Abrupt transitions in atmosphere jets through Reynolds stress resonance

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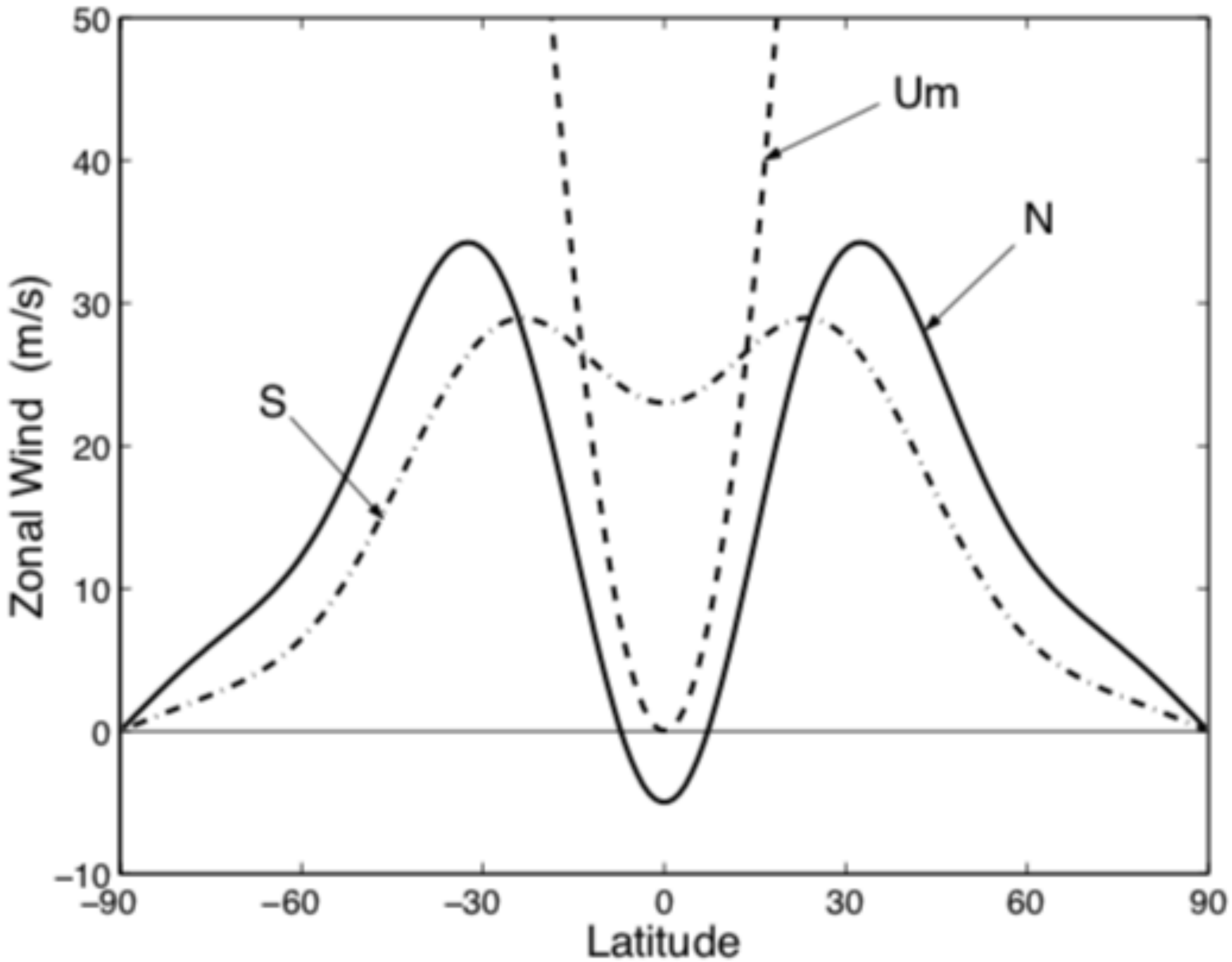
Workshop "Physics at the equator: from the lab to the stars", ENS de Lyon, October 2019.

I) Super-rotation

Super-rotation



Angular momentum:

$$M = a\cos\phi(\Omega a\cos\phi + u)$$

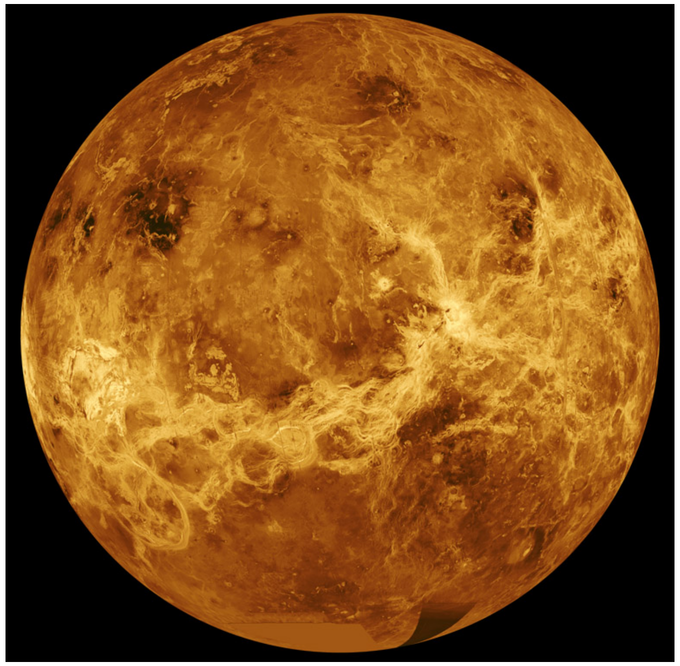
$$M > \Omega a^2 \iff u > \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_m$$

A planetary atmosphere super-rotates if it has prograde zonal flows with more angular momentum about the rotation axis than a fluid particle rotating at the speed of the planet at the equator $(u > U_m)$**.**

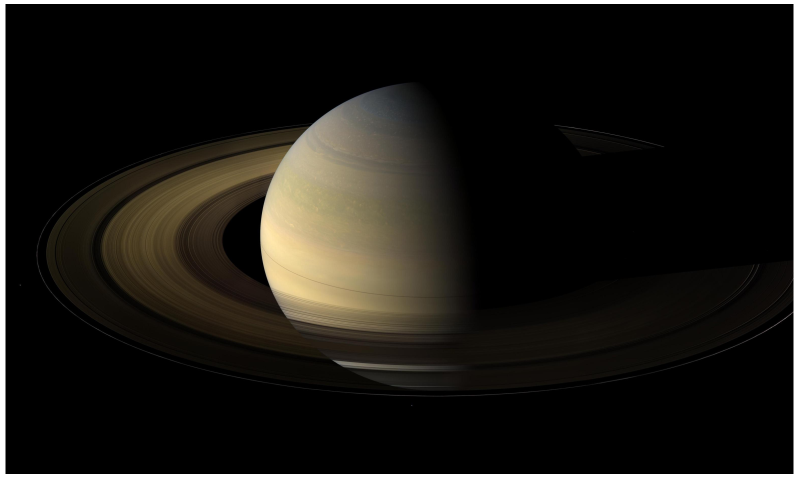
I. M. Held (1999) « Equatorial superstation in Earth-like atmospheric models »

