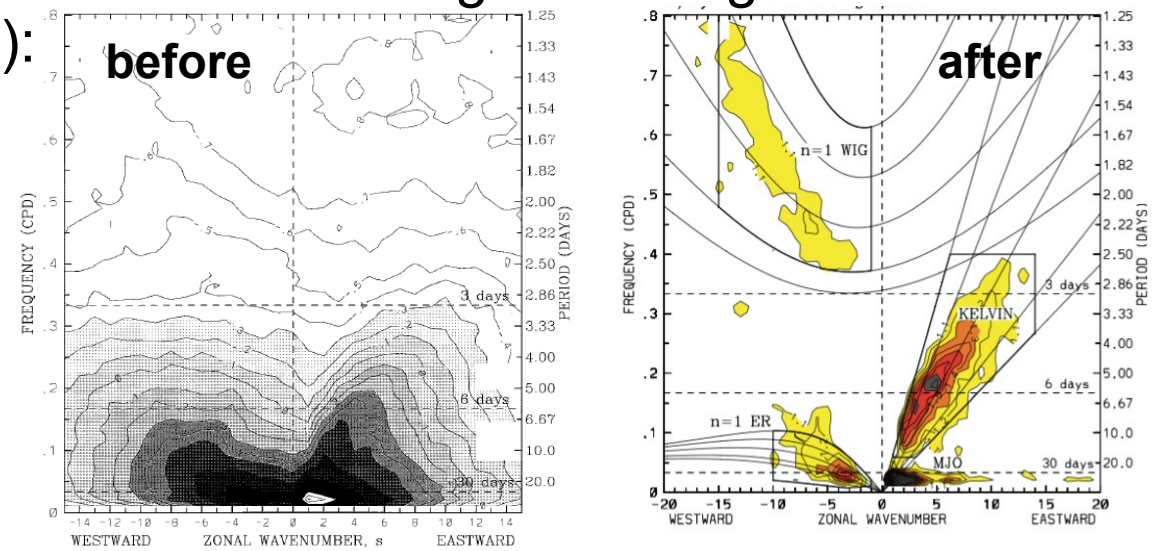
Dry - divergence forced

***tropics**, divergence forced*



Summary

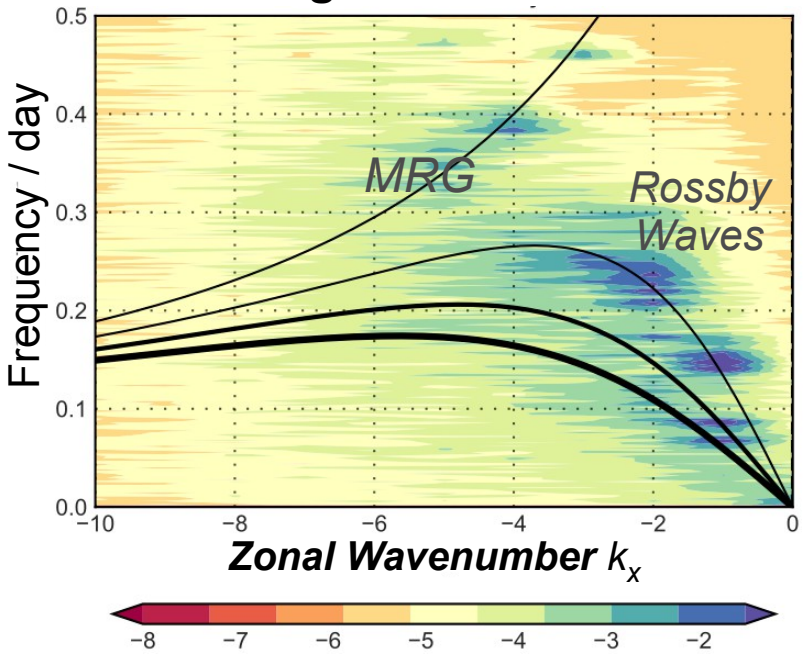
- We simulate a -5/3 **upscale** cascade with **equatorial waves (consistent** with *Suhas et al. 2015, 2017*)**.**
- Only **vorticity** & **moisture forcing** lead to **self-aggregation** & **upscale energy cascades.**
- Forcing moisture leads to **eastward propagating modes** & is possible out of previously dynamically passive low wind regions!
- Next step: Satellite observations exhibit more **continuous spectra** & **low-frequency** modes – before removing the background noise (*Wheeler & Kiladis, 1999*):



IV) Space-Time Spectra in Shallow Water

Exp. 1) ***Dry - vorticity forced***

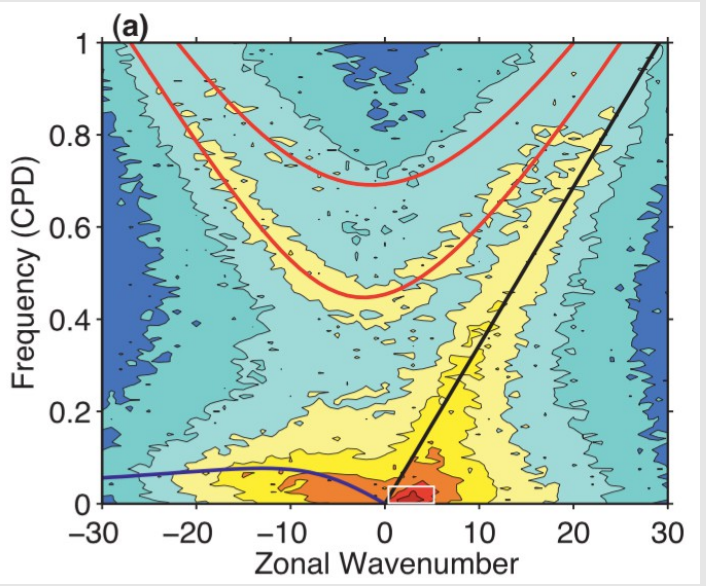
Space-Time Spectrum of Height Perturbation



- **Very discrete** in our shallow water experiments.
- **No low frequency** modes!