Super-rotation in the solar system

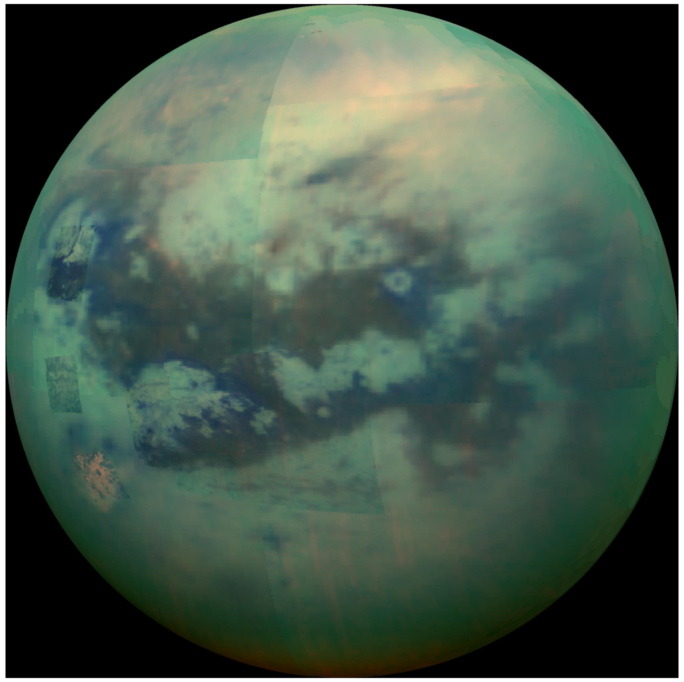
Venus



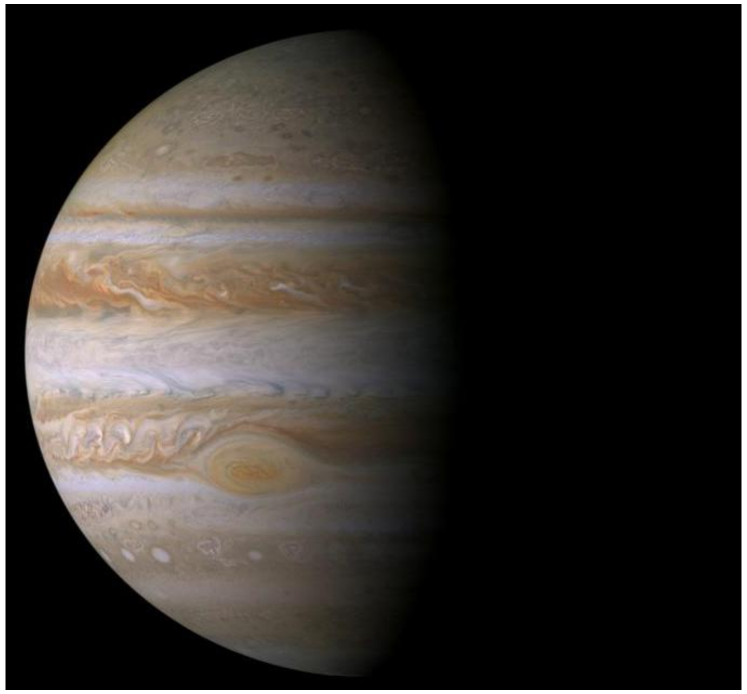
Saturn



Titan

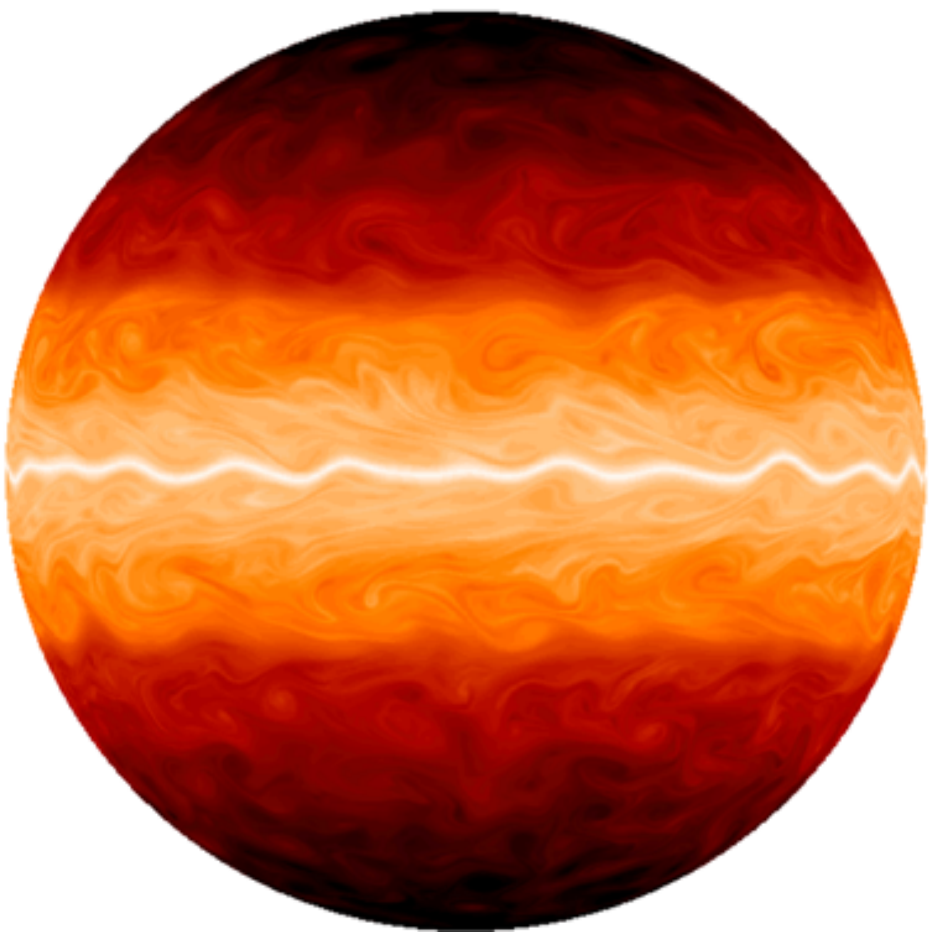


Jupiter



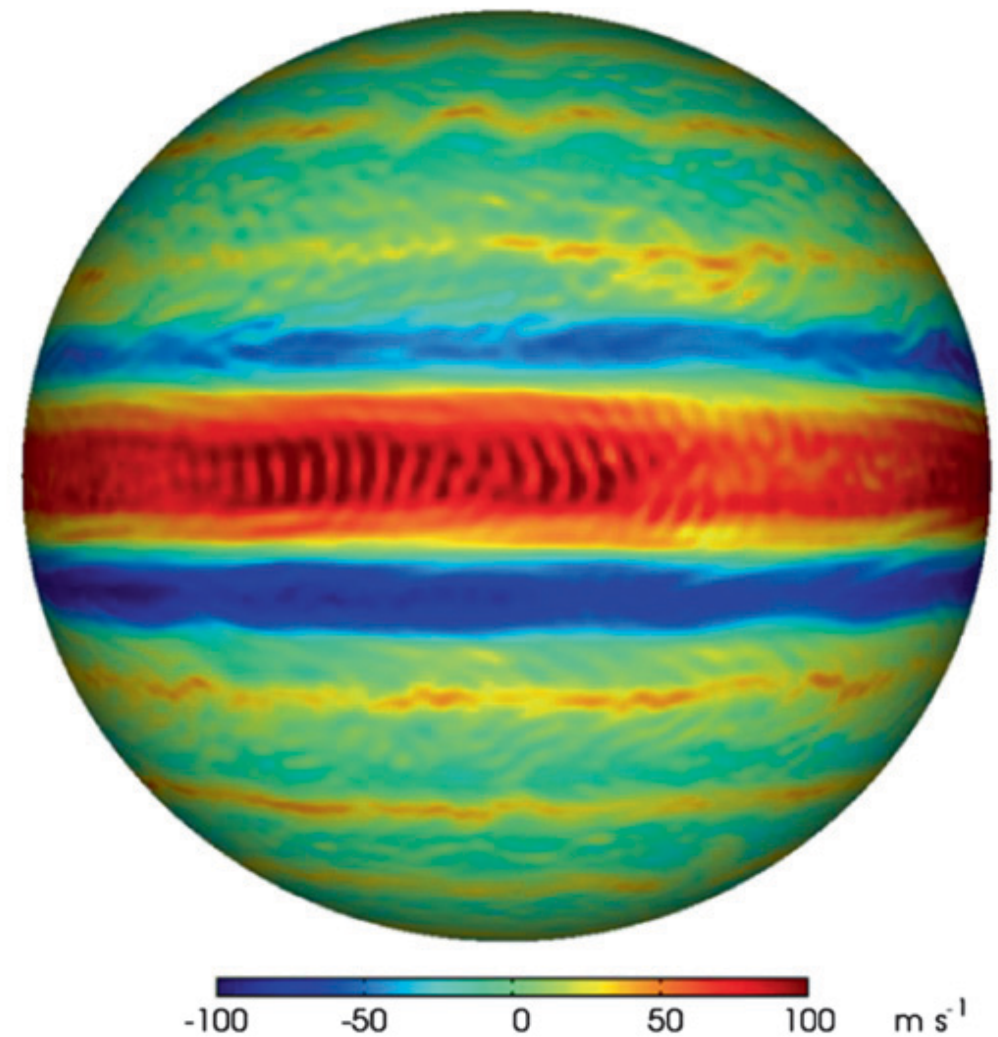
Super-rotation on extrasolar planets

Super-rotation in numerical simulations



Shallow Water Equations

R. K. Scott and L. Polvani (2008)



Primitive Equations

T. Schneider and J. Liu (2009)

Super-rotation requires convergence of angular momentum

$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F + D\,.$$

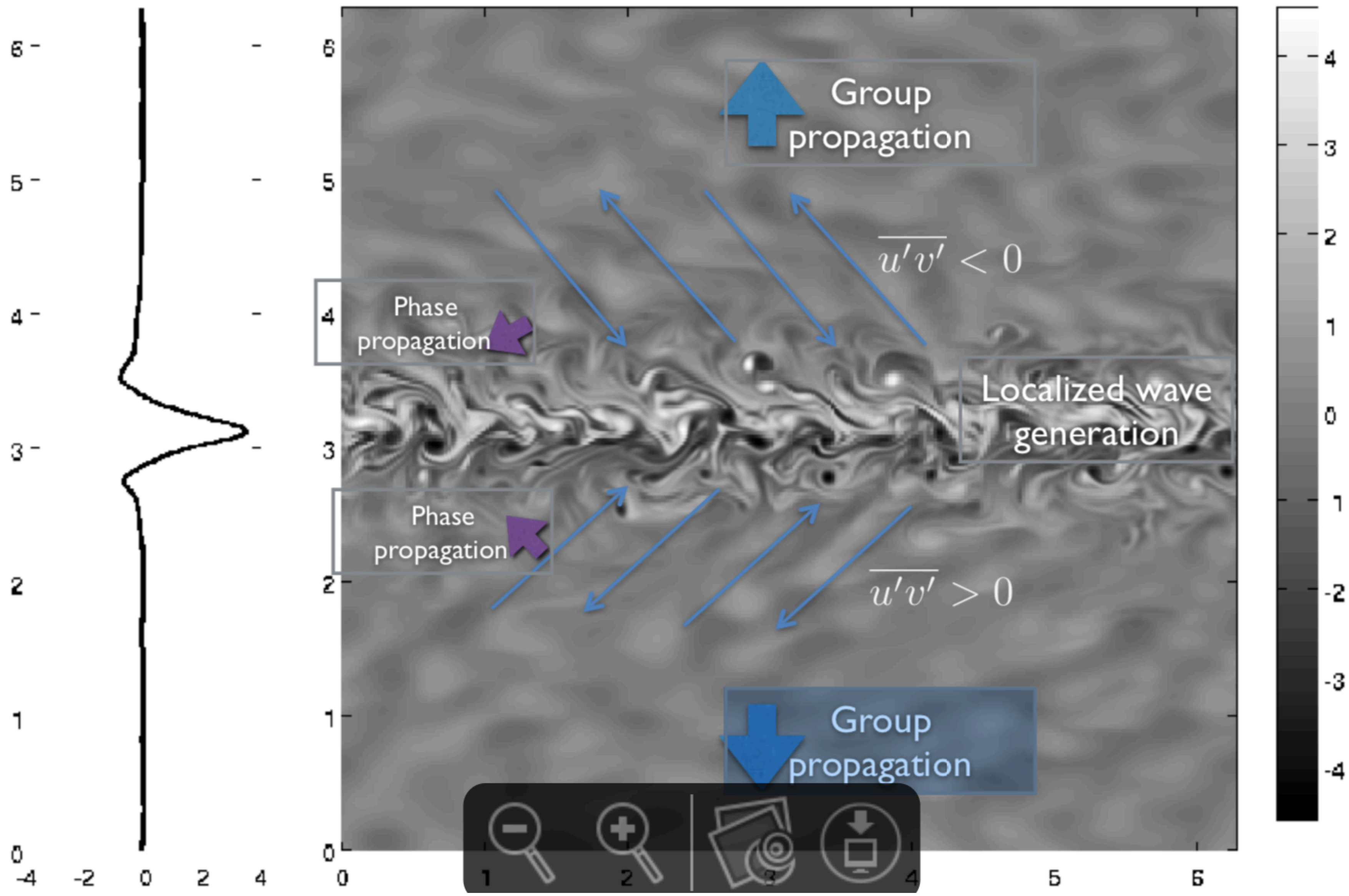
$$F = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{u'v'\cos\phi} - \frac{\partial}{\partial p}\overline{u'\omega'}\,.$$

Averaged zonal velocity equation

As the transport term conserves angular momentum, the dissipation terms mix it, and the boundary terms restore it to the boundary value, we have a maximum principle property, and super-rotation requires a source term in the zonally averaged equation through the Reynolds stress (eddy momentum flux convergence).

II) Super-rotation can be forced by Rossby–Kelvin waves

The usual suspect for a dynamical mechanism of momentum convergence



Localised forcing of potential vorticity mixing in a beta-plane dynamics

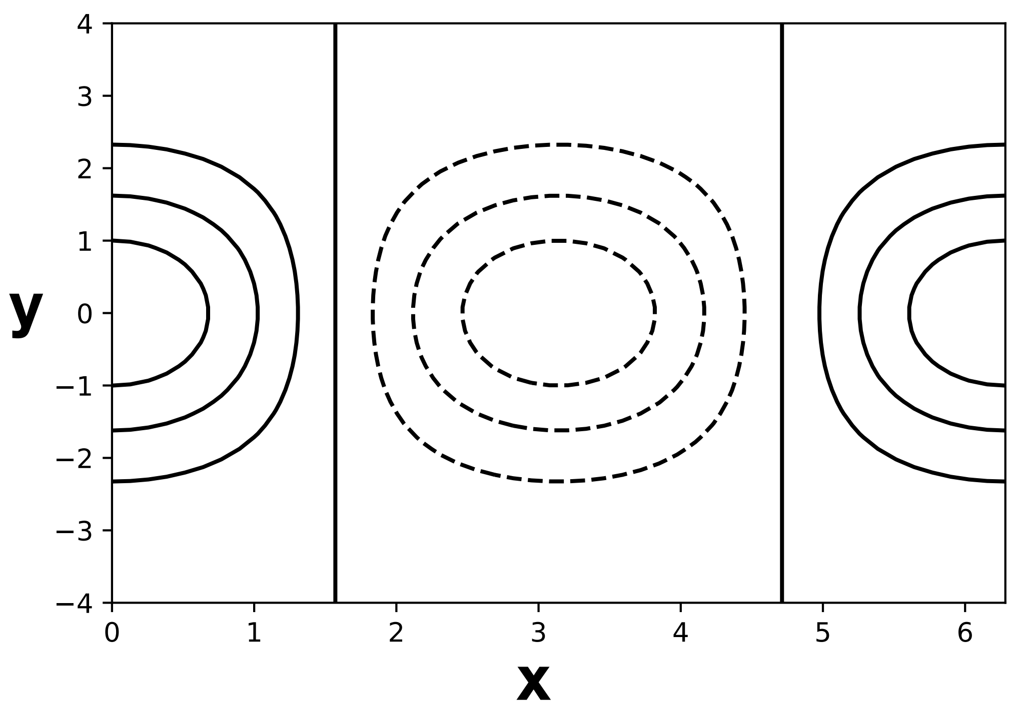
Localised sources of Rossby waves produces momentum convergence
(Mc Ewan, Thomson and Plumb, 1980, image from W. Young)

Locally forced Rossby-Kelvin waves at the equator

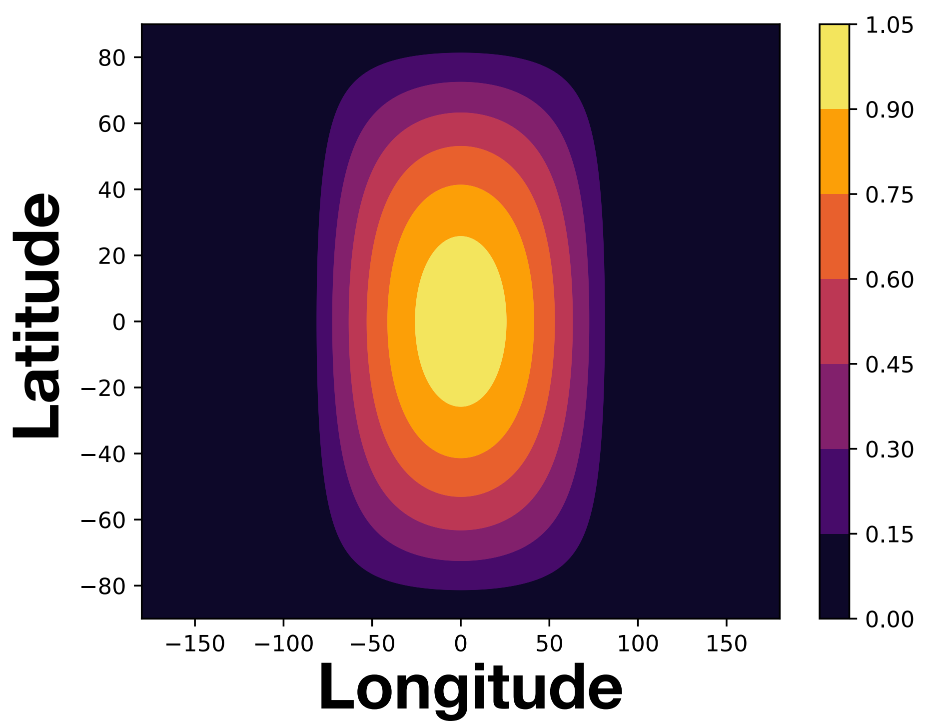
The Matsuno—Gill problem

$$\partial_t u - \beta y v + g\partial_x h = -\epsilon u,$$
$$\partial_t v + \beta y u + g\partial_y h = -\epsilon v,$$
$$\partial_t h + H\partial_x u + H\partial_y v = Q - h/\tau\,.$$

The forced and dissipated linearised Shallow Water equations on an equatorial β plane.



Forcing : $Q = Q_0 \cos(kx) e^{-y^2/2}$.



Top of atmosphere insolation

The Matsuno—Gill response (height and velocity)

$$\partial_t u - \beta y v + g\partial_x h = -\epsilon u,$$
$$\partial_t v + \beta y u + g\partial_y h = -\epsilon v,$$
$$\partial_t h + H\partial_x u + H\partial_y v = Q - h/\tau\,.$$
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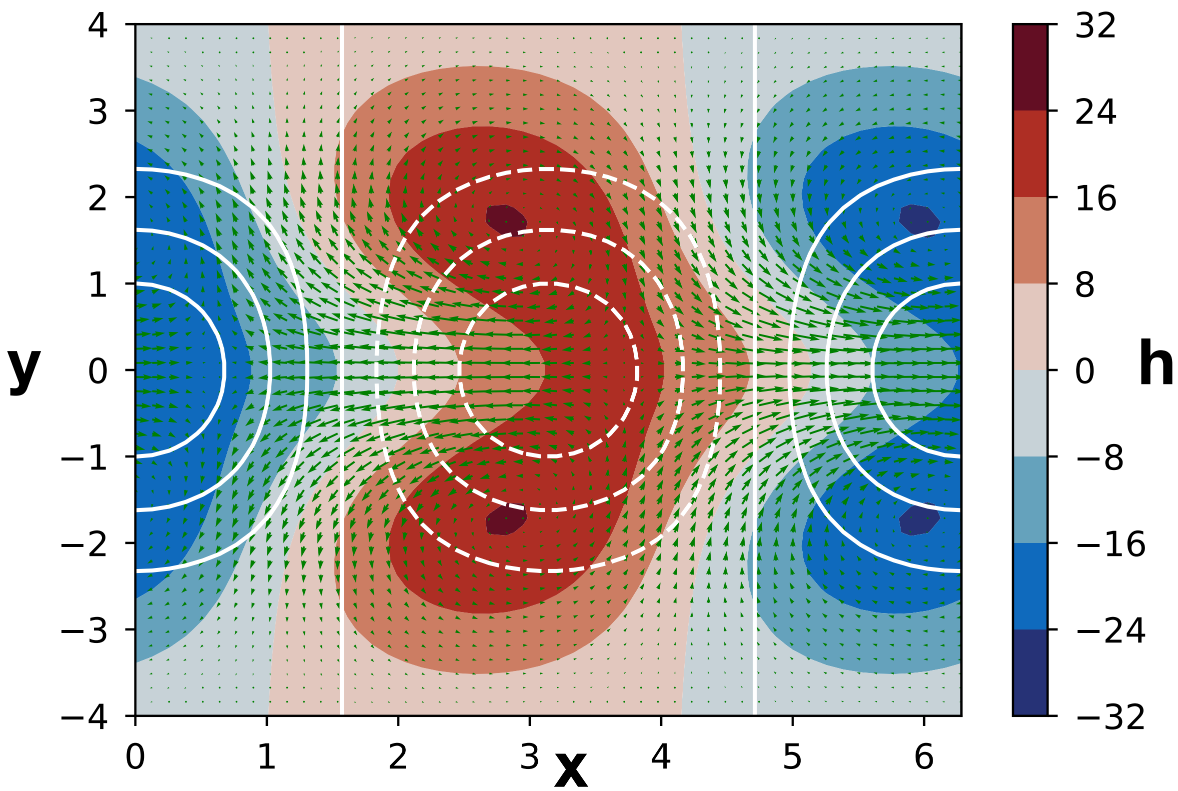
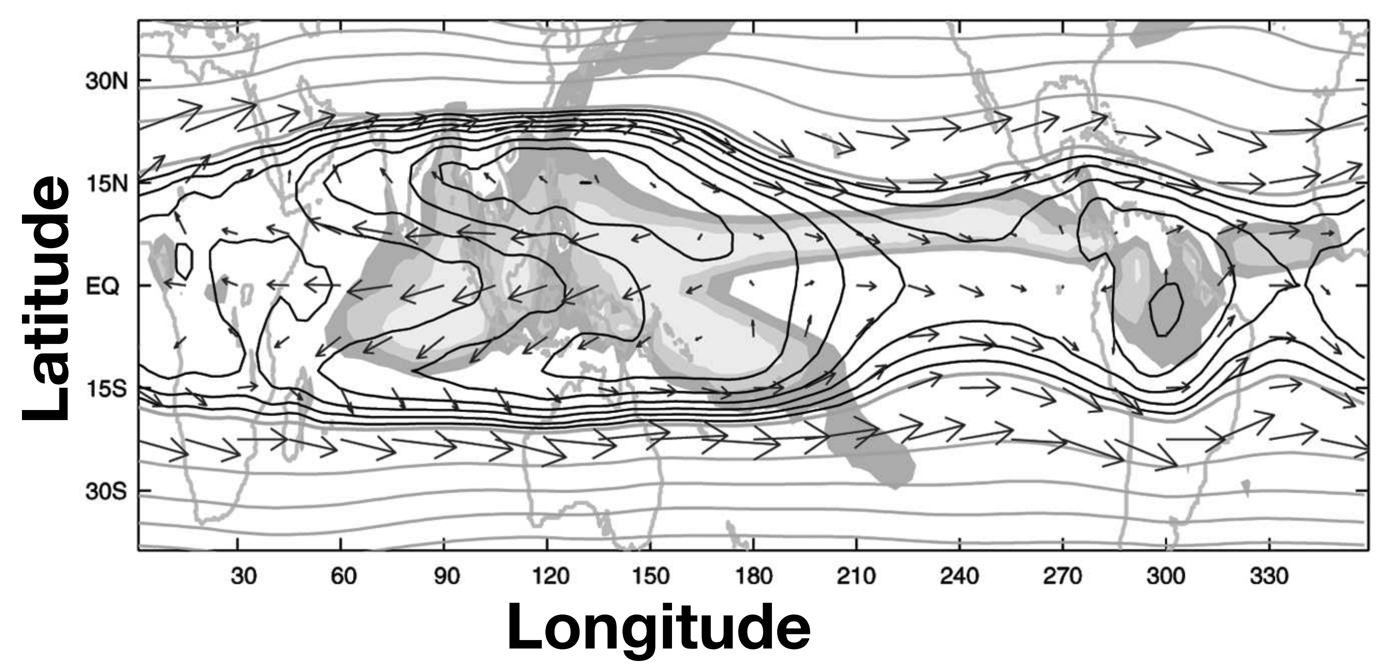