Yang & Ingersoll (2013)

Yang & Ingersoll (2013) **forced the flow** in a **non-linear** and **non-conservative** way!

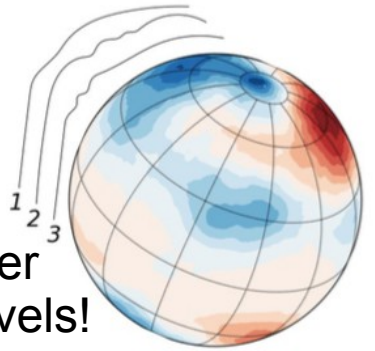


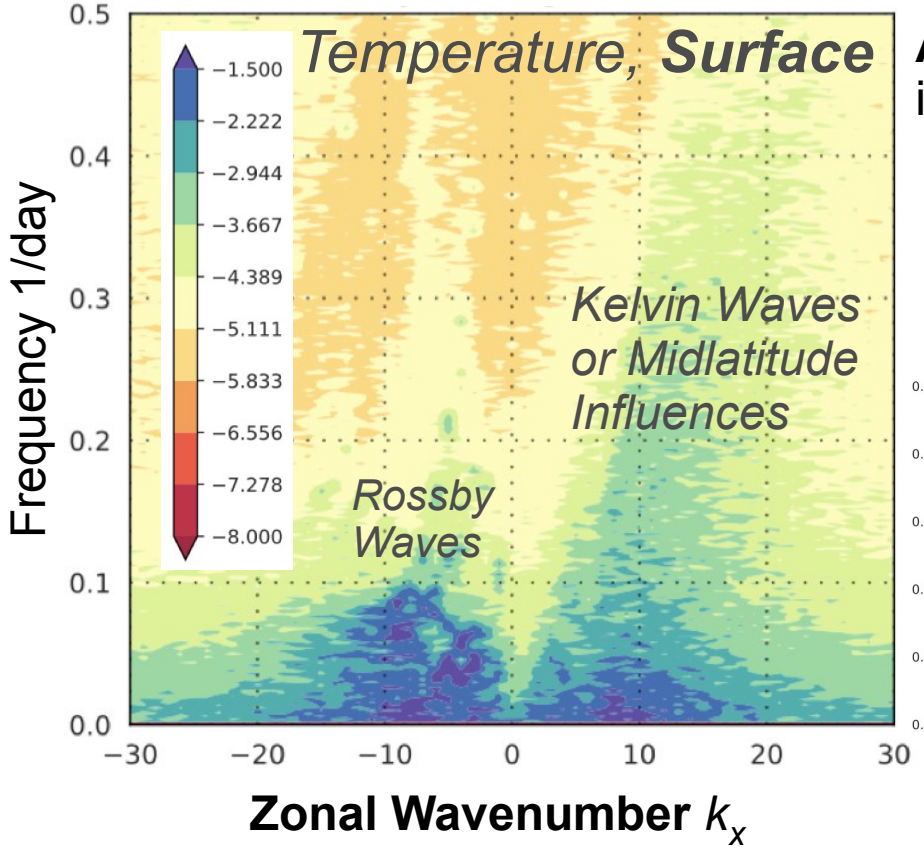
- **More continuous** spectrum.
- **Low frequency** modes!

IV) Space-Time Spectra in 3-level PTM

Experiment 4) **Dry - vorticity forced**

Relaxation to Held & Suarez Temperature $T_{eq}(y)$ in 3-levels.

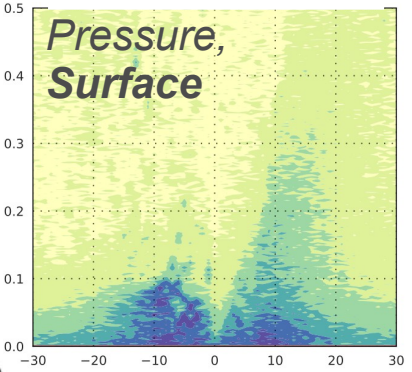


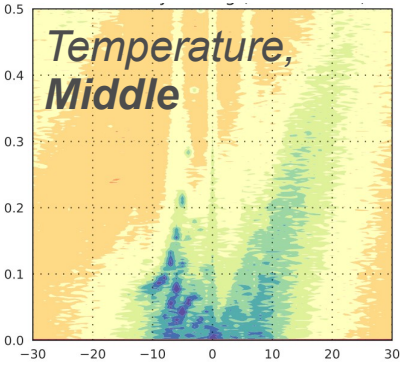


A crucial difference to shallow water is **convective coupling** between levels!

next step:

- **Add moisture** in 3-levels & vary physical parameterizations: convection scheme, radiation, boundary layer.





In 3-level flow we get **smoother** distributions with **low-frequency modes** (as in non-conservative forced experiments by *Yang & Ingersoll*).

Thank you for your attention!

Caltech