Matsuno—Gill response: height h (color map) and velocity (green arrows)
T. Matsuno (1966) A. E. Gill (1980)

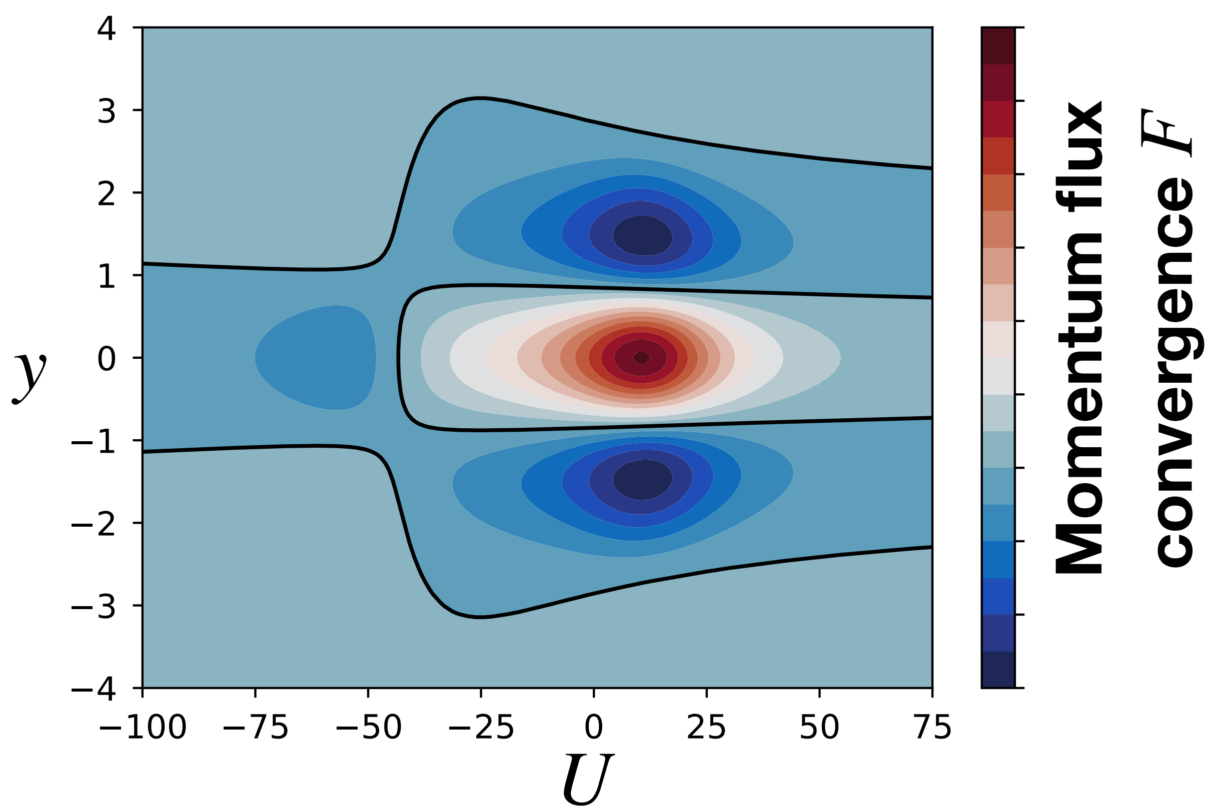


Reanalysis (NCEP) 150-hPa annual mean geopotential height (contours)
Dima, Wallace and Kraucunas (1966)

The Matsuno—Gill response (momentum flux convergence)

$$\partial_t u + U\partial_x u - \beta y v + g\partial_x h = -\epsilon u,$$
$$\partial_t v + U\partial_x v + \beta y u + g\partial_y h = -\epsilon v,$$
$$\partial_t h + U\partial_x h + H\partial_x u + H\partial_y v = Q - h/\tau\,.$$
$$\text{Forcing}: \quad Q = Q_0 \cos(kx) e^{-y^2/2}\,.$$

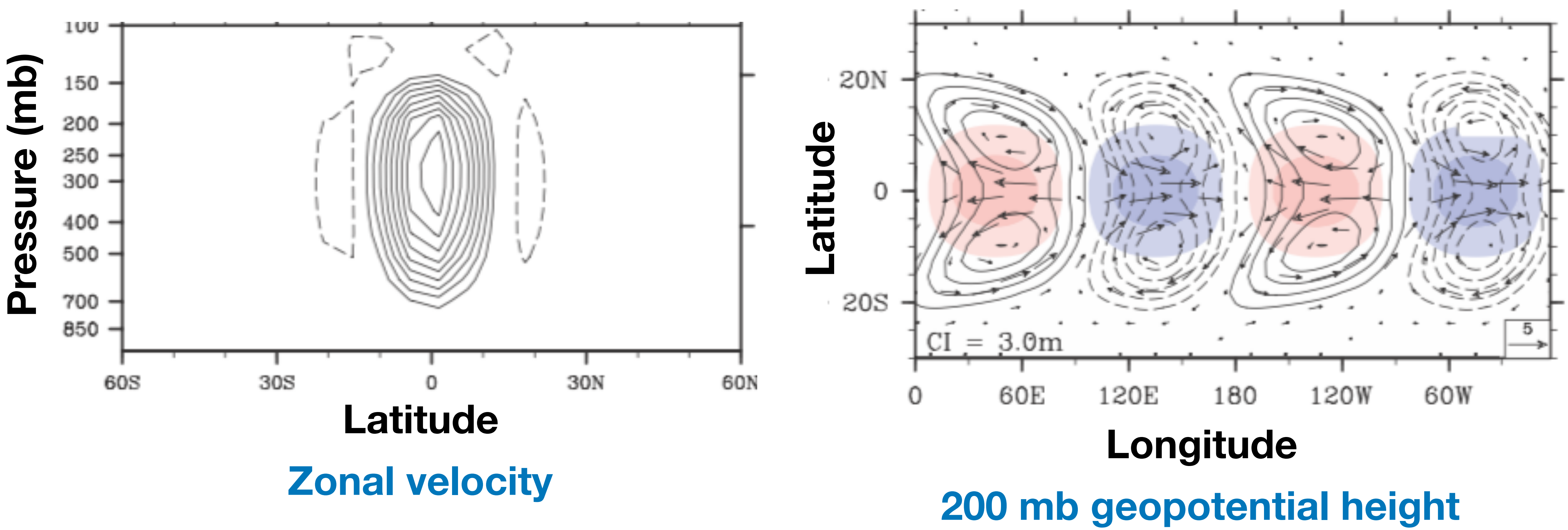
The forced and dissipated linearized Shallow Water equations on an equatorial β plane, with a mean zonal velocity $U\mathbf{e}_x$.



Matsuno—Gill momentum flux convergence versus mean zonal flow U
(Herbert, Caballero and Bouchet, 2019)

Rossby—Kelvin waves induces super-rotation

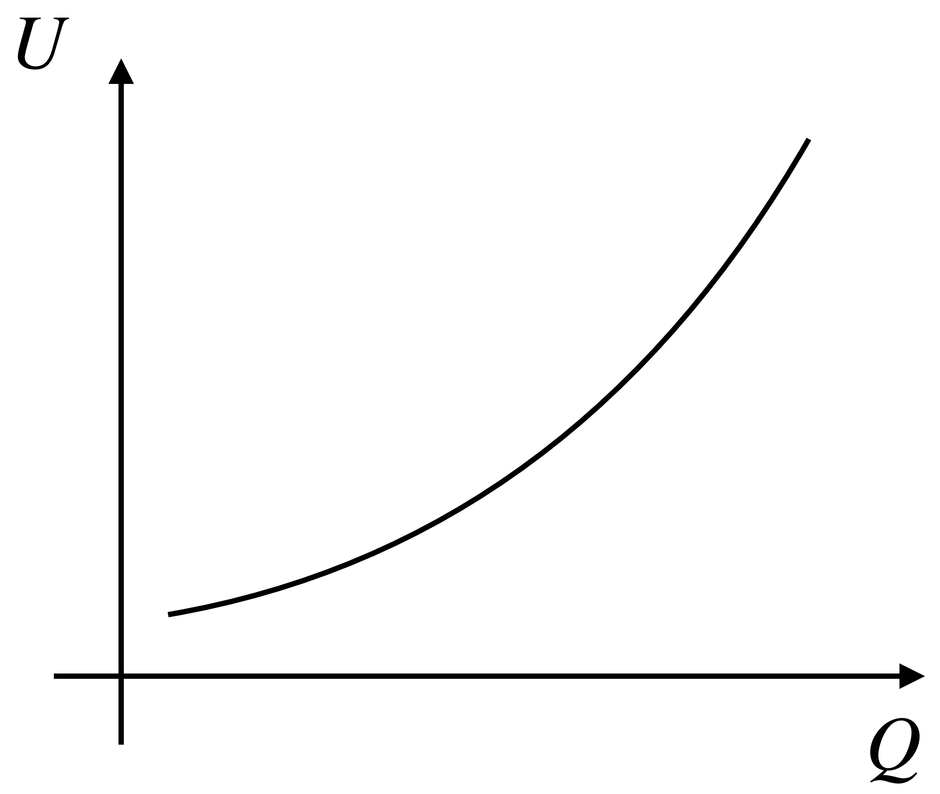
With CAM model



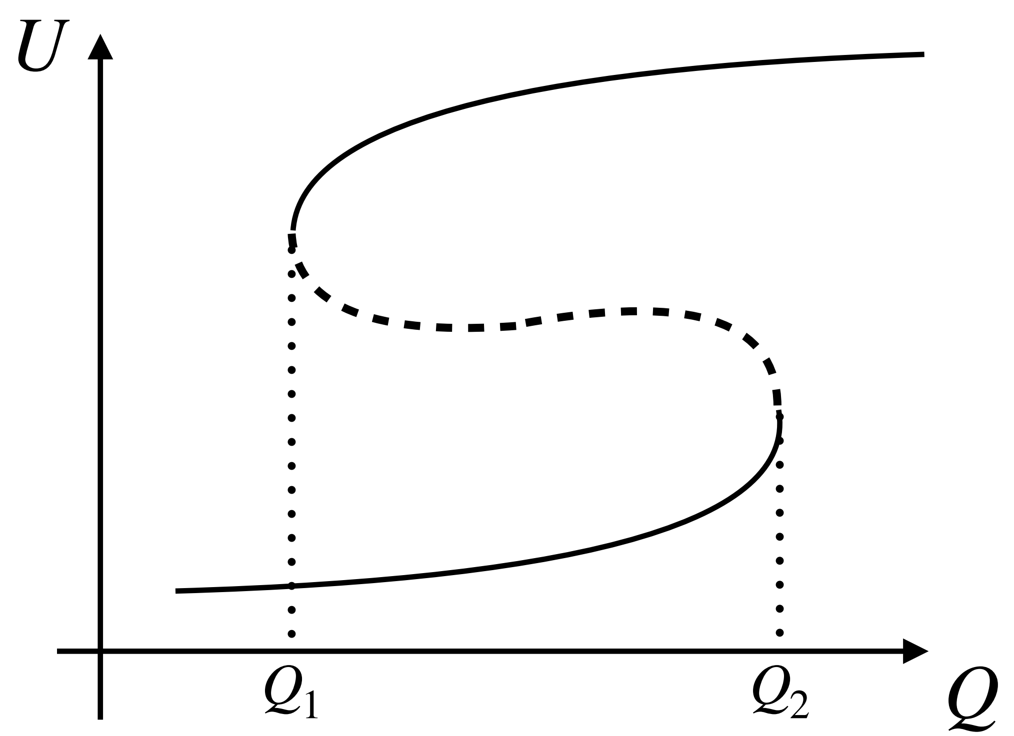
Arnold, Tziperman and Farrell (2011)

III) Are abrupt transitions to super-rotation possible?
Which physical parameters control the possible discontinuity of the transition?

First order (discontinuous) bifurcations and abrupt transitions



No bifurcation

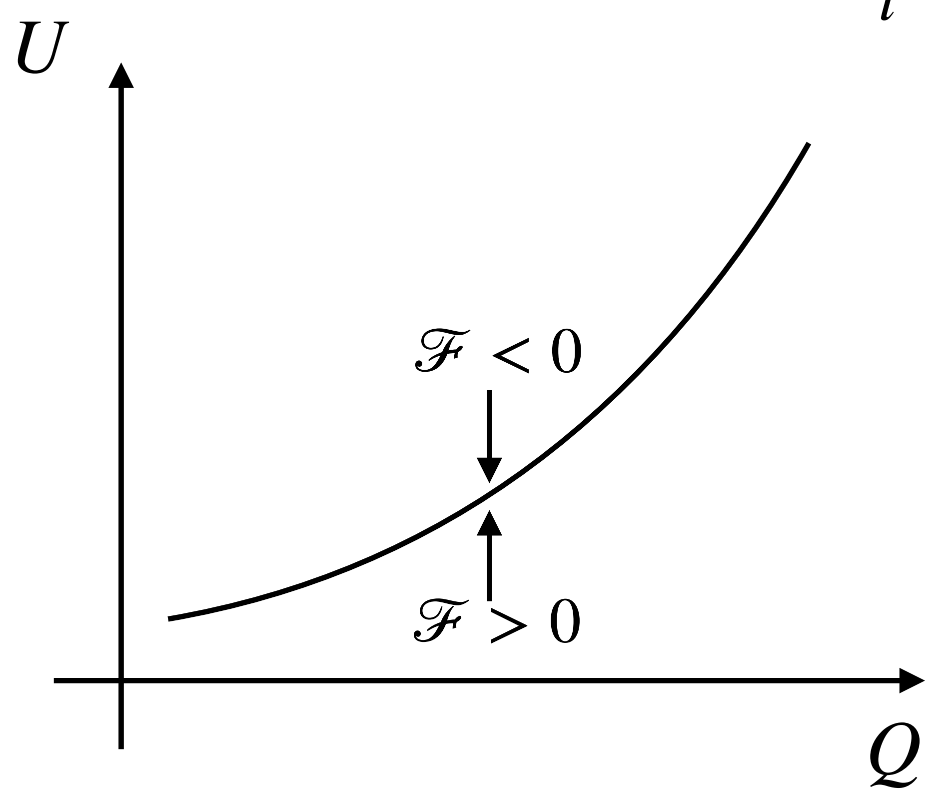


First order bifurcation

Abrupt transitions, first order transitions and bistability situations are related.

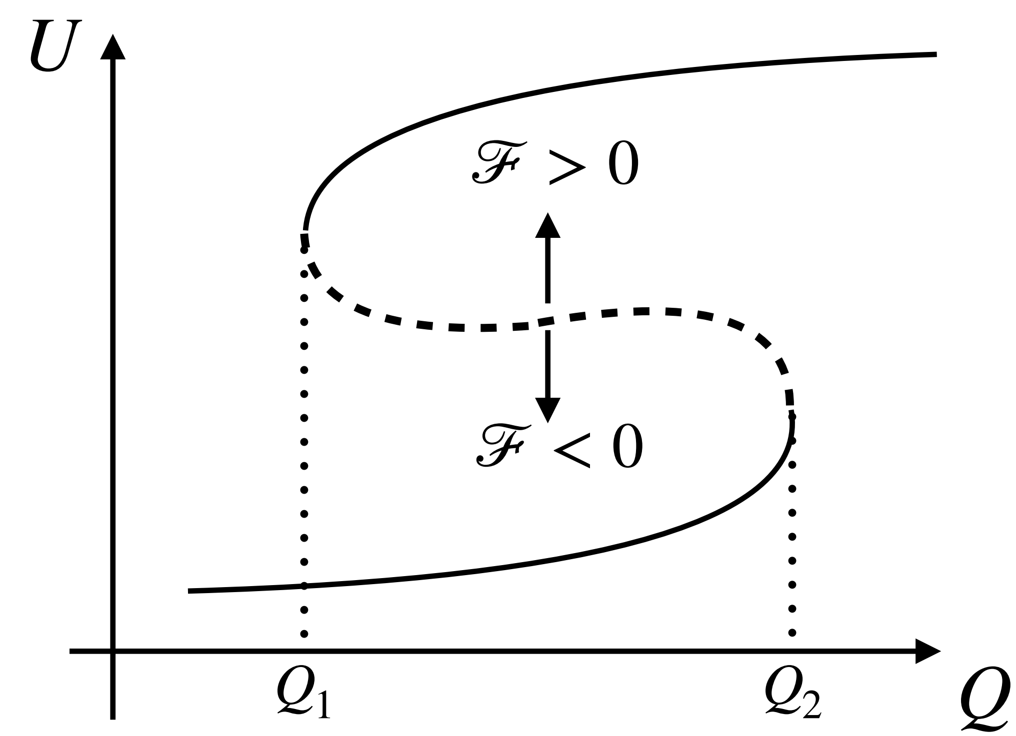
First order (discontinuous) bifurcations and positive feedbacks

$$\partial_t U = \mathscr{F}(U, Q)$$



No bifurcation

$\frac{\partial \mathscr{F}}{\partial U} < 0$: Negative feedback



First order bifurcation

$\frac{\partial \mathscr{F}}{\partial U} > 0$: Positive feedback

For abrupt transition and bistability to occur, there must exist a positive feedback.

Where could the positive feedback come from for super-rotation?

Two possibles positive feedbacks

1) Hadley cell

2) Momentum flux convergence

$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F \equiv \mathscr{F}[\bar{u}],$$

$$F = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{u'v'\cos\phi} - \frac{\partial}{\partial p}\overline{u'\omega'}.$$

Averaged zonal velocity equation

1. **Possible positive feedback through the Hadley cell (K. M. Shell and I. M. Held, 2004).**
2. **Possible positive feedback through momentum flux convergence induced by Rossby waves/jet resonance (Arnold, Tziperman and Farrell (2011).**

In which range of parameters are those positive feedback robust? Are they compatible? Are they robust to model complexity (will we see them in GCM and actual planets)?

Shallow Water model of the Hadley cell feedback

1D axisymmetric 1-1/2 layer shallow-water equations:

$$\frac{\partial u}{\partial t} + \frac{v}{a}\frac{\partial u}{\partial \phi} - \frac{uv\tan\phi}{a} = 2\Omega v \sin\phi + F + R - \epsilon u, \quad R = -\bar{Q}u/h\Theta(\bar{Q}),$$

$$\frac{\partial v}{\partial t} + \frac{v}{a}\frac{\partial v}{\partial \phi} + \frac{u^2\tan\phi}{a} = -2\Omega u \sin\phi - \frac{g^*}{a}\frac{\partial h}{\partial \phi} - \epsilon v,$$

$$\frac{\partial h}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial(hv\cos\phi)}{\partial \phi} = -\frac{h - h_{eq}}{\tau} \equiv \bar{Q}.$$

Zonal acceleration budget at equator:

$$F - \epsilon u + \frac{u}{h}\frac{h - h_{eq}}{\tau} = 0.$$

Layer thickness:

- Geostrophic balance with angular momentum conserving wind in the tropics
- Radiative equilibrium outside

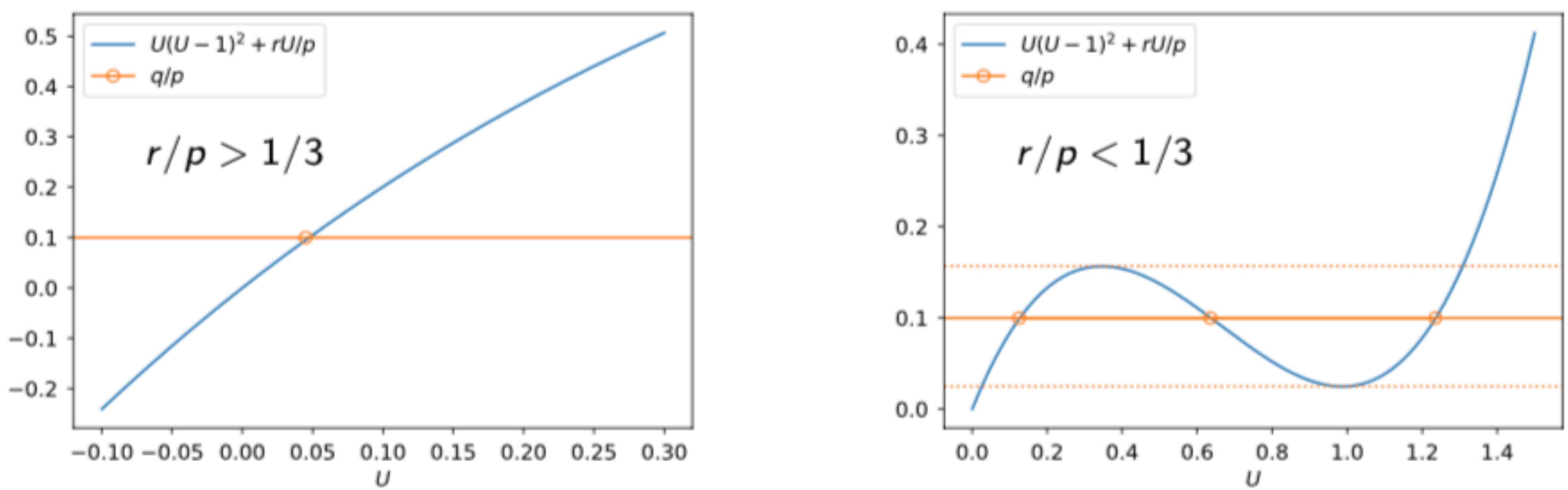
$$h - h_{eq} = -\frac{5}{18g^*}(u_{eq} - u)^2.$$

(K. M. Shell and I. M. Held, 2004)

Shallow Water model of the Hadley cell feedback

Zonal acceleration budget at equator:

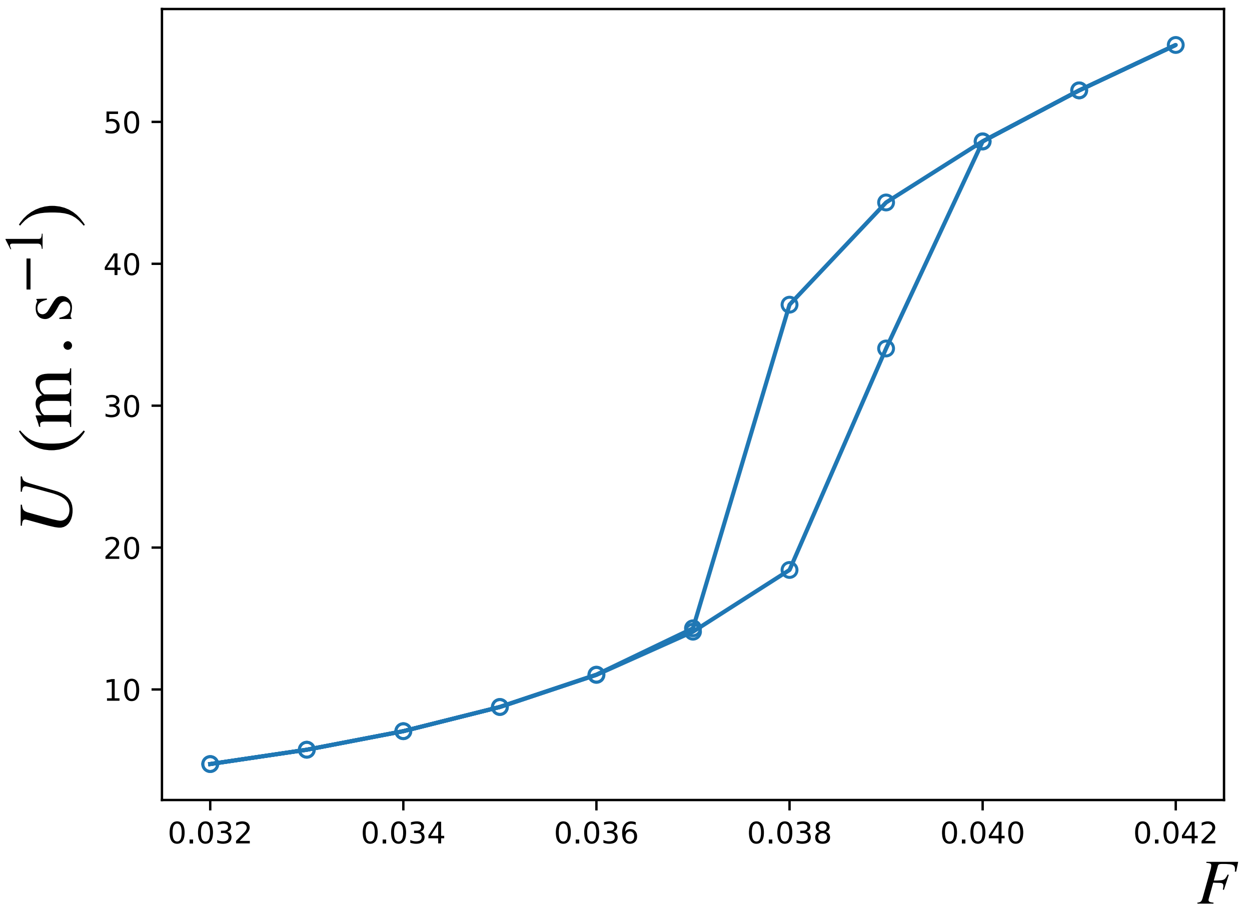
$$pU(U-1)^2 + rU - q = 0, \quad U = u/u_{eq}, \quad p = \frac{5u_{eq}^2}{18g^\star h_{eq}}, \quad q = \frac{F\tau}{u_{eq}}, \quad r = \epsilon\tau.$$



(K. M. Shell and I. M. Held, 2004)

Can we obtain abrupt transitions through the Hadley cell positive feedback in a more complex model ?

Discontinuous transitions through the Hadley cell feedback in an axisymmetric primitive equation model



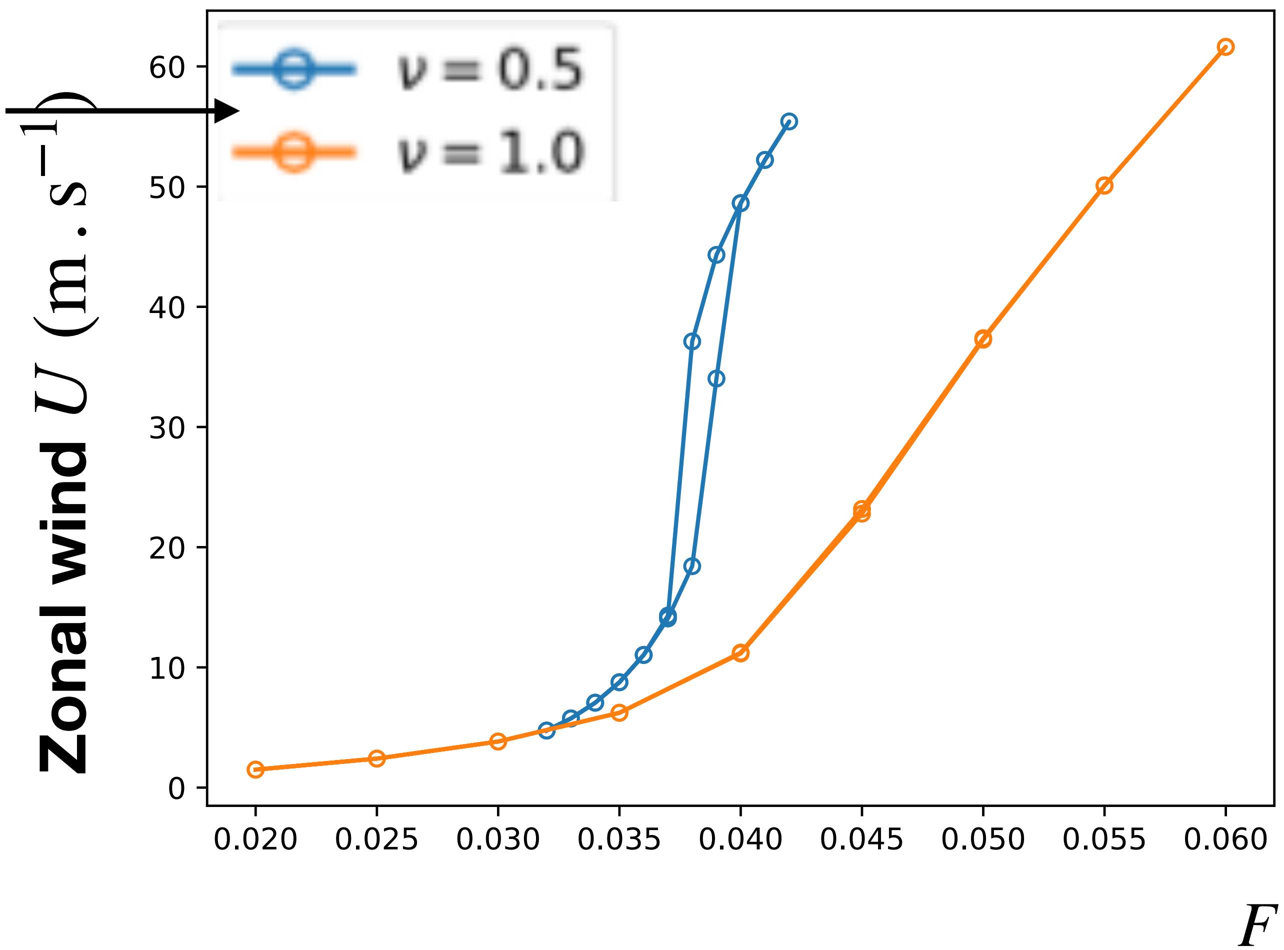
Hysteresis experiment for varying prescribed uniform momentum flux convergence in an axisymmetric model.

- **We obtained bistability in an axisymmetric primitive equation model, with prescribed constant eddy forcing.**
- **We observe a balance between the Hadley and the bulk dissipation through a constant eddy diffusivity.**

(Herbert, Caballero and Bouchet, 2019)

The Hadley cell feedback bistability mechanism seems not robust

Value of the bulk eddy diffusivity



- **In axisymmetric models, the bistability range is often extremely narrow, and very sensible to the bulk dissipation mechanism.**

- **Can we find a more robust positive feedback and bistability mechanism?**

(Herbert, Caballero and Bouchet, 2019)

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In which range of parameters are those positive feedback robust? Are they compatible? Are they robust to model complexity (will we see them in GCM and actual planets)?

The positive feedback through the Rossby wave/jet resonance

$$\partial_t u + U\partial_x u - \beta y v + g\partial_x h = -\epsilon u,$$
$$\partial_t v + U\partial_x v + \beta y u + g\partial_y h = -\epsilon v,$$
$$\partial_t h + U\partial_x h + H\partial_x u + H\partial_y v = Q - h/\tau\,.$$
$$\text{Forcing}:\quad Q = Q_0 \cos(kx) e^{-y^2/2}\,.$$

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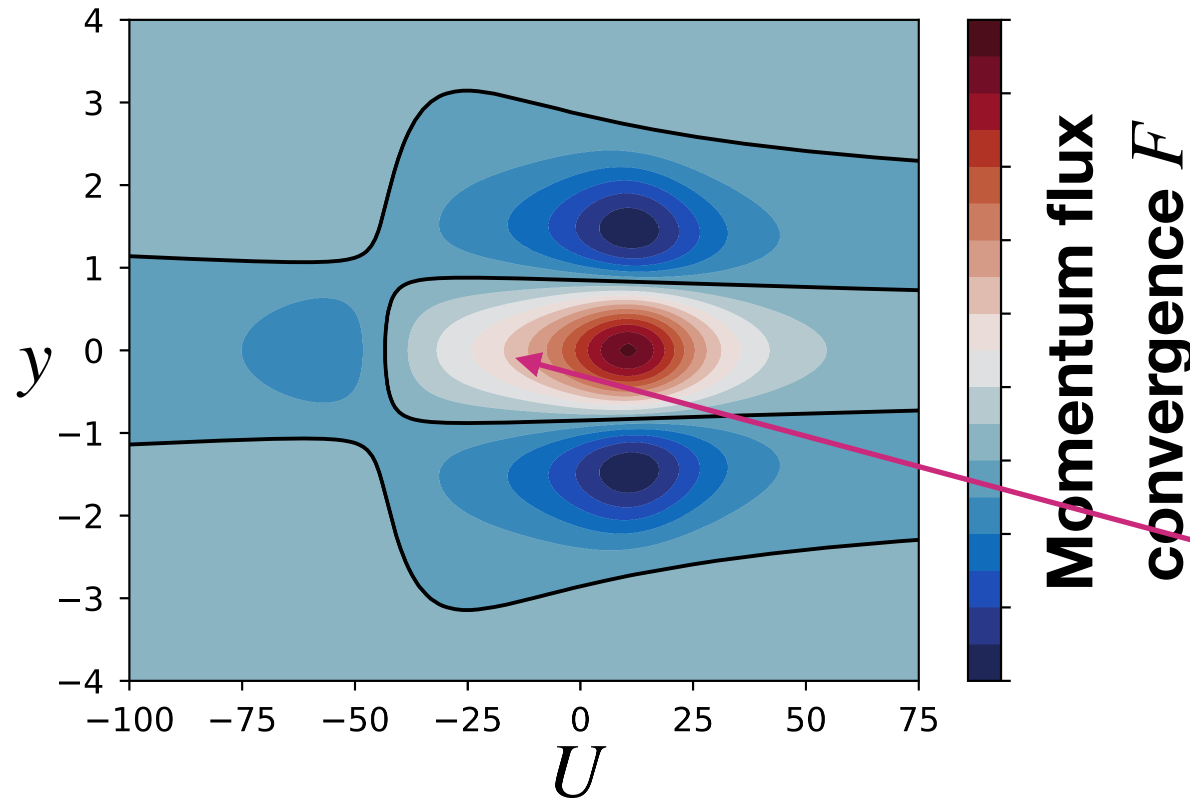


Momentum flux convergence at the equator

$$F = \frac{Q_0^2 \epsilon k^2 (c_K - c_R)(2U + c_K - 3c_R)}{12[\epsilon^2 + k^2(U + c_R)^2][\epsilon^2 + k^2(U + c_K)^2]}$$

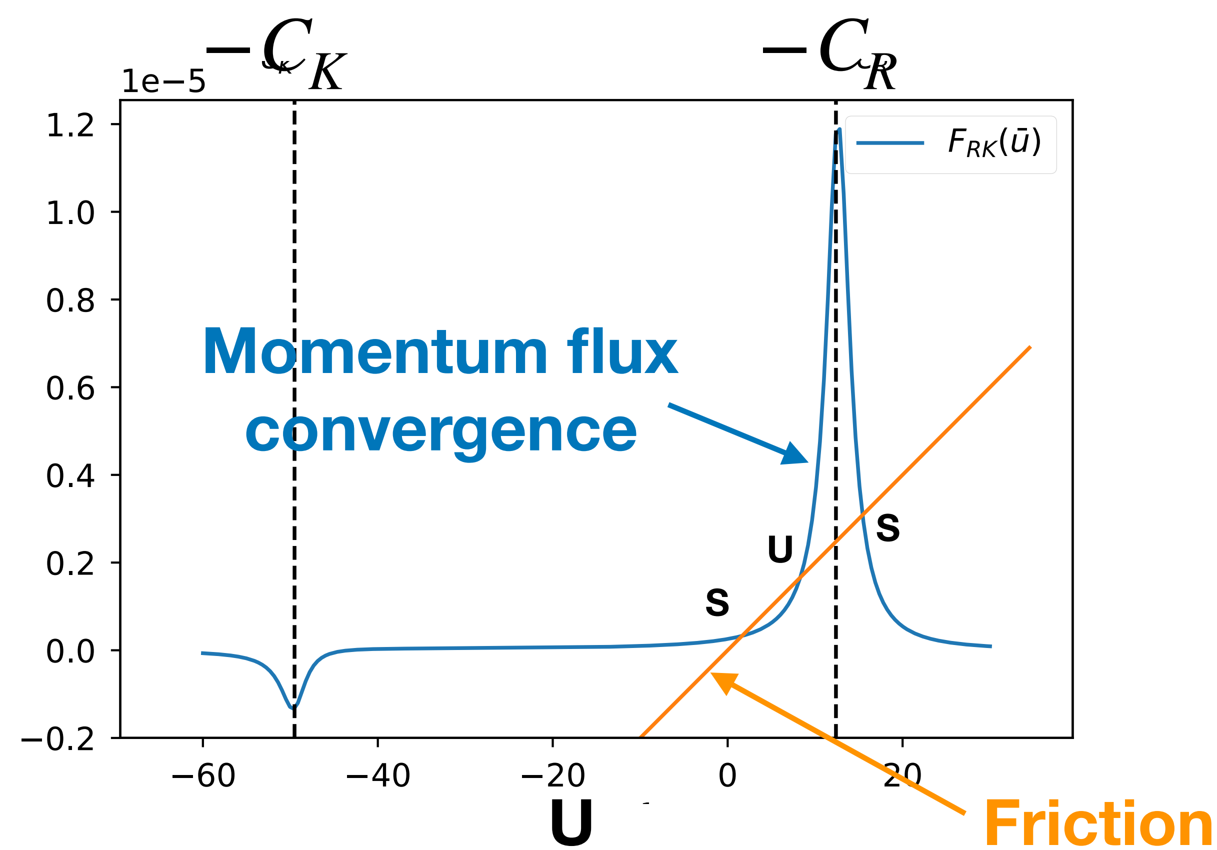
Positive feedback range: $\dfrac{\partial F}{\partial U} > 0$

Matsuno—Gill momentum flux convergence versus mean zonal flow U
(Herbert, Caballero and Bouchet, 2019)

Which parameter controls the positive feedback range through the Rossby wave/jet resonance

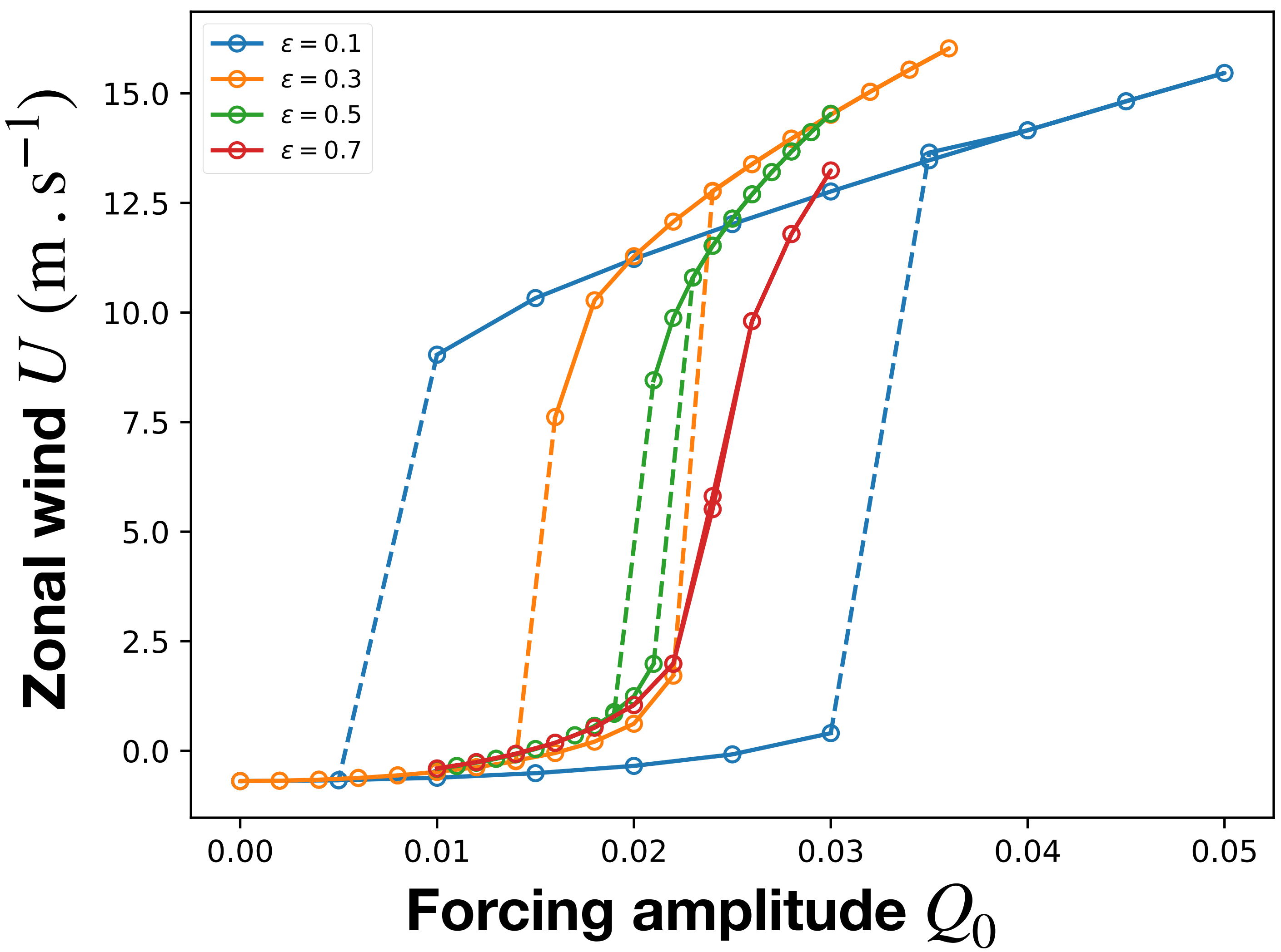
Momentum flux convergence at the equator

$$F = \frac{Q_0^2 \epsilon k^2 (c_K - c_R)(2U + c_K - 3c_R)}{12[\epsilon^2 + k^2(U + c_R)^2][\epsilon^2 + k^2(U + c_K)^2]}$$



- **The bistability range and abrupt transitions exist for narrow resonances: $kC_R > \alpha\epsilon$, where α is a non-dimensional number of order unity.**
- **The width of the bistability range is of order $\Delta U \simeq -C_R$.**

Bistability range for the axisymmetric model compared to the theoretical results

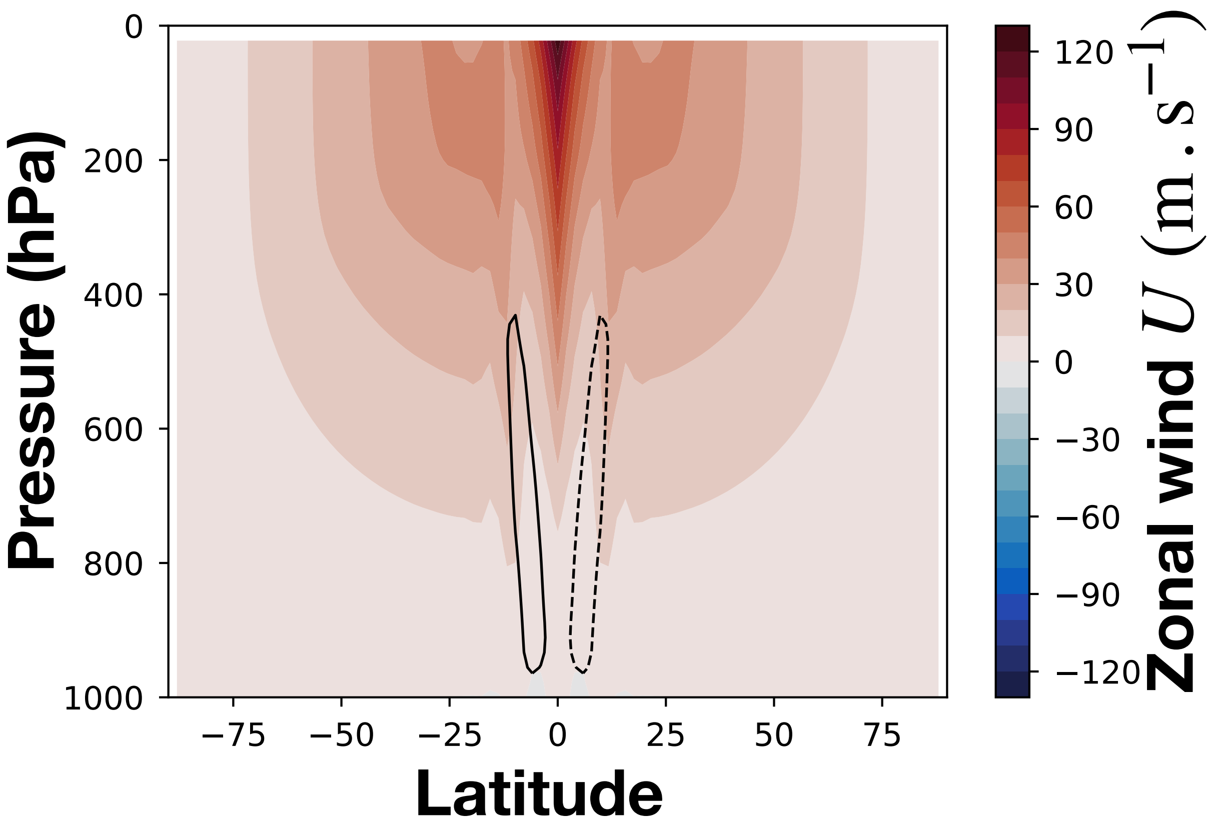


Hysteresis experiment, varying the amplitude of the Matsuno-Gill forcing, for different values of the resonance width ϵ.

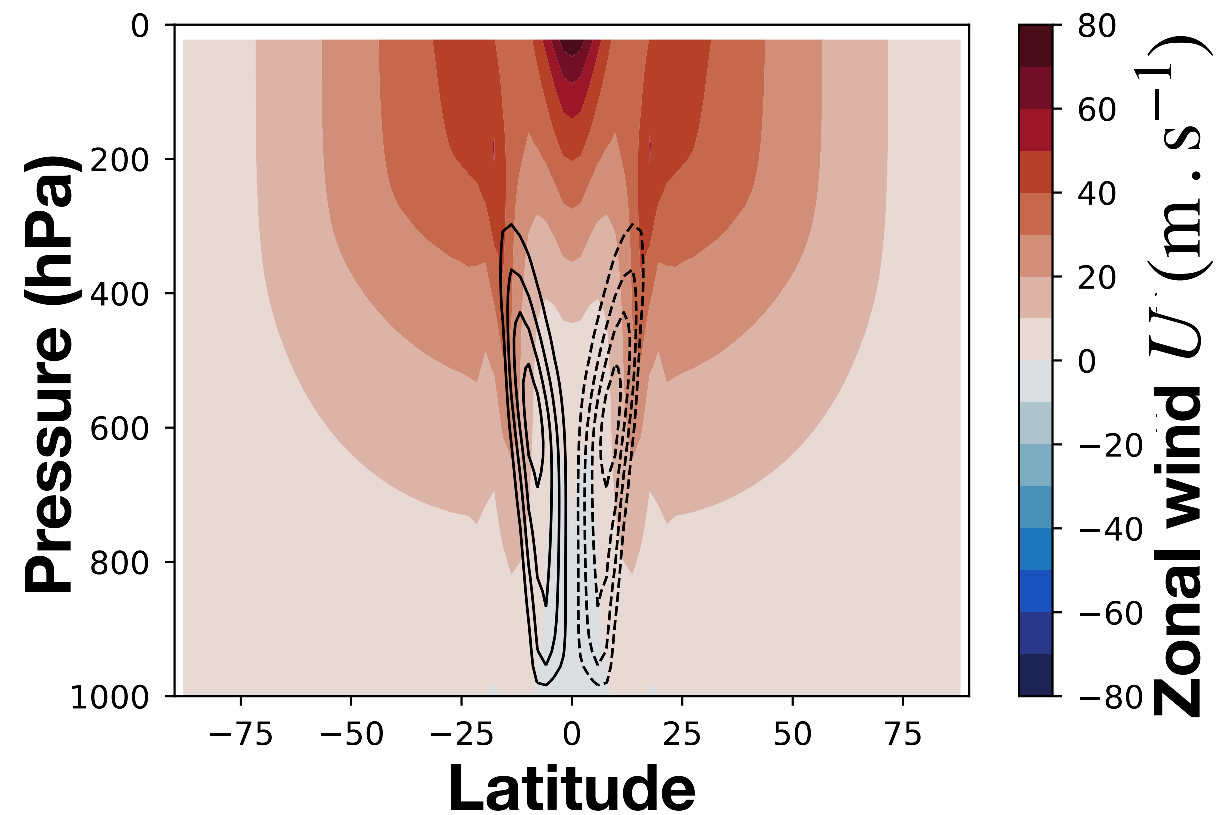
The bistability range is actually found for small ϵ/kC_R, has a width in terms of amplitude the decrease with ϵ/kC_R, and give a discontinuity of order $\Delta U \simeq -C_R$, as expected.

The normal and the super-rotating states

Normal state

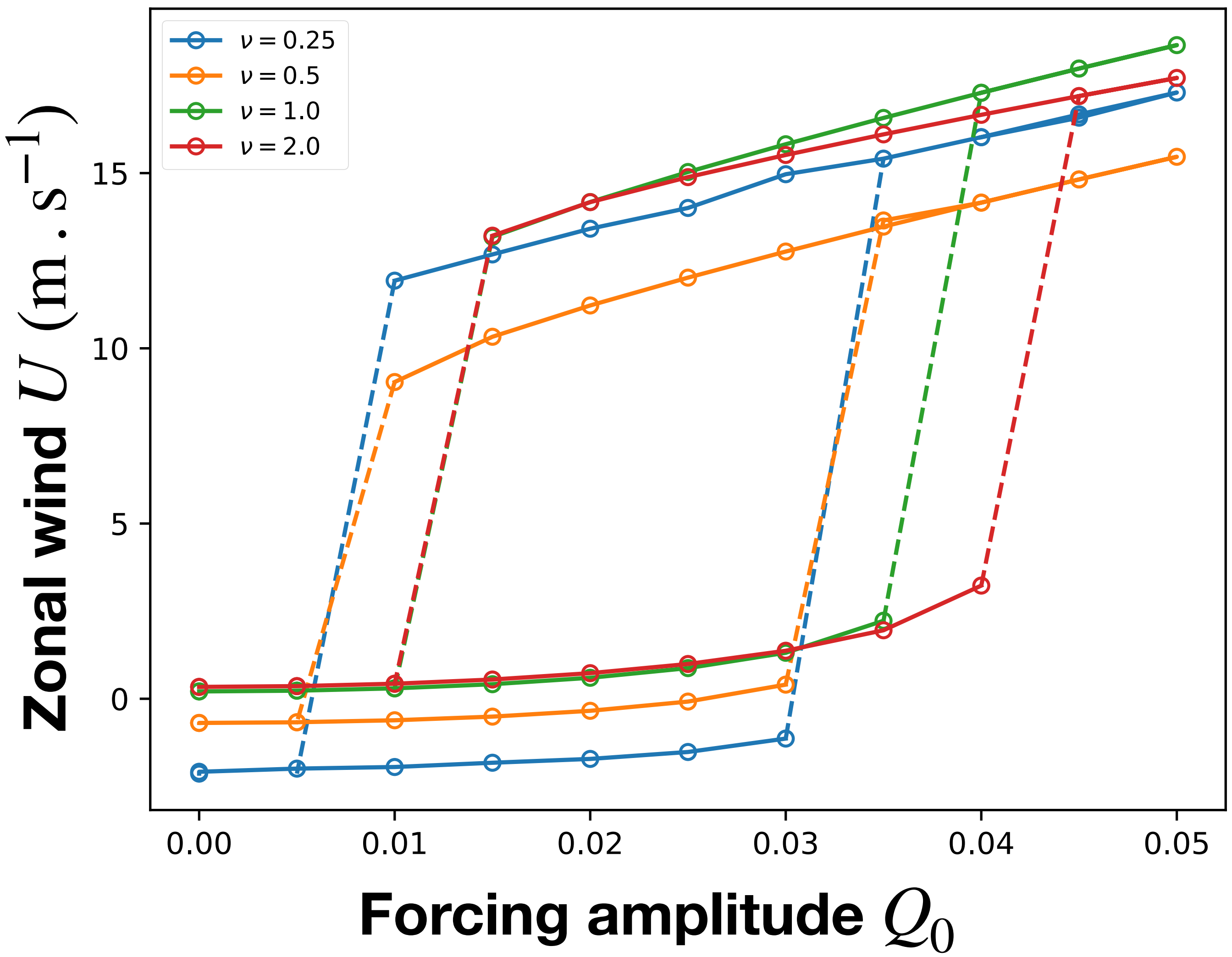


Super-rotating state



Zonal wind and transport contours for the axisymmetric primitive equation model .

Bistability range for the axisymmetric model compared to the theoretical results



Hysteresis experiment, varying the amplitude of the Matsuno-Gill forcing, for different values of the resonance bulk eddy diffusivity ν.

For resonant jet-waves positive feedbacks, the abrupt transition phenomena and the bistability range is robust to changes of the dissipation.

Conclusions

- We have theoretically characterized the range of parameters for bistability and abrupt transitions to the super-rotating state of the atmosphere.
- Those results have been verified in an axisymmetric primitive equation model. We are working on verifying them on a full GCM.
- The Hadley-cell positive feedback by himself seems not robust and highly dependent on the dissipation.
- Combined with the positive feedback of the Rossby wave/jet resonance, the bistability range is broad and robust. It is mainly characterized by the non dimensional parameter ϵ/kC_R that measures the dissipation of the Rossby —Kelvin waves.

(Herbert, Caballero and Bouchet, JAS 2019 and ArXiv:1905.12401)